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APPLICATION OF THREE NON-LINEAR ECONOMETRIC APPROACHES TO IDENTIFY BUSINESS CYCLES IN PERU

Gabriel Rodríguez

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Av. Universitaria 1801, Lima 32 – Perú.
Teléfono: (51-1) 626-2000 anexos 4950 - 4951
Fax: (51-1) 626-2874
econo@pucp.edu.pe
www.pucp.edu.pe/departamento/economia/

Encargada de la Serie: Giovanna Aguilar Andía
Departamento de Economía – Pontificia Universidad Católica del Perú,
gaguila@pucp.edu.pe

Rodríguez, Gabriel

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Application of Three Non-Linear Econometric Approaches to Identify Business Cycles in Peru

Gabriel Rodríguez

Pontificia Universidad Católica del Perú

Abstract

I use three non-linear econometric models to identify and analyze business cycles in the Peruvian economy for the period 1980:1-2008:4. The models are the Smooth Transition Autoregressive (STAR) model suggested by Teräsvirta (1994), the extended version of the Markov-Switching model proposed by Hamilton (1989), and the plucking model of Friedman (1964, 1993). The results indicate strong rejection of the null hypothesis of linearity. The majority of models identify quarters concentrated around 1988-1989 and 1990-1991 as recession times. Other important events which happened in the Peruvian economy (natural disaster in 1983, effects of the Asian and Russian crises in 1990s, terrorist activities in 1980s) are not selected except as atypical observations. Most of models also identify the period 1995:1-2008:4 as a very long and stable period of moderate-high growth rates. From the perspective of the Peruvian economic history and from a statistical point of view, the MSIAH(3) model is the preferred model.

Keywords: Nonlinearities, Asymmetries, STAR Model, Markov-Switching Model, Plucking Model, Recession Times

JEL Classification: C22, C52, E32.

Resumen

En este documento se usan tres modelos no lineales econométricos para identificar y analizar ciclos económicos en la economía Peruana para el período 1980:1-2008:4. Los modelos son el modelo autoregresivo de transición suave (STAR) propuesto por Terasvirta (1994), la versión extendida del modelo Markov Switching sugerido por Hamilton (1989), y el modelo Plucking de Friedman (1963, 1993). Los resultados indican fuerte rechazo de la hipótesis nula de linealidad. La mayoría de los modelos identifican trimestres concentrados alrededor de 1988-1989 y 1990-1991 como recesiones. Otros eventos importantes acontecidos en la economía Peruana (desastres naturales en 1983, efectos de las crisis Asiática y Rusa en 1990, actividades terroristas en los años 1980) no son seleccionadas excepto como observaciones atípicas. La mayoría de los modelos también identifican el período 1995:1-2008:4 como un periodo largo y estable de tasas de crecimiento moderadas y altas. Desde la perspectiva de la historia económica Peruana y desde un punto de vista estadístico, el modelo MSIAH(3) es el modelo seleccionado.

Palabras Claves: No Linealidades, Asimetrías, Modelo STAR, Modelo Markov Switching, Modelo Plucking, Recesiones

Classificación JEL: C22, C52, E32.

Application of Three Non-Linear Econometric Approaches to Identify Business Cycles in Peru[1]

Gabriel Rodríguez[2]

Pontificia Universidad Católica del Perú

1 Introduction

Econometric theory suggests that a number of important time-series variables should exhibit non-linear behavior. Two important features of the business cycles literature are the existence of nonlinearities and asymmetries. For example, Mitchell (1927) and Keynes (1936) noted that business contractions are briefer than business expansions, and they are also more sudden and violent. This fact was also found by Neftci (1984) when he analyzed the behavior of US unemployment rates. Therefore, business fluctuations are asymmetric and nonlinear.

There is a large number of nonlinear models; see, for example, Granger and Teräsvirta (1993) for a survey. A type of model to identify business cycles is an alternative model that assumes that the transition between regimes is caused by an observable variable which belong to the set of independent variables or some other exogenous variable. This model is the Smooth Transition Autoregressive (STAR) model proposed by Teräsvirta (1994). In this model, the transition variable determines the threshold level and speed (smoothness) parameters driving the transition between both regimes. The version where the named transition function assumes a logistic form appears to be adequate to analyze business cycles. One interesting empirical application is Teräsvirta and Anderson (1992).

Other type of models that assume that the transition between regimes is caused by exogenous but not observable variables (or unknown events) is the case of the Markov-Switching model, originally proposed by Hamilton

[1] I thank useful comments of the Editor and two anonymous referees on an earlier version of the paper. I am also very grateful to Francisco Nadal de Simone for useful e-mail and phone conversations and for important advise related to the plucking model. I also thank comments from participants to the XXIV Meeting of Economists of the Central Reserve Bank of Peru (December 2007). A very preliminary version of this paper appears as Working Paper 2007-007 of the Central Bank of Peru.

[2] Address for Correspondence: Gabriel Rodríguez, Department of Economics, Pontificia Universidad Católica del Perú, Av. Universitaria 1801, Lima 32, Lima, Perú, Telephone: +511-626-2000 (4998), Fax: +511-626-2874. E-Mail Address: gabriel.rodriguez@pucp.edu.pe.

(1989). In his original paper, Hamilton used an AR(4) specification in order to model the growth rate of the US output allowing for only a change in the mean. The results obtained allow to identify phases of recessions in very close concordance with the dates identified by the NBER which uses a large set of leading indicators. After it, the model has been largely used and extended to include regimen dependence both in variances and autoregressive parameters. Furthermore, the model has also been extended to the multivariate framework by Krolzig (1997). Some interesting applications are Goodwin (1993) and Bodman and Crosby (2000).

Friedman (1964, 1993) also noted asymmetry in real output behavior in that amplitude of real output contractions is strongly correlated with the succeeding expansion, but the amplitude of an expansion is not correlated with the amplitude of the succeeding contraction. This observation is the source of the very known "plucking" model of business cycles. Further evidence was found by Delong and Summers (1986), Falk (1986), and Sichel (1993).

Empirical evidence characterized by sudden jumps and slower declines of the real output gives support to the asymmetries suggested by some nonlinear models although in particular in favor of the plucking model. As Kim and Nelson (1999) argue, while these kind of asymmetries are consistent with the plucking model, they are also consistent with models where recessions are occasioned by infrequent permanent negative shocks as in the Markov-Switching models. What distinguishes the plucking model is the prediction that negative shocks are largely transitory, while positive shocks are largely permanent. Another important characteristic of the plucking model is the existence of an upper limit to the output, the so named potential output, which is set by the resources available in the economy.

Kim and Nelson (1999) have proposed an econometric specification of the plucking model. The approach offers more possibilities than standard linear models such as ARIMA models and the unobserved components models which can not account for asymmetries; see Watson (1986), Clark (1987).

Mills and Wang (2002) have applied the procedure of Kim and Nelson (1999) to the real output time series of the G-7 countries. Interesting performance of this approach has also been noted by Galvão (2002), where this model is one of the three models capable of reproducing the length of the U.S. business cycles. In this respect, see the special issue about business cycles published by *Empirical Economics* in 2002. Some further applications of the plucking model are Rodríguez (2005) and Nadal De Simone and Clarke (2007).

The number of empirical studies of business cycles applied to Latin

American countries is very scarce. One interesting exception is Mejía-Reyes (2004) where synchronization of business cycles in seven Latin American countries is analyzed using classical business cycles approach. This approach is also used by Castillo, Montoro and Tuesta (2006) in order to establish some stylized facts of the Peruvian economy. In this paper, I apply the above three described alternative non-linear models to the annual growth rates of the real output of Peru for the period 1980:1-2008:4. I consider that no other similar research has been done for this country. Some related work but more interested in estimations restricted to the output gap are Rodríguez (2010a, 2010b).

I consider that Peruvian time series could offer an interesting environment to verify theoretical and practical performance of the different models. In all the sample period, the behavior of the real output has been different. In the first half of the sample, growth rates have been very volatile and economy has been very instable in both economic and political terms. It is around 1992 that government started a process of structural reform with progressive impact in the performance of the economy. Furthermore, Central Bank officially adopted an inflation targeting monetary regime in 2002. All these changes have had positive consequences on the stability of the economy at the internal and external levels.

Some particular events happened in the sample period are worth to be mentioned. The first event happened in 1983 in the north of Peru where a serious natural disaster affected the agricultural sector of the economy. The second event is the presence of the terrorist group named *Shining Path* which started its armed actions in 1980 affecting many regions of the country and different sectors of the economy. The third event was the high inflation episodes happened in 1988 caused by different incoherent fiscal and monetary policies. The different fiscal and monetary programs applied failed to stop high inflation ended in a dramatic hyperinflation in 1990:2. This event was stopped in 1990:3 with a new government which dropped dramatically the monetary base and relaxed chaotic behavior of public prices.

All our estimations suggest presence and relevance of nonlinearities and some atypical observations (outliers) in the real output. Evaluation of some linear models suggests presence of autocorrelation in the residuals, squared residuals, presence of autoregressive conditional heteroscedasticity and strong dependence of the residuals. Nonlinear estimation appears to be needed.

The majority of estimates of the non-linear models identify quarters concentrated in 1988 and 1990 as recession times. Some events as the natural disasters happened in 1983 are only captured as atypical observations but they do not qualify as recession times. Furthermore, the Asian and Russian financial crises are not identified as recession times. It means that these events probably affected the financial sector of the economy but not the real output of the economy. Another potential explanation is that they affected real sector of the economy but negative growth rates are smaller compared with the levels identified by the models. The alternative hypothesis that armed and destructive actions of the terrorist group *Shining Path* could cause recession periods is difficult to accept according to the estimates. According to them, the destruction caused by this terrorist group does not cause recession times in the sense of recurrent negative growth rates.

Another similarity between models is that since 1994-1995, the economy appears to enter in a period of relative sustainable growth. It is supported by the various structural reforms of the economy in the labor market, financial sector, external sector, and adoption of the targeting inflation system. All these measures contributed to the adequate behavior of the economy with less volatility in the evolution of the major macroeconomic variables. Therefore all nonlinear models indicate long duration of the times with moderate-high growth rates.

From the point of view of the statistical evaluation of the different non-linear models, the MSIAH(3) model shows superior performance. According to the notation of the Section 2.2, it is a Markov-Switching model with intercept, autoregressive parameters and variance dependent of the regimes. In this case, the selected number of regimes is three. This model selects the recession times and normal times which are more consistent with visual inspection and with the real events happened in the Peruvian economy. At the moment of the testing evaluation, the model performs better than the other selected Markov-Switching models and the other competitive models.

The rest of the paper is organized in the following manner. Section 2 briefly describes the three alternative non-linear methods to be used in the estimations. Section 3 presents and discusses the results. Section 4 concludes.

2 Methodologies

In this section, the three non-linear econometric approaches used in the empirical section are briefly described. In all cases, y_t denotes the logarithm of the real output at period t and $\Delta_4 y_t$ denotes the annual growth rates of real output $(y_t - y_{t-4})$. The first model is the Smooth Transition Autoregressive (STAR) model proposed by Teräsvirta (1994). The second model is an extended version of the Markov-Switching Autoregressive (MS-AR) model originally suggested by Hamilton (1989). Finally, last model is the plucking model, theoretically proposed by Friedman (1964, 1993) and empirically implemented by Kim and Nelson (1999).

2.1 The Smooth Transition Autoregressive (STAR) Model

The Smooth Transition Autoregressive (STAR) model was proposed by Teräsvirta (1994). Following the same notation as in van Dijk, Franses, and Teräsvirta (2002), the smooth transition autoregressive (STAR) model for a time series $\Delta_4 y_t$ is represented by

$$\Delta_4 y_t = \phi_1' x_t[1 - F(v_t;\gamma,c)] + \phi_2' x_t[F(v_t;\gamma,c)] + \epsilon_t, \tag{1}$$

where $x_t = (1, \tilde{x}_t')$ with $\tilde{x}_t = (\Delta_4 y_{t-1}, ..., \Delta_4 y_{t-p})'$, $\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})'$, $i = 1$. The term ϵ_t is a martingale difference sequence with respect to the set of information up to and including time $t-1$ (denoted by Ω_{t-1}), and $F(v_t;\gamma,c)$ is named the transition function. In addition, it is assumed that the conditional variance of ε_t is constant, $E[\varepsilon_t^2|\Omega_{t-1}] = \sigma^2$.

The transition variable, denoted by v_t is an observable variable and it can be a lagged endogenous variable, $v_t = \Delta_4 y_{t-d}$ for $d > 0$ or it can also be an exogenous variable $v_t = z_t$ or a function of lagged endogenous variables $v_t = g(\tilde{x}_t;\alpha)$ for some function g, which depends on the $q \times 1$ parameter vector α. Lastly, the transition variable can be a linear time trend $v_t = t$ giving a model with smoothly changing parameters as discussed in Lin and Teräsvirta (1994).

Different choices for the transition function $F(v_t;\gamma,c)$ give rise to different types of regime-switching behavior. Two popular choices are the logistic and the exponential functions given origin to the so called Logistic STAR (LSTAR) and Exponential STAR (ESTAR) models, respectively.

Although a sequential testing procedure exists to decide which function should be selected, empirical evidence suggests more adequacy of the logistic function when identification of recession and normal times are the principal purpose[3]. The first-order logistic function is defined by

$$F(v_t;\gamma,c) = \{1 + \exp[-\gamma(v_t - c)]\}^{-1}, \tag{2}$$

[3] Application of the relevant statistics in the empirical section confirms this argument.

with $\gamma > 0$. In the LSTAR model, the parameter c in (1) or (2) can be interpreted as the threshold between the two regimes, in the sense that the logistic function changes monotonically between 0 and 1 as v_t increases. The parameter γ determines the smoothness of the change in the value of the logistic function and thus, the smoothness of the transition from one regime to the other. As γ becomes very large, the logistic function $F(v_t; \gamma, c)$ approaches the indicator function $I[v_t > c]$, defined as $I[A] = 1$ if A is true and $I[A] = 0$ otherwise, and, consequently, the change of $F(v_t; \gamma, c)$ from 0 to 1 becomes almost instantaneous at $v_t = c$.

Note that the transition function (2) is a special case of the general nth-order logistic function, defined by

$$F(v_t; \gamma, c) = \{1 + \exp[-\gamma \prod_{i=1}^{n} (v_t - c_i)]\}^{-1}, \tag{3}$$

where $\gamma > 0, c_1 \leqslant c_2 \leqslant ... \leqslant c_n$. This function can be used to obtain multiple switches between the two regimes; see for example application of Jansen and Teräsvirta (1996).

In the LSTAR models, the two regimes are associated with small and large values of the transition variable v_t relative to the parameter c. This type of regime-switching can be convenient for modelling, for example, business cycle asymmetries in distinguishing expansions and recessions. The LSTAR models has been successfully applied by Teräsvirta and Anderson (1992) and Teräsvirta, Tjøstheim and Granger (1994) to characterize the different dynamics of industrial production indexes in a number of OECD countries.

2.2 The Markov-Switching Model

Let Δy_t denotes the growth rate of real output in quarter t. The model of Hamilton (1989) specifies that Δy_t follows an autoregressive process of order 4, that is, an AR(4). Nonlinearity of the model arises because the process is subject to discrete shifts in the mean, between high-growth and low-growth states. These discrete shifts have their own dynamics, specified as a two-state first-order Markov process. The models is written as

$$\Delta y_t - \mu_{s_t} = \phi_1(\Delta y_{t-1} - \mu_{s_{t-1}}) - ... - \phi_4(\Delta y_{t-4} - \mu_{s_{t-4}}) + \epsilon_t, \tag{4}$$

where $\epsilon_t \sim i.i.d.\ N(0, \sigma^2)$. The variable that determines the change of the regime is an unobservable variable denoted by s_t such that $\Pr[s_t = 1|s_{t-1} = 1] = p$, $\Pr[s_t = 0|s_{t-1} = 1] = 1-p$, $\Pr[s_t = 0|s_{t-1} = 0] = q$, $\Pr[s_t = 1|s_{t-1} =$

$0] = 1 - q$. Therefore, $\mu_{s_t} = \alpha_0 + \alpha_1 s_t$, and $s_t = 1$ if high growth state, 0 otherwise.

The model is a non-linear combination of discrete and continuous dynamics. An attractive feature of the model is that no prior information regarding the dates of the two growth periods or the size of the two growth rates is required. In particular, note that the low-growth rate need not be negative. Derivation of the sample conditional log-likelihood $\sum_{t=1}^{T} \ln f[y_t|y_{t-1}, y_{t-2}, ...]$ is detailed in Hamilton (1989).

In notation of Krolzig (1997), the model of Hamilton (1989) is denoted by MSM(2)-AR(4), that is, a Markov-Switching model with a fourth autoregressive structure with changing mean between two regimes. However, other specifications are available. For example, a model where the mean, the variance and the autoregressive coefficients are regime dependent is denoted by MSMAH(m)-AR(k) where m indicates the number of states and k reflects the order of the autoregression. In some cases instead of modeling the mean as regime dependent parameter, it is considered the intercept as regime dependent. In these cases the notation is MSIAH(m)-AR(k) model. The models where the mean and the intercept are regime dependent offer different dynamics of adjustment of the variables after a change in regime.

2.3 The Plucking Model

Following the literature of unobserved components (Watson, 1986), it is possible to decompose y_t into a trend component and a transitory component, which are denoted as τ_t and c_t, respectively. That is,

$$y_t = \tau_t + c_t. \tag{5}$$

Adopting a similar notation as in Kim and Nelson (1999), I assume that shocks to the transitory component are a mixture of two different types of shocks, which will be denoted π_{s_t} and u_t, respectively. This allows us to account for regime shifts or asymmetric deviations of y_t from its trend component. In formal terms, the transitory component and the shocks affecting their behavior are specified as follow:

$$\begin{aligned}
c_t &= \phi_1 c_{t-1} + \phi_2 c_{t-2} + u_t^*, && (6)\\
u_t^* &= \pi_{s_t} + u_t, && (7)\\
\pi_{s_t} &= \pi s_t, && (8)\\
u_t &\sim N(0, \sigma^2_{u,s_t}), && (9)\\
\sigma^2_{u,s_t} &= \sigma^2_{u,0}(1 - s_t) + \sigma^2_{u,1} s_t, && (10)\\
s_t &= 0, 1, && (11)
\end{aligned}$$

where $\pi \neq 0$. In the above specification, the term u_t is the usual symmetric shock. The term π_{s_t} is an asymmetric and discrete shock which is dependent upon an unobserved variable denoted by s_t which is an indicator variable that determines the nature of the shocks to the economy (similar as in the Markov-Switching models previously described). When the economy is near the potential or trend output, it can be qualified as normal times. In this case, $s_t = 0$ which implies that $\pi_{s_t} = 0$. In the opposite situation, which could be qualified as a period of recession, the economy is hit by a transitory shock potentially with a negative expected value, that is, $\pi_{s_t} = \pi < 0$. In this case, aggregate or other disturbances are plucking the output down.

Note that equations (9) and (10) allow for the possibility that the variance of the symmetric shock u_t be different during normal and recession times. In order to account for a persistence of normal periods or periods of recession, it is assumed that s_t evolves according to a first-order Markov-Switching process as in Hamilton (1989). It means that

$$\begin{aligned} \Pr[s_t &= 1|s_{t-1} = 1] = q, \\ \Pr[s_t &= 0|s_{t-1} = 0] = p. \end{aligned}$$

As mentioned by Kim and Nelson (1999), the above specification for the transitory component of output shares the same idea with the literature on "stochastic frontier production function", initially motivated by Aigner, Lowell and Schmidt (1977); see also Goodwin and Sweeney (1993).

The model is completed with the specification of τ_t, the permanent component. In this respect, Friedman (1993) suggested that the potential output can be approximated by a pure random walk[4]. In this case, all possible sorts of shocks can produce disturbances on it. In formal terms, this means that the permanent component can be specified as follows:

$$\begin{aligned} \tau_t &= g_{t-1} + \tau_{t-1} + v_t, & (12) \\ g_t &= g_{t-1} + w_t, & (13) \\ v_t &\sim N(0, \sigma^2_{v,s_t}), & (14) \\ w_t &\sim N(0, \sigma^2_w), & (15) \\ \sigma^2_{v,s_t} &= \sigma^2_{v,0}(1 - s_t) + \sigma^2_{v,1} s_t, & (16) \end{aligned}$$

where the stochastic trend component τ_t is subject to two kinds of permanent shocks: to its level v_t and to its growth rate w_t. Thus, equations (12)-(15) allow for productivity shocks. Note that it is allowed for the possibility that the variance of the shock to the level may be different during

[4] Friedman (1993) named this component "the ceiling maximum feasible output."

normal and recession times. However, variance of the shock to the growth rate is not likely to be systematically different during the normal and the recession times.

Notice that the model expressed by (5)-(16) nests the model suggested by Clark (1987). As it is well known, the model of Clark (1987) does not account for asymmetries, which in the context of the specification (5)-(16) implies that $p = q = 0$, $\pi = 0$. Furthermore, in the model of Clark (1987), we have $\sigma_{v,0} = \sigma_{v,1} = \sigma_v$, and $\sigma_{u,0} = \sigma_{u,1} = \sigma_u$.

3 Empirical Results

3.1 Data and Preliminaries

In all next estimations, y_t denotes the logarithm of the real output at period t and $\Delta_4 y_t$ denotes the annual growth rates of real output $(y_t - y_{t-4})$. The statistical information is quarterly covering the period 1980:1-2008:4 and it is obtained from the statistics of the Central Bank of Peru.

Figure 1 shows the evolution of the logarithm of the non-seasonal adjusted real output, and the annual growth rates. One clear evidence of visual inspection of the Figure 1 is the different behavior of the real output before and after 1990-1991. The first period is characterized for high volatility in growth rates and instability in domestic economy. The second period is associated with reduced volatility and a more stable economy. In fact, important structural reforms were applied since 1992. Emerging economies as Peru are characterized for a larger exposition to changes in its economic structure and consequently there is less stability in its economic cycles. The structural reforms were addressed to obtain commercial openness, larger development of the capital and financial markets, more flexibility in the labor market and a more efficiency of the monetary and fiscal policy. Furthermore in 2002 a fully-fledged-inflation targeting regime was officially adopted. All these measures implied reduction in volatility of nominal variables, a change in the correlation between growth rate of money and inflation, and reduction of levels of inflation and nominal interest rate.

It is worth to mention that during the sample period under analysis, some important events happened in the Peruvian economy. The first event was a natural disaster happened in 1983[5] in the north of Peru which affected dramatically the agricultural sector of the economy. Second, the years 80s were characterized by recurrent episodes of very high inflation which ended

[5] This natural disaster is currently known as the *child phenomenon*.

with a dramatic hyperinflation in August 1990. Third, in the 1990s, the financial sector of the economy was affected by external financial crises happened in Asia and Russia. Fourth, a terrorist group named *Shining Path* started its armed actions in 1980s. This group caused important social and economic losses to the country, and one of the most important achievements of the government of Fujimori was the control of this terrorist group in 1992.

3.2 Results

In the next lines I proceed in the following manner. Firstly, in order to justify non-linear estimations, I propose and estimate some linear models. These linear models are submitted to different statistical tests to verify necessity of non-linear estimations. Secondly, for each class of the proposed non-linear models, I will estimate two or three specifications. Actually, I will present the best two or three models of their respective category. Analysis of the different results and application of different statistical tests will determine the selection of the best model.

Table 1a presents estimates of the best three linear models I may found. The first model is an AR(2) model where all coefficients are statistically significant. The second model is an ARMA(2,2) model where the first moving-average coefficient is statistically insignificant and it has been deleted. Final linear model is an AR(2) model augmented with four dummy variables capturing atypical observations. These atypical observations are 1988:4, 1990:2, 1990:3, 1991:4, and they are closely related to the application of fiscal-monetary programs to stop high inflation episodes. This model has been selected using the automatic procedure *PcGets* created by Hendry and Krolzig (2001).

The three linear models show very similar low levels of persistence (0.705, 0.540 and 0.62). It means that when negative shocks hit the economy, its effects decay relatively fast as indicated by the sum of the autoregressive coefficients. Furthermore, all three models presents complex roots inside of the unit circle indicating pseudo-cyclical behavior. According to these criteria, the AR(2) model augmented with additive outliers is the best linear model.

Table 1b shows statistical tests used in order to evaluate residuals of the previously three estimated models. There is the Lagrange Multiplier (LM(j)) statistic to verify presence of autocorrelation. I am also presenting the ARCH statistic to observe presence of autoregressive conditional heteroscedasticity in the residuals. In order to test for normality of the residuals, I use the Jarque and Bera (JB) statistic. Finally, I use the BDS statistic proposed by Brock, Dechert, Scheinkman, and LeBaron (1996) which is a portmanteau test for time based dependence in a series. It can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos.

According to the values presented in Table 1b, none linear model approves satisfactorily all testing evaluation. The simple AR(2) model suffers severely of autocorrelation as indicated by the statistic $LM(j)$. Furthermore, this model presents evidence of non normality and dependence of the residuals. The ARMA(2,2) model presents very similar results and therefore it is not recommended. The presence of additive outliers which are not modeled in the AR(2) and ARMA(2,2) models could explain the presence of autocorrelation in the squared residuals and violation of the normality of the residuals. In this sense the model AR(2) augmented with four dummies for four additive outliers does not suffer of these problems. This model presents some problems of autocorrelation at long lags but essentially its principal drawback is that residuals show dependence. A further issue related to the linear models is the instability of the estimated parameters as indicated by the application of the statistic of Chow or the different statistics proposed by Andrews (1993)[6]. According to it, all these linear models are not recommended.

The second set of estimates corresponds to the LSTAR models. Econometric modeling with STAR models has to begin with testing the null hypothesis of linearity against the alternative hypothesis of a STAR model. However, there are complications due to the presence of unidentified nuisance parameters under the null hypothesis. One way to express the null hypothesis is $\gamma = 0$ in equation (1). In this case, the parameter c, ϕ_1 and ϕ_2 are the unidentified parameters. The solution proposed in the literature is a set of LM type statistics which replace the transition function $F(v_t; \gamma, c)$ by a suitable Taylor series approximation. When the alternative hypothesis is a LSTAR model Lukkonnen, Saikkonen and Teräsvirta (1988) have proposed the respective LM type statistics. On the other side, Saikkonen and Lukkonen (1988) and Escribano and Jordá (1999) have suggested similar statistics when the alternative hypothesis is an ESTAR model. The advantage of this approach is that the model under the alternative hypothesis is not needed to be estimated and asymptotic theory is available (χ^2_{df} distribution). In order to correct for sample size, I follow the suggestion to use the F statistics; see also Teräsvirta (1998). The different F statistics suggest strong rejection of the null hypothesis of linearity. Furthermore, the rejections suggest that a

[6]Results available upon request.

LSTAR specification is preferred[7].

Two models are presented and both include two thresholds, therefore they are denoted by LSTAR(2). In the first model, the transition variable is $\Delta_4 y_{t-1}$ while in the second model, the variable $\Delta_4 y_{t-4}$ has this role. According to the estimates of γ, c_1 and c_2, the behavior of both models is not very different. Parameter γ is larger in the second model which shows an abrupt switch between both regimes. Only based on the information criteria, the first model should be selected. This model indicates that the first regime is stationary while the second one is nonstationary and persistence is higher in the first regimen compared to the second regime. Unlike this model, the second model presents roots inside of the unit circle indicating stationarity in both regimes. Level of persistence is around 0.75 in the first regime of both models. However, level of persistence is very low (0.09 and 0.26 in first and second model, respectively) in the second regime. It shows that duration of transitory shocks (negative or positive) in this regime are extremely brief. It is worth to say that for both models, roots of the autoregressive polynomials are complex indicating pseudo-cyclical behavior.

Figure 2a shows the evolution of the transition function for the sample period. The picture suggests only few periods where Peruvian economy has experimented very large positive and negative growth rates. The period 1986:4-1987:2 reflects the high growth rates experimented in the first half of the first government of President A. Garcia which were associated with high levels of fiscal spending. However, potential output does not follow same pattern of growth. Another identified period is the first three quarters of 1989. The period 1994:2-1995:2 contains also observations of high growth rates. An important observation is that this model does not identify any recession period (more than two consecutive negative growth rates) which represents a serious drawback even when information criteria suggests suitability. The alternative model (a LSTAR(2) model with $\Delta_4 y_{t-4}$ as transition variable) detects some different periods. The brief periods 1987:3-1988:1, 1991:3-1991:4, 1995:1-1996:2 are identified as observations where growth rates have been particularly high. As before, the very high growth rates observed during government of President A. Garcia are again obtained. However, unlike previous model, the present model selects 1989:4-1990:3 as an extreme regime with negative growth rates. It is interesting and makes more sense because during these quarters the Peruvian economy experimented negative growth rates aggravated with an hyperinflation pe-

[7] The F statistics are not shown in order to save space, but they are available upon request.

riod never experimented before. Furthermore, these quarters are associated with high influence and destructive activity of the terrorism group *Shinning Path*.

Figure 2b show evolution of the transition function respect to the transition variable and they indicate similar behavior. Figures also indicate that observations smaller than c_1 and greater than c_2 are scarce. It could suggest that Peruvian economy has experimented a few number of observations with very negative or positive growth rates confirming previous obtained results. In consequence, most of observations are between both thresholds. However, both estimated thresholds are (in absolute values) very large which indicate that the interval $[\widehat{c}_1, \widehat{c}_2]$ is broad suggesting that Peruvian economy has experimented large (negative or positive) growth rates. Observing the picture we may see that most of observations are concentrated to the right of the zero growth rate confirming that this economy has grown in a sustained way for a long period (1995-2008). Most of the observations are clearly concentrated around 0.0% and 9.0% (around 50.0%). Furthermore, comparing both pictures, the model where $\Delta_4 y_{t-4}$ is the transition variable presents more abrupt change between both regimes. There is almost no observations to the left of c_1. It suggests that when economy has been in recession times, the growth rates have been very negative but economy was not in this regime too much time.

In order to examine performance of the STAR models, Eirtheim and Teräsvirta (1996) suggest the use of LM type tests which include a LM type statistic of no autocorrelation in the residuals[8], a LM type statistic of no remaining nonlinearity[9], and the LM type statistic of parameter constancy[10]. Furthermore, I include the statistic JB to test the null hypothesis of normality in the residuals. Table 2b shows the results. The first panel indicates that the null hypothesis of the LM type statistic for no autocorrelation is rejected

[8] The LM type statistic for serial independence is distributed as a χ^2_k. It is a generalization of the LM test for serial correlation in an AR(k) model as suggested by Godfrey (1979).

[9] In Table 2b, I present F, F_2, F_3, and F_4 which denote the different Taylor approximations used to calculate the respective LM type statistics. All variables, except the selected transition variable, are tested as the potential source of remaining nonlinearity. See Eirtheim and Teräsvirta (1996), and Teräsvirta (1998).

[10] In the Table 2b, I present F_i $(i = 1, 2, 3)$ which indicates a LM type statistic against a STAR model with transition function H_i $(i = 1, 2, 3)$. H_i indicates a transition function of the form:

$$H_i(t; \gamma, c_i) = [1+\exp[-\gamma \Pi_{i=1}^{3}(t - c_i)]]^{-1} - 0.5,$$

where $c_1 \leq c_2 \leq c_3$; see Eirtheim and Teräsvirta (1996), and Teräsvirta (1998) for further details.

from the third lag and on. In the second panel, the LM type statistic of no remaining nonlinearity indicates rejection of the null hypothesis in favor of the alternative hypothesis of nonlinearity associated to an additive multiple regime model (an additive STAR model) for the second model (where the transition variable is $\Delta_4 y_{t-4}$). The first model does not reject (at 5.0%) the null hypothesis of no remaining nonlinearity. However, the LM type statistic of parameter constancy suggests a rejection of the null hypothesis in favor of a smoothly changing parameter model for the first model (where the transition variable is $\Delta_4 y_{t-1}$)[11]. In other words, despite careful parameterization of nonlinearity, the parameter constancy is still a problem. It is true that in comparative terms, the LSTAR models offer an useful alternative to the previous estimated linear models as a consequence of its ability to show a switching dynamics between regimes. For example, the presence of outliers is captured in some sense and consequently normality of the residuals is not longer rejected. Rejection of the LM type statistics above mentioned do not recommend validity of this kind of models. Furthermore, the results suggest that volatility is an issue in the previous estimations. Therefore an interesting possibility is to introduce an heteroscedastic behavior in order to capture different volatilities. I follow this way in the following lines.

Now, the category of non-linear models to be estimated is the autoregressive Markov-Switching model. All different MSIAH(m)-AR(k) for $m = 2, 3$ and $k = 1, 2, 3, 4$ have been estimated. In order to illustrate suitability of this methodology, I select the best estimations according to the information criteria; see Table 3a. Following notation established in previous section, the first model is a MSIAH(2) model. All parameters are statistically significant except the third lag in both regimes. Given this fact, a second model is estimated with exactly the same structure as the first model except that now third lag has been dropped from both regimes. In the third model, a third regime is introduced and this models is denoted MSIAH(3). In this case, all parameters are significant except fourth lag in the second regime. In all three cases, the null hypothesis of linearity is strongly rejected as it may be observed from the statistic of Davies (1987)[12]. Furthermore, comparing the

[11] It corresponds to the case where $v_t = t$, that is, a time varying STAR model. It implies that y_t follows a STAR model at all times with a smooth change in the autoregressive parameters in both regimes.

[12] In the context of Markov-Switching models, the usual tests (Likelihood Ratio, Wald, and Lagrange Multiplier) do not have the standard asymptotic distribution. The problem comes from two sources: under the null hypothesis, some parameters are not identified and the scores are identically zero. To overcome this problem, Davies (1987) starts with the idea of giving a range of values to the parameters under the alternative hypothesis, thus avoiding the problems of estimating them, and construct some statistics based on the

three models using the information criteria, the MSIAH(2) model is slightly selected while the MSIAH(3) model is very close to it, in special according to the AIC.

Estimates of the standard deviation of the MSIAH(2) model suggest that second regime (moderate-high growth rates) has been six times the standard deviation of the first regimen (recession times). According to the MSIAH(3) model, the second regime (high growth rates) is still more volatile than the first regime (recessions) while last regime (moderate growth rates) presents similar volatility than the first regime (recession times). According to the set of roots implied for the AR(4) process, the first regime of the MSIAH(2) model appears to be nonstationary. Second regime is stationary and the level of persistence is around 0.653. The model MSIAH(3) is also nonstationary in the first regime but second and third regimes are stationary. In all cases, the presence of complex roots assures pseudo cyclical behavior. Last Markov-Switching model (see last two columns of Table 3a) has roots inside of the unit circle indicating stationarity. The level of persistence is around 0.650 in both regimes.

Figure 3a shows probabilities to be in recession times ($s_t = 1$) estimated by the MSIAH(2) model. The identified periods are 1985:2-1985:3, 1988:3-1988:4, 1990:2-1990:3 and 1992:2-1992:3. What is important to note is that all these periods have a duration of only two quarters indicating very fast reverting behavior. The expected duration of the first regime is 1.40 quarters which is equivalent to 11.0% of the total number of observations. The second regime has an expected duration of 11.95 quarters.

Another point is the fact that the model identifies 1983:1, 1991:4 and 1993:4 (only one quarters) as quarters where output presented very negative growth rates. Because duration is only of one quarter, they cannot be considered as recession episodes according to standard or practical rules. These observations may be qualified as outliers. For example 1983:1 is related with natural disaster happened in the north of Peru. However, according to the model, this only observation is an atypical growth rate but it cannot be interpreted as part of a recession regime.

In Figure 3b, probabilities of $s_t = 1$ obtained from the alternative MSIAH(2) are shown. The calculated expected durations are 9.35 and 12.80 quarters for first and second regimes, respectively. It indicates that recession and normal times have 43.8% and 56.2% of the total number of observations.

value of the objective function obtained with these given parameter values. Therefore, we obtain an upper bound for the significance level of the likelihood ratio statistic under the null hypothesis consisting of the model with the lower number of states. For further and technical details, see the annex of Garcia and Perron (1996).

In fact, the figure shows that this model selects too many observations qualified as recession quarters. One evident case is the period 1985:2-1992:4 identified as a recession period which is very difficult to conciliate with real data because it has long duration and also because it includes some periods as 1987 where economy showed high growth rates contradicting findings of the model[13].

Estimates of the MSIAH(3) model indicate that the expected durations of each regime are 1.68, 3.06 and 7.94 quarters, respectively. In terms of the total number of observations, it implies durations of 10.7%, 38.8%, and 50.4%, respectively. Figure 3c presents the recession times ($s_t = 1$) identified by the MSIAH(3) model. According to this model, 1988:3-1988:4 and 1990:2-1991:1 are observations identified as recessions. The dates are in close concordance with the real evidence. For example 1988:3-1988:4 are related to the negative growth rates of output related to the application of fiscal-monetary policies applied in these quarters in order to stabilize high inflation. On another side, the period 1990:2-1991:1 are quarters with negative growth rates caused by the hard stabilization program applied to stop hyperinflation where monetary authority reduced strongly monetary base[14]. Other measures applied with the same goal were credit contraction, and unfreezing public prices. All these measures drive to the larger recession. Notice that the model does not identify negative growth rates of 1983 related to the natural disaster happened in the north of Peru. Actually, the model identifies 1983:1 as an observation with a very negative growth rate with high probability to be in $s_t = 1$. However, it is only one quarter and consequently, it does not enter in the traditional and practical definition of a recession of (at least) two negative consecutive growth rates. Similar arguments are applied to the cases of 1987:3 and 1991:4. In other words, the model selects these observations as outliers or atypical observations but not as recession times.

Table 3b shows different statistics to evaluate residuals of the Markov-Switching models previously estimated. First observation is that performance of all three Markov-Switching models is more satisfactory than previous estimated models. The statistic LM(j) does not suggest presence of autocorrelation in the residuals of the three models. The presence of

[13] I decided to still include this model because it is a good example of a good fitting but clearly wrong in selection of the recession times (among other things). It is a very simple modification of the MSIAH(2) and it is persented in the third column of Tables 3a and 3b. However this simple modification shows strong changes in major results. It illustrates that carefulness is needed at the moment of selecting models.

[14] This program is well known as the *Fujimori Plan*.

autoregressive conditional heteroscedasticity can not be rejected both for the MSIAH(2) and the modified MSIAH(2) models. However, the model MSIAH(3) can not reject the null hypothesis of absence of ARCH effects in the residuals. Given that the only difference between models is the presence of an additional regime, it suggests that inclusion of a third regime is adequate. The null hypothesis of normality can not be rejected for the three models. It suggests that presence of additive outliers or other kind of structural change is well captured by the presence of different regimes. Finally, serial independence can not be rejected for any of the three Markov-Switching models. In summary, all statistical tests indicate adequacy of the MSIAH(3) model.

Table 4a presents estimates of three versions of the plucking model. Each column corresponds to a gradual reduction of the number of estimated parameters. The first column (denoted plucking 1) corresponds to the unrestricted plucking model. Second column (denoted plucking 2) shows estimates of the plucking model where the hypothesis $\sigma_{v_0} = \sigma_{v_1}$ has been imposed. A p-value close to unity indicates that this null hypothesis can not be rejected. Last column (denoted plucking 3) presents estimates of a plucking model where the null hypothesis $\sigma_{v_0} = \sigma_{v_1} = 0$ has been imposed. The null hypothesis is not rejected. The results show clearly that all estimates are almost the same in the three columns of Table 4a. It means that nothing is lost imposing the above restrictions.

It is worth to mention that many other restrictions were tested and all these restrictions were strongly rejected. For example, restricting the symmetric transitory shock to be $\sigma_{u_0} = \sigma_{u_1} = 0$ is strongly rejected which indicates the relative significance of the symmetric shock. Finally, a symmetric trend and cycle alternative model was also estimated. It implies estimating the plucking model with the restrictions $\pi = 0$, $\sigma_{u_0} = \sigma_{u_1}$, and $\sigma_{v_0} = \sigma_{v_1}$ which is equivalent to the model of Clark (1987). This set of hypothesis is strongly rejected (p-value of 0.012) suggesting relative importance of the asymmetric discrete shocks and different behavior of volatilities of the symmetric shock. Only according to this metric, the plucking model fits real output data better than the symmetric trend-plus-cycle model of Clark (1987)[15]. Therefore, given the above results, the preferred model is the model denoted by plucking 3 (third column in Table 4a).

The sum of the autoregressive coefficients is around 0.71 and the roots of the second-degree polynomial implies pseudo-cyclical behavior. The sum of the autoregressive coefficients for the transitory component of output

[15] The results of the different tested hypothesis is available upon request.

fall when asymmetry is accounted for, and thus it tends to be statistically lower in the plucking model than in the model of Clark, for example (0.92). According to Simone de Nadal and Clarke (2007), it is important because if the sum of the autoregressive coefficients is close to one, output shocks could be erroneously considered as permanent (or very persistent) if estimated output behavior is restricted to be symmetric when it is in fact asymmetric.

Therefore, the estimates also indicate that once a negative transitory shock hits the economy, its effects decay relatively fast, as indicated by the relatively low value of the sum of the autoregressive coefficients. This result is also found in previous empirical research as Kim and Nelson (1999), Mills and Wang (2001), Rodríguez (2005), Nadal de Simone and Clarke (2007).

Regarding the asymmetric shock (π), it is negative as expected in all columns of the Table 4a. The asymmetric shock appears to explain more of the variance of the cyclical component than the symmetric shock. Our estimates also indicate that the stochastic trend component is not affected by significant permanent "normal" (σ_{v_0}) or recessive (σ_{v_1}) shocks. Observing the estimates of Table 4a, we find that these parameters are not statistically significant and therefore they are deleted in final third column of Table 4a. However, accounting for asymmetry, makes the shock to the trend growth (σ_w) component statistically significant[16].

Figure 4a shows the evolution of the current and potential output and the corresponding estimated cycles according to the preferred plucking model (plucking 3). The data seem to confirm Friedman's view that the economy is most of the time at its potential level (since 1994-1995) but the evidence does not show that output is plucked down from time to time. What we observe is a very large period where output appears to be plucked down and after it, a long period where economy is close to the potential output.

Figure 4b presents the probabilities to be in recession times related to the selected plucking model (plucking 3). Of course, evolution of these probabilities are strongly correlated with the previous picture. The model suggests that 1988, 1993, 1994 are periods where economy has been in recession times or in a very low growth rates regime. For the previous years, the probabili-

[16]Strictly speaking, the plucking model (and also the model of Clark (1987)) specifies the real output as an I(2) process. It is well known, however, that real output is a trend stationary process or an I(1) process. If the variance of the shock to the trend growth component is not statistically different from zero or it is very small, this should not pose a major misspecification problems. In our case, this parameter is statistically significant but is always a small value. It should not pose major misspecifications problems. All models were estimated without restricting growth to have zero variance. In fact, when model was estimated imposing this restriction, a strong rejection was obtained.

ties are relatively more volatile and 1984-1995 appears as a low growth rates period. Since 1995 until end of the sample, probabilities are almost zero. It suggests that for these quarters Peruvian economy has been working very close to the potential output. The duration of this period appears to be very large.

Table 4b shows testing of the residuals. All three variations of the plucking model fail to pass satisfactorily any of the statistical tests. The residuals are strongly contaminated by autocorrelation, non normality and dependence.

The results indicate that even when the plucking model appears to be appropriate and appealing in theoretical terms, the behavior of the estimated residuals invalidates this model. Furthermore, the duration of the recession times and their associated probabilities appear to be too large and difficult to conciliate with simple analysis of the growth rates.

A summary of the principal results is needed here. The majority of models identify recession times in quarters concentrated in 1988-1989 and 1990-1991. Furthermore, they identify some other very negative growth rates with duration of only one quarter (as the natural disaster of 1983). These observations, following classical definitions, are classified as atypical observations but not as recession times.

On another side, some of these apparently good-fitting models have bad performance at the moment of the statistical evaluation. The MSIAH(3) model selects recession times and normal times in a relative adequate manner. The selected periods are consistent with visual inspection and with the real events happened in the Peruvian economy. At the moment of the testing evaluation, this model performs better than the other estimated Markov-Switching models and the other competitive models. Therefore, I consider that it is the preferred model.

I would like to finish this section comparing recession times identified in this paper with alternative procedures. A very well known procedure to identify recession times is to account for more than two consecutive negative growth rates as an indicator of recession times. According to this procedure, the identified recession times are 1982:3-1984:1, 1985:3-1986:1, 1988:1-1989:3, 1990:3-1991:2, 1992:2-1992:3, 1998:2-1999:1, 2000:4-2001:2. Therefore, we have more quarters in recession times compared with estimates of this paper. Using a shorter sample, Castillo, Montoro and Tuesta (2006) found three completed cycles where troughs are identified around 1983-1984, 1989-1990, and 2001, respectively. On another side and using classical business approach, Mejía-Reyes (2004) detects following troughs for Peru: 1983:4, 1988:4, 1990:4, 1992:3, 1997:2. It is clear that similarities exist in the selection of the recession times. More specifically, we may say that dates of recession times identified in this paper are included in the more large number of recession times identified for other alternative procedures.

A natural question is to know reasons why the methods applied in this paper select a reduced number of recession times. Or why the other methods could overestimate the number of quarters of recession times. My conjecture is that Peruvian economy has experimented very negative (and very positive) growth rates which could affect the selection of recession times by any method. The estimated non-linear models correctly identify these observation but they are qualified as atypical observations. From another side, because there are very negative growth rates, moderate negative growth rates could not be selected as recession times. For example, the natural disaster of 1983 affected seriously agricultural sector and consequently real output. However, this phenomenon is selected as an atypical observation because it has one quarter of duration. The only explanation for this fact is that, even when growth rates around 1983 are highly negative, these rates are smaller compared to the extreme values experimented in other moments of the economy. Another example could be the financial crises in Asia and Russia. Apparently, these events have affected economy. However the negative growth rates are smaller compared with other negative growth rates experimented for the economy. Therefore, the negative growth rates caused by external financial crises are not detected because they are “small”.

4 Conclusions

In order to identify and analyze business cycles, three alternative non-linear models have been estimated. The first class is the Logistic Smooth Transition Autoregressive (LSTAR) model suggested by Teräsvirta (1994) where one observable variable drives the switching behavior between the two regimes. The second model is an extended version of the Markov-Switching model proposed by Hamilton (1989) where an unobservable variable determines the switching behavior between m regimes in a probabilistic way. The third model is the unobserved components model theoretically suggested by Friedman (1964, 1993) and econometrically specified by Kim and Nelson (1999). The new feature of this model is the presence of an asymmetric shock that explain the presence of recession times. In the rest of the time, economy is operating close to the potential output.

The majority of the estimated models identify periods of recessions as quarters concentrated around 1988 and 1990. Some events as the natural disasters happened in 1983 are only captured as atypical observations but they do not qualify as recession times. Furthermore, the Asian and Russian financial crises are not identified as recession times. It means that these events probably affected the financial sector of the economy but not affected the real output of the economy. Another alternative explanation is that the negative effects of these events were smaller compared to the negative impact of other events. The alternative hypothesis that armed and destructive actions of the terrorist group *Shining Path* could cause recession periods is difficult to accept according to the quantitative results.

Another similarity between models is that since 1994-1995 until end of the sample, the economy appears to enter in a period of relative sustainable growth. It is supported by the various structural reforms applied in the labor market, financial sector, external sector, and the adoption of the targeting inflation system. All these measures contributed to the adequate behavior of the economy since 1995 with less volatility in the evolution of the major macroeconomic variables. Therefore, all non-linear models identify a long duration regime characterized by moderate-high growth rates.

From the point of view of the statistical evaluation of the different non-linear models, the MSIAH(3) model shows superior performance. This model selects the recession times and normal times in a relative adequate manner. The selected periods are consistent with visual inspection and with the real events happened in the Peruvian economy. At the moment of the testing evaluation, the model performs better than the other Markov-Switching models and the other competitive models. The model does not present evidence of autocorrelation in the residuals and the squared residuals. Furthermore, there is no presence of non normality, and the residuals are independent.

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Table 1a. Estimates of Linear Models

Coefficient	AR(2)		ARMA(2,2)		AR(2) + Additive Outliers	
	Estimate	(p-value)	Estimate	(p-value)	Estimate	(p-value)
c	0.008	(0.078)	0.012	(0.058)	0.009	(0.005)
ϕ_1	1.184	(0.000)	1.056	(0.000)	1.006	(0.000)
ϕ_2	-0.479	(0.000)	-0.516	(0.000)		
ϕ_3					-0.382	(0.000)
θ_1						
θ_2			0.452	(0.000)		
DU_{1t}					-0.137	(0.000)
DU_{2t}					-0.199	(0.000)
DU_{3t}					-0.192	(0.000)
DU_{4t}					-0.153	(0.000)
Other Information						
$\overline{R}^2$	0.718		0.722		0.833	
AIC	-3.437		-3.443		-3.924	
HQ	-3.407		-3.402		-3.854	
SC	-3.363		-3.343		-3.750	

$DU_{1t}= 1$ if $t = 1988:4$, $DU_{2t}= 1$ if $t = 1990:2$, $DU_{3t}= 1$ if $t = 1990:3$, $DU_{4t}= 1$ if $t = 1991:4$

Table 1b. Evaluation of Linear Models

Statistic	AR(2)		ARMA(2,2)		AR(2) + Additive Outliers	
	Value	(p-value)	Value	(p-value)	Value	(p-value)
LM Test for Autocorrelation in Residuals						
LM(1)	0.098	(0.754)	5.724	(0.018)	1.379	(0.243)
LM(2)	0.499	(0.608)	7.038	(0.001)	0.825	(0.441)
LM(3)	4.325	(0.006)	5.485	(0.002)	2.175	(0.096)
LM(4)	5.658	(0.000)	4.443	(0.002)	2.832	(0.028)
LM(8)	2.935	(0.006)	2.797	(0.008)	2.734	(0.009)
Autoregressive Conditional Heteroscedasticity in Residuals						
ARCH(1)	16.847	(0.000)	22.611	(0.000)	0.356	(0.552)
ARCH(2)	8.283	(0.000)	11.386	(0.000)	0.321	(0.726)
Normality Test of Residuals						
JB	38.580	(0.000)	70.029	(0.000)	0.379	(0.827)
Independence Test of Residuals						
BDS(m=2,0.7)	3.363	(0.000)	3.998	(0.000)	0.953	(0.340)
BDS(m=3,0.7)	4.386	(0.000)	5.099	(0.000)	1.431	(0.152)
BDS(m=4,0.7)	5.007	(0.000)	6.004	(0.000)	1.738	(0.082)
BDS(m=5,0.7)	5.321	(0.000)	6.332	(0.000)	2.371	(0.018)
BDS(m=6,0.7)	6.237	(0.000)	6.765	(0.000)	3.064	(0.002)

The statistic LM(j) tests the null hypothesis of no autocorrelation of order j in the residuals; ARCH(j) tests the presence of Autoregressive conditional heteroscedasticity in the residuals; JB is the Jarque and Bera statistic to test the null hypothesis of normality in residuals; and BDS tests the null hypothesis of independence in residuals.

Table 2a. Estimates of LSTAR Models

Coefficient	LSTAR(2)		LSTAR(2)	
	$v_t = \Delta_4 y_{t-1}$		$v_t = \Delta_4 y_{t-4}$	
	Estimate	(p-value)	Estimate	(p-value)
		First Regime		
μ_1	0.011	(0.021)	0.008	(0.062)
ϕ_{11}	1.430	(0.000)	1.071	(0.000)
ϕ_{12}	-0.698	(0.000)		
ϕ_{13}			-0.304	(0.000)
ϕ_{14}				
		Second Regime		
μ_2	-0.020	(0.141)		
ϕ_{21}	-0.765	(0.000)		
ϕ_{22}	0.859	(0.000)	-0.652	(0.000)
ϕ_{23}				
ϕ_{24}			0.386	(0.000)
		Other parameters		
γ	3.112	(0.395)	13.258	(0.578)
c_1	-0.176	(0.000)	-0.117	(0.000)
c_2	0.106	0.000	0.092	0.000
		Other Information		
$\overline{R}^2$	0.775		0.787	
AIC	-3.867		-3.845	
HQ	-3.726		-3.603	
SC	-3.519		-3.249	

The model is $\Delta_4 y_t = \phi_1' x_t[1 - F(v_t;\gamma,c)] + \phi_2' x_t[F(v_t;\gamma,c)] + \epsilon_t$, where $x_t = (1, \Delta_4 y_{t-1}, \Delta_4 y_{t-2}, \Delta_4 y_{t-3}, \Delta_4 y_{t-4})$, and $F(v_t;\gamma,c) = \{1 + \exp[-\gamma \Pi_{i=1}^{2}(v_t - c_i)]\}^{-1}$. The Model 1 uses $v_t = \Delta_4 y_{t-1}$ and Model 2 uses $v_t = \Delta_4 y_{t-4}$.

Table 2b. Evaluation of LSTAR Models

LM Test of No Autocorrelation in Residuals				
	LSTAR(2), $v_t = \Delta_4 y_{t-1}$		LSTAR(2), $v_t = \Delta_4 y_{t-4}$	
Lag	Value	(p-value)	Value	(p-value)
1	0.210	(0.647)	0.001	(0.978)
2	0.277	(0.758)	1.201	(0.305)
3	3.146	(0.028)	4.083	(0.001)
4	6.623	(0.000)	6.959	(0.000)
5	4.833	(0.000)	5.827	(0.000)
6	4.092	(0.001)	4.710	(0.000)
7	3.987	(0.001)	5.282	(0.000)
8	3.523	(0.002)	4.815	(0.000)
LM Test of No Remaining Nonlinearity				
	LSTAR(2), $v_t = \Delta_4 y_{t-1}$	LSTAR(2), $v_t = \Delta_4 y_{t-4}$		
	Transition Variable (p-values)			
Statistic	$\Delta_4 y_{t-2}$	$\Delta_4 y_{t-1}$	$\Delta_4 y_{t-2}$	$\Delta_4 y_{t-3}$
F	0.092	0.000	0.034	0.035
F_4	0.645	0.061	0.503	0.107
F_3	0.111	0.267	0.437	0.273
F_2	0.057	0.000	0.001	0.016
LM Test of Parameter Constancy				
	LSTAR(2), $v_t = \Delta_4 y_{t-1}$		LSTAR(2), $v_t = \Delta_4 y_{t-4}$	
Statistic	Value	(p-value)	Value	(p-value)
$F_1(H_1)$	2.394	(0.034)	1.285	(0.277)
$F_2(H_2)$	1.867	(0.050)	0.637	(0.778)
$F_3(H_3)$	1.600	(0.079)	0.949	(0.515)
Normality Test of Residuals				
JB	2.173	(0.337)	4.156	(0.125)

Description of the different LM type statistics (no autocorrelation, no remaining nonlinearity, and parameter constancy) are given in the text (see footnotes 7, 8, and 9); JB is the Jarque and Bera statistic to test the null hypothesis of normality in residuals.

Table 3a. Estimates of Markov-Switching Models

Coefficient	MSIAH(2)		MSIAH(3)		MSIAH(2)	
	Estimate	t-value	Estimate	t-value	Estimate	t-value
First Regime ($s_t = 1$)						
μ_1	-0.088	-41.615	-0.067	-7.886	-0.005	-0.593
ϕ_{11}	1.466	56.409	1.553	13.697	1.236	9.059
ϕ_{12}	-0.262	-6.379	-1.406	-6.203	-0.645	-4.385
ϕ_{13}	-0.035	-0.999	1.347	5.351		
ϕ_{14}	0.448	17.235	-0.293	-1.817	0.086	0.894
σ_1	0.005		0.019		0.053	
Second Regime ($s_t = 2$)						
μ_2	0.019	6.133	0.009	1.727	0.021	4.716
ϕ_{21}	1.175	14.833	1.293	12.587	0.687	5.228
ϕ_{22}	-0.311	-2.567	-0.317	-1.765	0.275	2.324
ϕ_{23}	-0.063	-0.515	-0.432	-2.694		
ϕ_{24}	-0.147	-1.812	0.054	0.561	-0.310	-4.832
σ_2	0.028		0.027		0.017	
Third Regime ($s_t = 3$)						
μ_3			0.028	6.839		
ϕ_{31}			0.484	4.293		
ϕ_{32}			0.248	1.965		
ϕ_{33}			0.267	2.305		
ϕ_{34}			-0.399	-4.427		
σ			0.016			
Other Information						
Log Likelihood	222.809		231.626		213.960	
AIC	-3.867		-3.845		-3.740	
HQ	-3.726		-3.603		-3.619	
SC	-3.519		-3.249		-3.442	
LR Test	67.220		84.854		49.906	
p-value (Davies)	0.000		0.000		0.000	

The models are $\Delta_4 y_t = \delta_{s_t} + \phi_{s_t,1}\Delta_4 y_{t-1} + ... + \phi_{s_t,4}\Delta_4 y_{t-4} + \epsilon_t$, where $\epsilon_t \sim i.i.d.\ N(0, \sigma^2_{s_t})$. In models MSIAH(2), $s_t = 1$ or $s_t = 2$, while in models MSIAH(3), $s_t = 1$, $s_t = 2$, or $s_t = 3$. Last row (denoted by *Davies*) is the Davies' (1987) upper bound test where the null hypothesis is linearity; see the text for further details (footnote 10).

Table 3b. Evaluation of Markov-Switching Models

Statistic	MSIAH(2)		MSIAH(3)		MSIAH(2)	
	Value	(p-value)	Value	(p-value)	Value	(p-value)
LM Test for Autocorrelation in Residuals						
LM(1)	0.596	(0.442)	0.420	(0.518)	0.004	(0.946)
LM(2)	0.638	(0.530)	0.663	(0.517)	0.019	(0.981)
LM(3)	0.798	(0.497)	0.646	(0.587)	0.814	(0.488)
LM(4)	0.601	(0.662)	0.655	(0.624)	1.603	(0.179)
LM(8)	1.621	(0.128)	0.859	(0.558)	0.904	(0.516)
Autoregressive Conditional Heteroscedasticity in Residuals						
ARCH(1)	0.077	(0.077)	0.173	(0.678)	4.505	(0.036)
ARCH(2)	0.042	(0.042)	1.195	(0.307)	3.44	(0.035)
Normality Test of Residuals						
JB	2.527	(0.282)	1.424	(0.491)	1.689	(0.429)
Independence Test of Residuals						
BDS(m=2,0.7)	0.066	(0.947)	0.032	(0.974)	0.217	(0.827)
BDS(m=3,0.7)	0.351	(0.725)	1.203	(0.229)	1.377	(0.168)
BDS(m=4,0.7)	0.154	(0.877)	0.878	(0.379)	0.921	(0.357)
BDS(m=5,0.7)	0.316	(0.752)	0.487	(0.626)	1.014	(0.310)
BDS(m=6,0.7)	0.856	(0.392)	0.265	(0.791)	1.163	(0.245)

The statistic LM(j) tests the null hypothesis of no autocorrelation of order j in the residuals; ARCH(j) tests the presence of Autoregressive conditional heteroscedasticity in the residuals; JB is the Jarque and Bera statistic to test the null hypothesis of normality in residuals; and BDS tests the null hypothesis of independence in residuals.

Table 4a. Estimates of Plucking Models

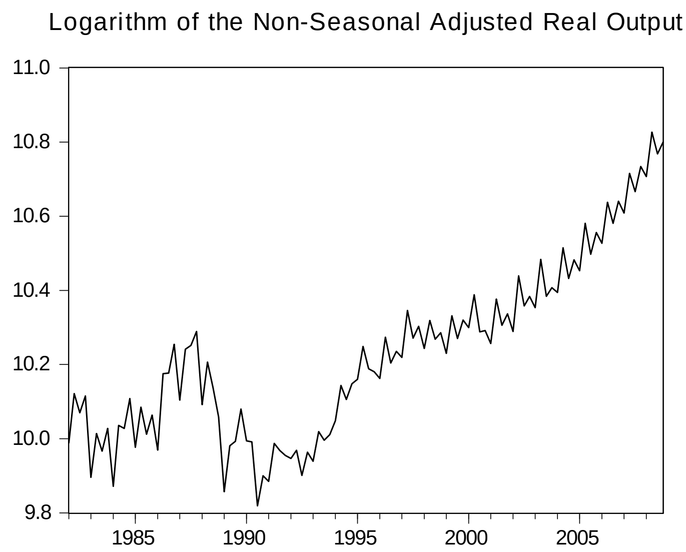
Coefficient	Plucking 1		Plucking 2		Plucking 3	
	Estimate	t-value	Estimate	t-value	Estimate	t-value
q	0.94420	27.218	0.94421	27.368	0.9442	27.368
p	0.97519	54.971	0.97519	55.095	0.9752	55.096
ϕ_1	0.96407	10.124	0.96408	10.105	0.9641	10.095
ϕ_2	-0.25342	-2.993	-0.25343	-2.995	–0.2534	-2.984
σ_{u_0}	0.01210	11.634	0.01210	12.100	0.0121	12.100
σ_{u_1}	0.03497	8.699	0.03497	8.742	0.0349	8.725
σ_{v_0}	0.00002	0.031	0.00005	0.038		
σ_{v_1}	0.00016	0.024				
σ_w	0.00173	2.790	0.00173	2.883	0.0017	2.833
π	-0.05367	-5.329	-0.05366	-5.366	-0.0537	-5.370
Log Likelihood	249.809		249.809		249.809	

The complete specification of the model is given by expressions (5)-(16) in the text.

Table 4b. Evaluation of Plucking Models

Statistic	Plucking 1		Plucking 2		Plucking 3	
	Value	(p-value)	Value	(p-value)	Value	(p-value)
LM Test for Autocorrelation in Residuals						
LM(1)	42.654	(0.000)	42.652	(0.000)	42.654	(0.000)
LM(2)	21.438	(0.000)	21.431	(0.000)	21.438	(0.000)
LM(3)	15.587	(0.000)	15.586	(0.000)	15.587	(0.000)
LM(4)	11.583	(0.000)	11.583	(0.000)	11.583	(0.000)
LM(8)	7.489	(0.000)	7.488	(0.000)	7.489	(0.000)
Autoregressive Conditional Heteroscedasticity in Residuals						
ARCH(1)	30.148	(0.000)	30.149	(0.000)	30.148	(0.000)
ARCH(2)	19.189	(0.000)	19.189	(0.000)	19.189	(0.000)
Normality Test of Residuals						
JB	175.912	(0.000)	175.906	(0.000)	175.904	(0.000)
Independence Test of Residuals						
BDS(m=2,0.7)	5.334	(0.000)	5.334	(0.000)	5.334	(0.000)
BDS(m=3,0.7)	7.297	(0.000)	7.297	(0.000)	7.297	(0.000)
BDS(m=4,0.7)	8.461	(0.000)	8.461	(0.000)	8.461	(0.000)
BDS(m=5,0.7)	9.614	(0.000)	9.614	(0.000)	9.614	(0.000)
BDS(m=6,0.7)	10.713	(0.000)	10.713	(0.000)	10.713	(0.000)

The statistic LM(j) tests the null hypothesis of no autocorrelation of order j in the residuals; ARCH(j) tests the presence of Autoregressive conditional heteroscedasticity in the residuals; JB is the Jarque and Bera statistic to test the null hypothesis of normality in residuals; and BDS tests the null hypothesis of independence in residuals.



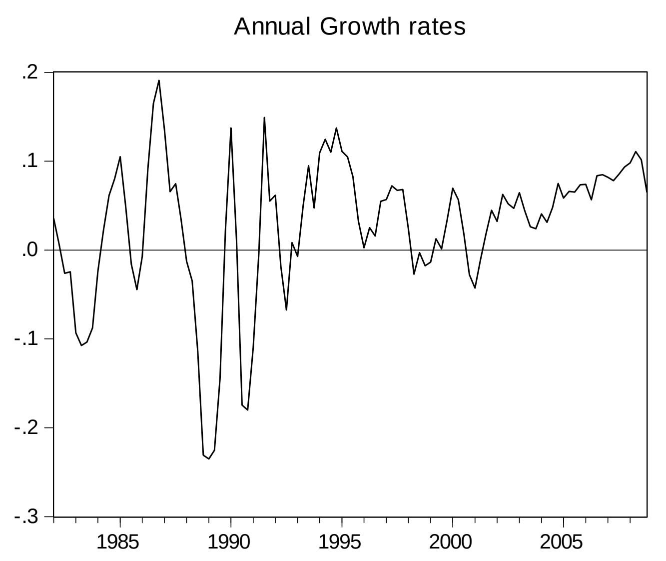


Figure 1. Logarithm of the Non-Seasonal Adjusted Real Output and Annual Growth Rates

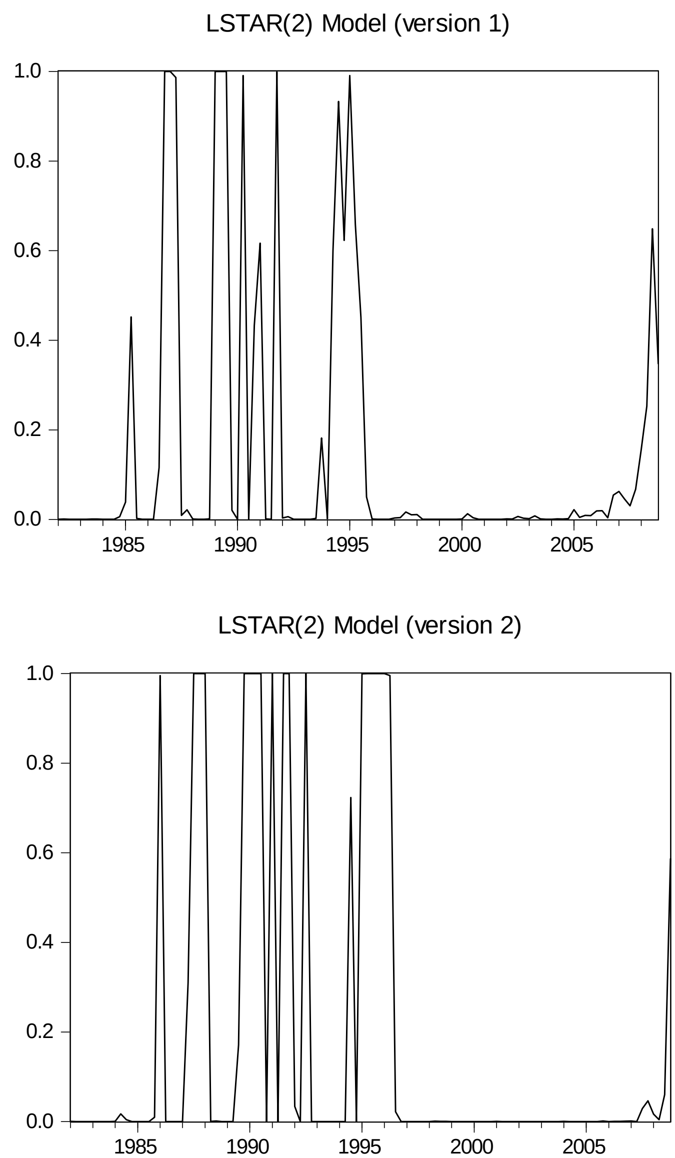
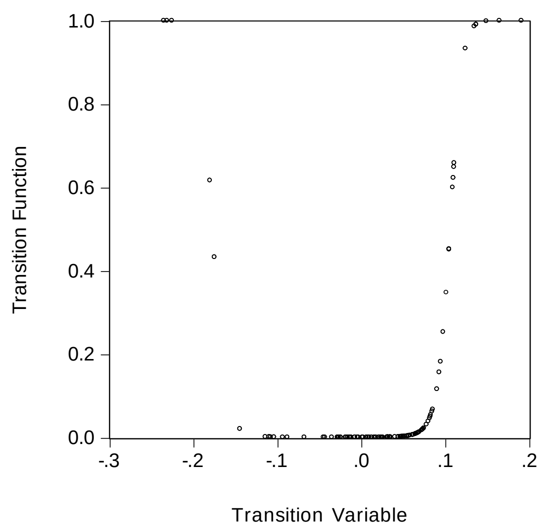
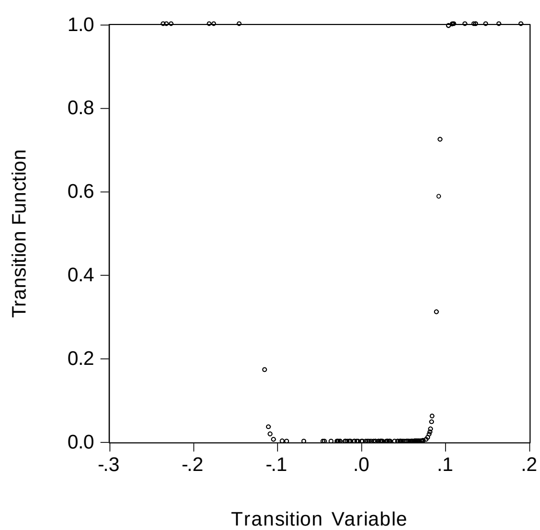


Figure 2a. Transition Function against Time; LSTAR(2) model with Transition Variable $\Delta_4 y_{t-1}$ (version 1) and LSTAR(2) model with Transition Variable $\Delta_4 y_{t-4}$ (version 2)

LSTAR(2) Model (version 1)





LSTAR(2) Model (version 2)

Figure 2b. Transition Function against Transition Variable; LSTAR(2) model with Transition Variable $\Delta_4 y_{t-1}$ (version 1) and LSTAR(2) model with Transition Variable $\Delta_4 y_{t-4}$ (version 2)

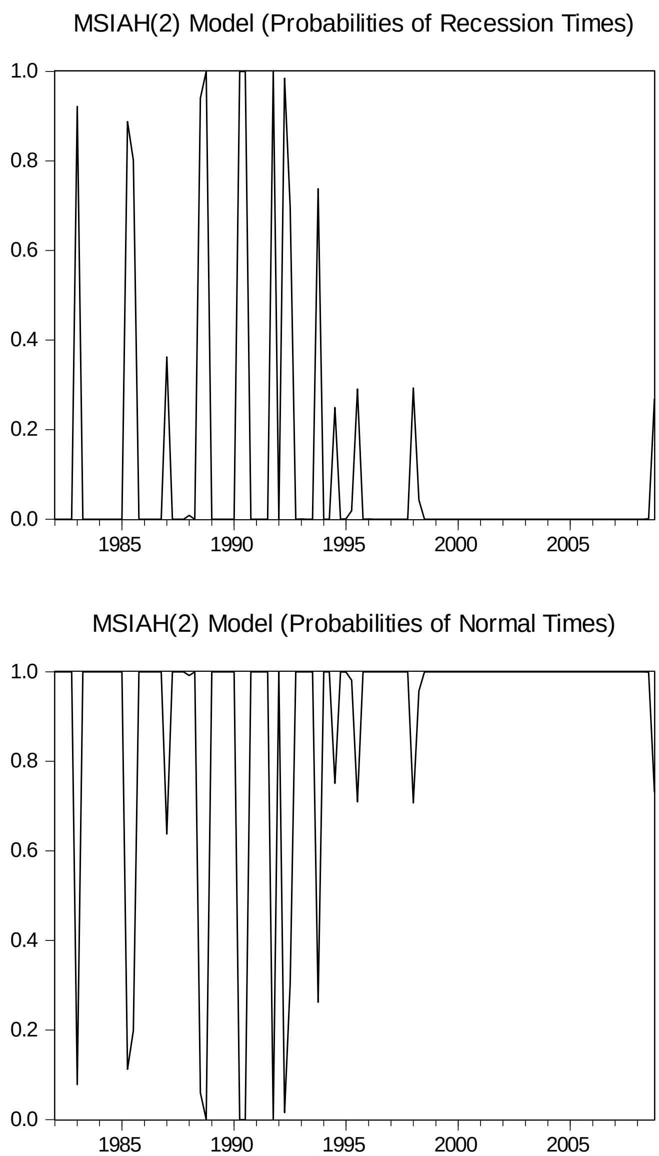
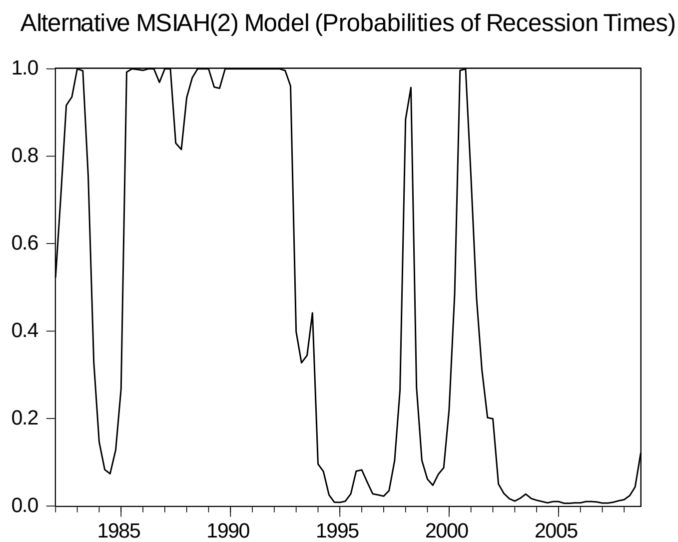


Figure 3a. Probabilities to be in Recession Times ($s_t = 1$) for the MSIAH(2) Model



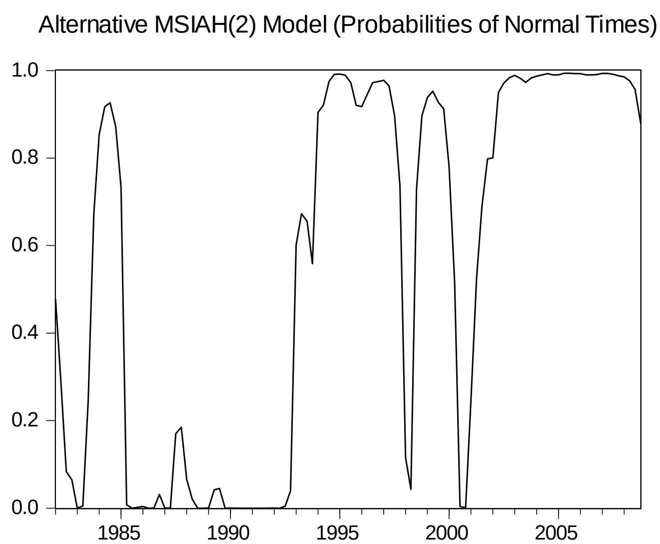


Figure 3b. Probabilities to be in Recession Times ($s_t = 1$)and Normal Times ($s_t = 2$) for the Alternative MSIAH(2) Model

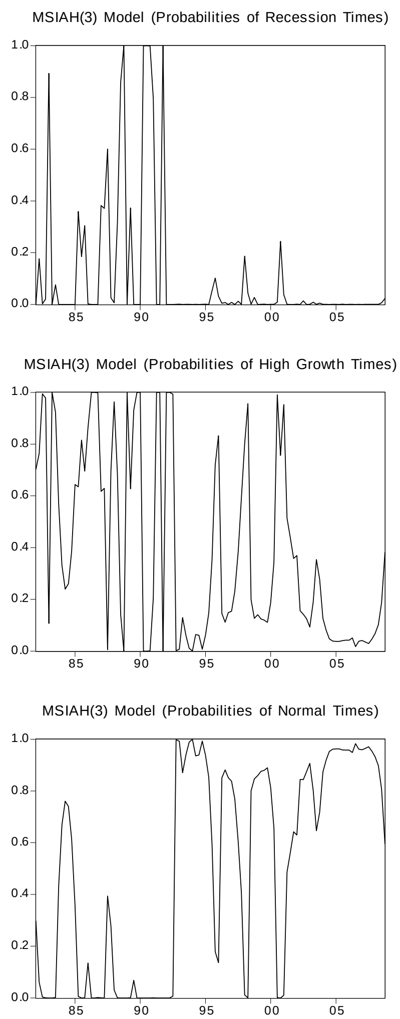


Figure 3c. Probabilities to be in Recession Times ($s_t = 1$), High Hrowth Times (($s_t = 2$), and Normal Times ($s_t = 3$) for the MSIAH(3) Model

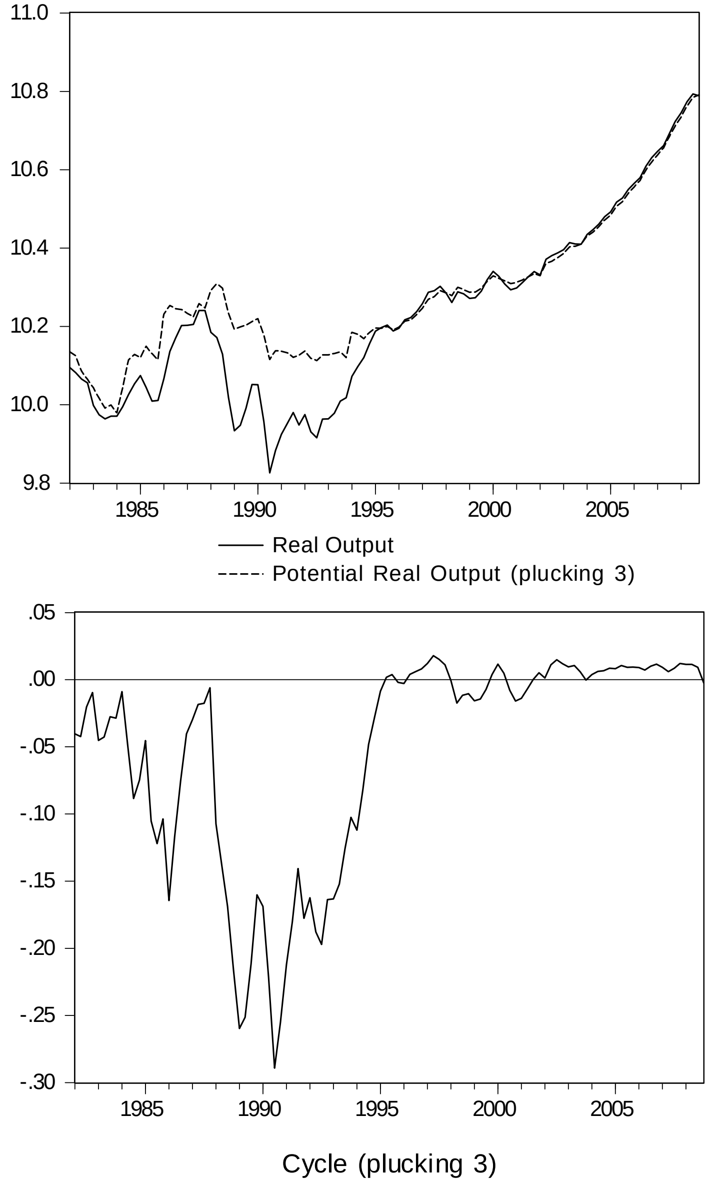


Figure 4a. Real Output and Potential Real Output of the selected Plucking Model (plucking 3); Transitory Component of the selected Plucking Model (plucking 3)

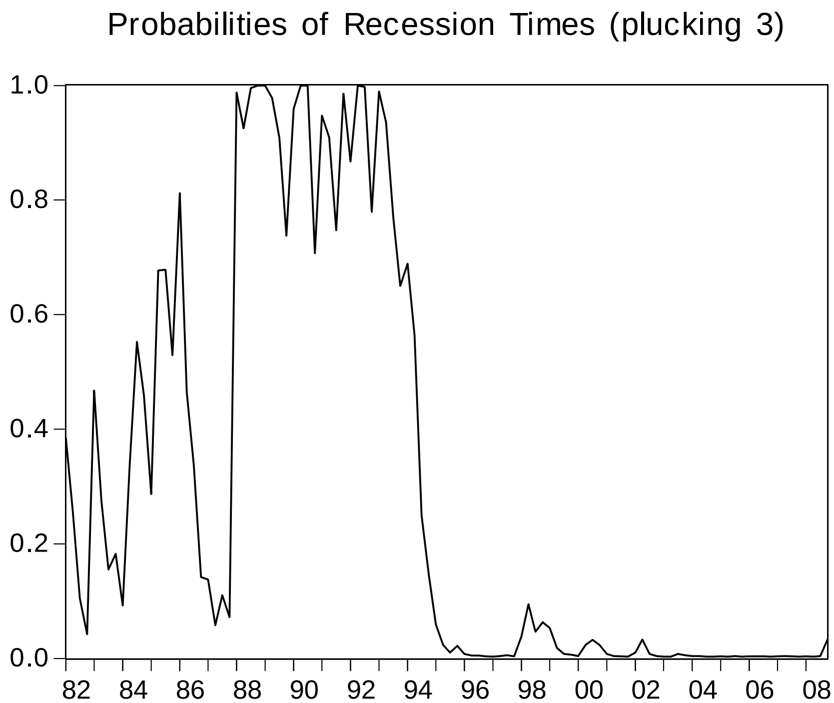


Figure 4b. Probabilities to be in Recession Times ($s_t = 1$) of the selected Plucking Model (plucking 3)