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How Investors Face Financial Risk: Loss Aversion and Wealth Allocation

by

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Abstract

We studied how the capital allocation decisions and the loss aversion of nonprofessional investors change subject to behavioral factors. The optimal wealth allocation between risky and risk-free assets results within a value-at-risk (VaR) portfolio model, which involves assessing risk individually according to an extended prospect-theory framework. We showed how the past performance and the portfolio evaluation frequency affect investor behavior and prove myopic loss aversion holds across different evaluation frequencies. We also illustrated that 1 year is the optimal evaluation horizon at which, under practical constraints, maximization of risky holdings occurs. Finally, we presented evidence that indicates that researchers using standard VaR significance levels may be underestimating the loss aversion of individual investors.

Keywords: Prospect Theory, Myopic Loss Aversion, Value-at-Risk, Portfolio Evaluation, Capital Allocation

Introduction

According to the top-down strategy evident in portfolio theory, portfolio optimization involves a threefold decision procedure:[3] The first step, the capital allocation decision, reflects the choice between risky and risk-free assets. The second step, the so-called asset allocation decision, includes further selection of different classes of risky assets. The third step, the security allocation decision, entails the selection of specific securities to be held within each particular risky asset class selected. In practice, professional portfolio managers usually make the last two decisions with no intervention from their nonprofessional clients. In contrast, in terms of the first decision, the input of the nonprofessional investors is necessary so that portfolio managers can determine the capital allocation that best fits the individual risk profiles of their clients.

In this paper, nonprofessional investors refer to those people whose main occupation does not concern financial investments and/or who lack the necessary knowledge, expertise, time, or any combination of these factors to make more sophisticated investment decisions preferring to rely on the help of professional portfolio managers in devising an optimal mix of risky assets[4]. Such a definition omits issues, such as investors' technical training, experience, and expertise, from the analysis. The focus of the paper is on the decision process of nonprofessional investors. Although of indisputable practical importance, the literature reflects a lack of research on this process. The extensive research on portfolio optimization illustrates more sophisticated details, such as choosing among different categories of risky assets and securities (asset and security allocation), usually the responsibility of professional portfolio managers.

One can think of the decision process of nonprofessional investors as unfolding in two main actions: First, they determine the total sum of money for investment in financial markets (in technical terms, the budget constraint)[5]. Second, to split this money optimally among different financial instruments, they ask for professional advice (i.e., they commit the technical details of the optimization of their portfolio to professional managers who dispose of sufficient resources to this end). Professional portfolio managers will then ask their clients to provide them with information about the level of risk they are ready to bear (the risk constraint)[6]. Acting on this information, portfolio managers finally derive the optimal capital allocation for their particular clients. In this context, what is important for nonprofessional investors is simply how the portfolio managers will split their wealth optimally between risky and risk-free assets[7].

In this paper, we extend the portfolio optimization setting in Campbell, Huisman, and Koedijk (2001), where risk is quantified in the form of value-at-risk (VaR)[8], by explicitly accounting for the formation of what we denote as the individual VaR (VaR*). Our VaR* relies on subjective perceptions of the nonprofessional investors and is formulated in line with the extended prospect theory in Barberis, Huang, and Santos (2001). This paper represents a descriptive (not a prescriptive) approach to the nonprofessional investors' capital allocation decisions.

Our contribution to current literature includes both a theoretical and an empirical component. We make two main theoretical contributions: First, we revisit the traditional capital allocation procedure, which delivers values of the optimal risky and risk-free investments without accounting for the intrinsic quantification of financial risk borne by investors. We introduce the concept of investors' subjective VaR*, which is contingent upon their loss aversion, the past performance of the risky portfolio, the current value of the risky investment, and the expected risk premium. We then show how VaR* flows into the portfolio optimization undertaken by the professional manager in the form of a risk constraint. Second, we introduce an extended measure, termed as the global first-order risk aversion (gRA), which provides additional information on the loss attitude of nonprofessional investors. The gRA indicates the sensitivity—in terms of first-order changes—of the prospective value (the perception of the utility generated by financial investments) to the variation of expected returns, as such presenting a more complete description of loss aversion than the simple coefficient of loss aversion (λ) of Kahneman and Tversky (1979) and Tversky and Kahneman (1992).

The empirical contributions of this paper come from the simulation of the theoretical part using Standard and Poor's (S&P) 500 index and U.S. 3-month treasury bills between 1982 and 2006. Past performance appears to drive the current perception of the risky portfolio. Investors allocate on average between 0% and 35% of their wealth to risky assets, where the main source of this substantial fluctuation is the portfolio evaluation frequency. The proportion of risky investments decreases quickly when investors check portfolio performance more often than once a year, which is in accordance with the notion of myopic loss aversion introduced in Benartzi and Thaler (1995).

In addition, assuming that only evaluation horizons of less than 1 year are of practical relevance, a twofold segmentation that depends on the portfolio evaluation frequency of both the prospective value and the global first-order risk aversion indicates 1 year as being optimal for generating positive attitudes towards risky investments. This result is in line with the empirical findings of Brunnermeier and Nagel (2008), who concluded that "households rebalance only very slowly following inflows and outflows or capital gains and losses" (p. 714). Finally, studying the equivalence between the traditional VaR approach and the estimates in our VaR* framework, such as equivalent significance levels, loss-aversion coefficients, and investments in risky assets, illustrates that the actual reluctance towards financial losses of nonprofessional investors might be underestimated.

The following section presents the main theoretical considerations. We start with a brief review of the optimal portfolio selection model with exogenous VaR* by Campbell et al. (2001) and then take on the value function formulation in Barberis et al. (2001). Next, we introduce the notion of VaR*. Finally, concentrating on how individual investors perceive the value of the risky portfolio, we derive the prospective value and introduce our extended measure of loss aversion. In the subsequent section, we illustrate the implementation of our theoretical model and discuss the impact of the evaluation frequency and of the past performance on the wealth percentages invested in the risky portfolio. In addition, we extensively investigate the evolution of the prospective value and of the extended loss-attitude measure subject to various evaluation frequencies. We further restate our model in terms of previous research with VaR risk constraints. The final section includes a summary of the results and the conclusion. Graphs and further findings appear in the Appendix.

Theoretical Model

The main theoretical considerations of our work appear in this section. On the one hand, we based our setting on the portfolio selection model of Campbell et al. (2001), which includes VaR as the measure of risk. On the other hand, we incorporated the individual perception of risky projects as captured in the extended prospect theory framework of Barberis et al. (2001). In this section, we detail the construction of our measure of individual loss aversion VaR* and its implications for the wealth allocation decisions of nonprofessional investors. We also add to the formal representation of the investors' attitude towards financial losses by introducing the notion of gRA and discussing how the evalu ation frequency of the risky performance influences this attitude.

Optimal portfolio selection with "exogenous" VaR

Campbell et al. (2001) developed a procedure that allows investors to choose among (risky and risk-free) financial assets to maximize the generated expected returns. The choice is subject to a twofold restriction: First, there is a limited budget for risky investments, although borrowing or lending extra money is possible at the fixed market interest rate[9] (the budget constraint); second, there is a maximum acceptable risk following from risky holdings (the risk constraint). This risk is quantified in the form of VaR. Specifically, the maximum expected loss from holding the risky portfolio should not exceed what we refer to as the exogenous VaR (VaR^{ex}).

In other words, VaR^{ex} stands for the risk level that the nonprofessional client is disposed to bear, communicated to the portfolio manager in the form of a single fixed number. In most portfolio optimization models and in particular in Campbell et al. (2001), managers do not account for how VaR^{ex} forms in the clients' perception. They consider VaR^{ex} a constraining threshold level, exogenous to the optimization problem[10].

The model of Campbell et al. (2001) delivers, first, the optimal weights w_p^{ott} of the risky portfolio component, and, second, the optimal investment in risk-free assets B_t. The latter stands for the optimal sum to be borrowed ($B_t >$ 0) or lent ($B_t < 0$) at the fixed risk-free gross return rate Rf and yields the following expression:

$$B_t = \frac{VaR^{ex} + VaR_t}{R_f - q_t(w_t^{opt}, \alpha)}, \tag{1}$$

Where $q_t(w_p^{ott},\alpha)$ represents the quantile of the distribution of portfolio gross returns $R_{t+1}(w_t)$ at the confidence level $1 - \alpha$ (or significance level α), and VaR_t is the portfolio VaR^{11}. Thus, the value of the risky investment at time t+1 can be written as follows:

$$S_{t+1} = (W_t + B_t) R_{t+1}, \tag{2}$$

where W_t is the initial wealth and Rt+1 the portfolio gross returns.

Because we consider that nonprofessional investors are mainly concerned with how to split their money between risky and risk-free assets, the optimal investments in risk-free and risky assets in Equations 1 and 2 represented fundamental variables in our model.

The individual loss level VaR*

Originating from the main ideas of the above portfolio allocation setting, we took our model a step further by asking how nonprofessional investors actually arrive at their desired level of loss aversion. In this context, we elaborated on the construction of an individual loss level, which we denoted as VaR*, and on its implications for the wealth allocation between risky and risk-free assets. Formally, we can think of VaR* replacing VaR^{ex} in the above optimization procedure, with respect to which it remains an exogenous constraint. However, the value of this risk constraint results from individual behavioral parameters. By explicitly accounting for its formation, we endogenized the risk measure VaR*[12].

The value function

Investors' desires and attitudes—and hence their subjective level VaR*—depend on their perception of the financial investments' value. The prospect theory (PT) in Kahneman and Tversky (1979) and Tversky and Kahneman (1992) illustrates how we can formalize individual perceptions of financial performance by means of the value function v[13]. Accordingly, human minds take for actual carriers of value not the absolute outcomes of a project but their changes, which are defined as departures from an individual reference point. The deviations above (below) this reference are the gains (losses). Thus, the value function is kinked at the reference point and exhibits distinct profiles in the domains of gains and losses, being steeper for losses (a property known as loss aversion). The value function also shows diminishing sensitivity in both domains (i.e., concave for gains but convex for losses).

Barberis et al. (2001) noted that past performance of risky investments could additionally influence individual perceptions. The past performance is evident in the cushion concept. Formally, the cushion corresponds to the difference between the current value of the risky investment S_t and a historical benchmark level of the risky value Z_t (e.g., the price at which the assets were purchased, a more recent value of the risky holdings, or a combination). When the difference is positive (negative), investors made money (registered losses) by investing in risky assets in the past.

Our approach depended on the extended formulation of the value function proposed in Equations 15 and 16 in Barberis et al. (2001). In the following, we refer to $x_t = R_{t+1} - R_{ft}$ as the risk premium, to $S_t - Z_t$ as the (absolute) cushion, and to $z_t = Z_t / S_t$ as the relative cushion. The positive (negative) past performance corresponds to a positive (negative) cushion that can be termed as $Z_t \leq S_t$ ($Z_t > S_t$) or equivalently as $z_t \leq 1$ ($z_t > 1$). The value function takes different courses depending on the past performance and can appear as follows[13]:

$$v_{t+1} = \begin{cases} v_{t+1}^{\text{prior gains}}, \; for \; S_t \geq Z_t \\ v_{t+1}^{\text{prior losses}}, \; for \; S_t < Z_t, \end{cases} \tag{3}$$

where

$$v_{t+1}^{\text{prior gains}} = \begin{cases} S_t x_{t+1} & , \text{for } S_t x_{t+1} + (S_t - Z_t) R_{ft} \geq 0 \\ \lambda S_t x_{t+1} + (\lambda - 1)(S_t - Z_t) R_{ft} & , \text{for } S_t x_{t+1} + (S_t - Z_t) R_{ft} < 0, \end{cases} \quad (4)$$

and

$$v_{t+1}^{\text{prior losses}} = \begin{cases} S_t x_{t+1} & , \text{for } x_{t+1} \geq 0 \\ \lambda S_t x_{t+1} + k(Z_t - S_t) x_{t+1} & , \text{for } x_{t+1} < 0. \end{cases} \quad (5)$$

The parameter λ in Equations 4 and 5 is the coefficient of loss aversion. According to PT, investors are loss averse when $\lambda > 1$, while $\lambda = 1$ points to loss neutrality. The parameter $k \geq 0$ captures the influence of previous losses on the perception of current ones: The larger the previous losses, the more painful the next losses become. We denoted the parameter as the sensitivity to past losses.

Note that the gain branches of both value functions in Equations 4 and 5 are invariable to the past performance z_t. The loss branches are distinct. However, they both contain a first term $\lambda S_t(R_{t+1} - R_{ft})$ that resembles the original PT but also a second revealing the impact of the cushion $S_t - Z_t$. In addition, the reference point shifts according to the past performance.

The derivation of VaR*

The risk-free investment from Equation 1 depends on, among others, the risk level indicated by the nonprofessional client to the portfolio manager. This level is the result of individual perceptions of financial losses, which, in line with PT, can substantially differ from the actual losses. In this section, we define a new measure of the individually accepted (or desired) loss level that we denoted as VaR*.

We started from the literal definition of VaR*, which is the maximum loss that an individual investing in risky assets can expect a priori. We concentrated on the terms *loss, individual,* and *maximum* encompassed by this definition. First, VaR* quantifies losses. According to PT, what actually counts for individual (nonprofessional) investors is not the absolute magnitude of a loss but rather the subjectively perceived magnitude, as captured by the value function described above. Hence, VaR* should rely on the subjective value of losses expressed in the loss branches of the value functions in Equations 4 and 5. VaR* thus depends on individual features, originating in the subjective view, over gains and losses and can vary over time, for instance with the past performance of risky investments. Moreover, we are looking for a maximal value such that, in calculating VaR*, investors must ascribe a maximal occurrence probability to the losses in the value function. Finally, VaR* should correspond to the concept of VAR and hence represent a quantile, namely, according to the above considerations, a quantile of the subjective loss distribution.

Therefore, we suggest the following formal definition for the individual loss level:

$$\begin{aligned} VaR_{t+1}^{*} &= E_t\left[loss-value_{t+1}\right] - \varphi\sqrt{Var_t\left[loss-value_{t+1}\right]} \\ &= \lambda S_t E_t\left[x_{t+1}\right] + \left(\zeta_t(\lambda-1)R_{ft} + (\zeta_t - 1)kE_t\left[x_{t+1}\right]\right)(S_t - Z_t), \end{aligned} \quad (6)$$

where loss-value stands for the subjective value ascribed to financial losses according to the loss branch of the value functions in Equations 4 and 5, and the subjectively perceived losses are assumed to follow a distribution (e.g., normal or Student's *t*) with the lower quantile φ. Moreover, $E_t[x_t+1] = E_t[R_t+1] - R_{ft}$ denotes the expected risk premium. Obtaining the last expression in Equation 6 requires using the simplifying notation

$$\zeta_t = \sqrt{\pi_t(1-\psi_t)}\left(\sqrt{\pi_t(1-\psi_t)} - \varphi\sqrt{1-\pi_t(1-\psi_t)}\right)$$

Once the nonprofessional investors decide on a desired VaR*, they communicate it to the portfolio manager. The client indication represents an exogenous risk level that corresponds to VaR^{ex} in Equation 1 and is applied to determine the optimal level of borrowing or lending B_t. When VaR* is lower in absolute value than the portfolio VaR, B_t is negative, which formalizes the profile of more risk-averse investors who prefer to increase the proportion of wealth invested in risk-free assets. In contrast, for a VaR* higher than VaR in absolute value, investors augment their risky investments by borrowing extra money (i.e., they are less risk averse). Thus, analyzing the evolution of B_t (or equivalently of S_t/W_t, as conducted in the subsequent application) can shed some light on the behavior of nonprofessional investors confronted with financial losses.

A further interesting topic of investigation lies in estimating the equivalent loss aversion coefficient λ*t that can be obtained for a fixed $\overline{VaR^*}$ under the traditional approach[16]. Common assumptions of this approach are significance levels of 1% , 5%, or 10% and no dependency on past performance $k = 0$. The formula of λ* is then immediate from the definition of VaR* (see Equation 6) and yields the following[17]:

$$\lambda_{t+1}^{*} = \frac{\overline{VaR^*} + \varsigma_t R_{ft}(S_t - Z_t)}{S_t E_t[x_{t+1}] + \varsigma_t R_{ft}(S_t - Z_t)} \quad (7)$$

The prospective value of risky investments

The estimation of the individually maximum acceptable loss level VaR* represents only the first step in our analysis. The step dictates the optimal choice of the nonprofessional investors in terms of wealth percentages allocated between risky and risk-free assets. We were also interested in the attitude of nonprofessional investors towards financial losses in general. This attitude influences the level of the individual VaR*. The attitude results from the perception of the utility generated by financial investments[18]. The corresponding PT concept of (subjectively) expected utility is the so-called prospective value V. In our framework, we formulated the prospective value of the risky portfolio as the following probability-weighted sum of expected risky performance[19]:

$$
\begin{aligned}
V_{t+1} &= \pi E_t\left[v_{t+1}^{\text{prior gains}}\right] + (1-\pi_t) E_t\left[v_{t+1}^{\text{prior losses}}\right] \\
&= \left(\pi_t \psi_t + (1-\pi_t)\omega_t + (\pi_t(1-\psi_t) + (1-\pi_t)(1-\omega_t))\lambda\right) S_t E_t\left[x_{t+1}\right] \\
&+ \left(\pi_t(1-\psi_t)(\lambda-1)R_{ft} - (1-\pi_t)(1-\omega_t)kE_t\left[x_{t+1}\right]\right)(S_t - Z_t).
\end{aligned} \tag{8}
$$

We can further decompose the prospective value into two parts, the expressions of which are apparent in Equation 8: The first part captures the expected risky value relative to the safe bank investment $StEt[xt+1]$, subsequently denoted as the PT effect. The second part covers the influence of the cushion $S_t - Z_t$, which we referred to as the cushion effect.

We introduced a further notion referring to the investors' attitudes towards financial risks to capture more complex dependencies than the simple coefficient of loss aversion λ. According to PT, loss aversion corresponds to risk aversion of the first order in the loss domain. In the same spirit, we termed the first derivative of the prospective value with respect to the expected risk premium as global first-order risk aversion (gRA), which yields the following[20]:

$$
\begin{aligned}
gRA_t &= \frac{\partial V_{t+1}}{\partial E_t\left[x_{t+1}\right]} \\
&= \left(\pi_t \psi_t + (1-\pi_t)\omega_t + (\pi_t(1-\psi_t) + (1-\pi_t)(1-\omega_t))\lambda\right) S_t - (1-\pi_t)(1-\omega_t)k(S_t - Z_t)
\end{aligned} \tag{9}
$$

Thus, gRA indicates the sensitivity—in terms of first-order changes—of the prospective value to the variation of expected returns (or equivalently to the expected risk premium)[21]. Because gRA directly reflects changes in the prospective value, which is proportional to the attractiveness of financial investments, higher gRA values point to a more relaxed loss attitude.

The impact of portfolio evaluation frequency

In line with the concept of myopic loss aversion (mLA), introduced in Benartzi and Thaler (1995), frequent evaluations of financial performance (i.e., myopia or narrow framing[22]) and the reluctance to experience losses can dramatically affect the risk perception, and hence the subjective desirability of risky investments emerges[23]. We were interested in testing for mLA in our framework and, more generally, in observing how wealth allocation decisions and loss attitudes vary at different portfolio evaluation frequencies. To this end, the subsequent application section reflects an examination of how the wealth allocation to risky and risk-free assets given by S_t and B_t, the prospective value V, and the extended measure of the loss attitude gRA change at various evaluation horizons τ or, equivalently, at various evaluation frequencies 1/τ. In particular, we worked with τ values ranging from 1 day to 8 years, with a focus on short durations, which we considered more plausible in practice.

The evaluation frequency affected our variables, and hence investors' decisions and attitudes, in two ways: First, through expected returns, which are themselves directly influenced by the evaluation frequency (the direct transmission mechanism). Second, through past returns, which influence several model variables (such as the cushions, the past and current gain probabilities, etc.) to be indirectly dependent on the evaluation frequency (the indirect transmission mechanisms).

Theoretically, we could study the direct dependence (i.e., on returns) by holding all model parameters, besides current return expectations, invariable to the evaluation frequency. However, this is technically impossible because the evaluation frequency indirectly affects multiple other parameters. Nevertheless, by eliminating the current returns, we can discard the direct effect because gRA represents by definition a derivative with respect to expected returns, where the direct impact is no longer contained. Consequently, studying how the prospective value and gRA vary with respect to the evaluation frequency amounts to examining the total and the indirect mechanism, respectively, evident in the application section.

The application section will present the analysis of a further related issue: Given that the portfolio evaluation frequency appears to affect investor perceptions of financial losses (and thus the level of risky investments), could the reverse causality hold as well? In other words, for a certain loss aversion value (at time t), we wanted to determine whether an evaluation frequency existed that was optimal in the sense that it led to the most relaxed attitude towards risky investments. If so, financial advisors, whose interest is to attract clients to raise capital, could recommend that their clients undertake performance checks with this optimal frequency to maximize their risky investments and the budget at the advisors' disposal[24]. We searched for the optimal evaluation frequency τ* in terms of the maximization of first the perceived risky value V(τ*) and second the loss acceptance gRA(τ*).

Application

We implemented the above theoretical results using daily values of the S&P 500 index (corrected for dividends and stock splits) and of U.S. 3-month treasury bill (T-bill) nominal returns. These two financial instruments, the stock index and the T-bill, served as proxies for the risky and the risk-free investment, respectively. Both data series ranged from 01/02/1962 to 03/09/2006 (11,005 observations). Descriptive statistics are apparent in Table 1.

Table 1
Log Difference of the S&P 500 Index

	S&P 500		3-month T-bill	
	Evaluation frequency		Evaluation frequency	
	Quarterly	Yearly	Quarterly	Yearly
Mean	0.017	0.066	0.017	0.073
Median	0.018	0.071	0.017	0.070
Std Dev	0.079	0.136	0.006	0.026
Kurtosis	2.661	-0.9659	0.623	0.974
Skewness	-0.671	-0.205	0.951	1.042
Max	0.290	0.345	0.036	0.142
Min	-0.302	-0.207	0.009	0.037
Obs	175	43	175	43

Note. The table presents descriptive statistics of the log-difference R_t of the S&P 500 index, corrected for dividends and stock splits, and of the 3-month T-bill log-returns R_f for quarterly and yearly portfolio evaluations. The first index formed a proxy for the risky investment, and the second one reflected a proxy for the risk-free investment. The data series ranged from 01/02/1962 to 03/09/2006.

Because of the financial reform in 1979, which significantly changed the trading conditions, the early 1980s marked the beginning of a new era of financial markets. We therefore reckoned that only the second part of the data would be relevant for inferring current market evolutions and divided our sample into two parts: The active data set (from 03/01/1982[25] to 03/09/2006, reflecting 6,010 observations) and the inactive data set (the first part of the sample from 01/02/1962 to 03/01/1982). The basis of the subsequent investigations was the active set, while the previous observations provided a basis for estimating the empirical mean and the standard deviation of the portfolio returns at date zero of trade (03/01/1982). The data contain an outlier, corresponding to the October 1987 market crash, which may have distorted the results. Because the market data served in our work merely as support for simulating trading behaviors, which we viewed as more general, we smoothed the outlier by replacing it with the mean of the 10 before and after data points[26].

We considered that nonprofessional investors perceive risky investments according to the value functions in Equations 4 and 5 and calculated their maximum expected loss level according to Equation 6. The active data set allowed us to run the model based on the previous sections and to derive the desired VaR* as well as the wealth proportion invested in the risky portfolio (i.e., in the S&P 500 index). The remaining money was assumed to be automatically invested in the risk-free 3-month T-bill. Moreover, we assumed that investors started trading with an even initial wealth allocation between the risky portfolio and the risk-free asset[27]. We also took the number of investors to be constant (i.e., no investors entered or exited the market during the trading interval)[28].

We constructed daily, weekly, monthly, up to 11 months (increasing a month at a time), and yearly as well as further lower frequency returns ranging from 1 to 8 years (with a 1-year increment). The case relied on values considered in Barberis et al. (2001) for the loss-aversion coefficient and the sensitivity to past losses, namely $\lambda = 2.25$ and $k = 3$[29]. The expected portfolio gross returns were taken to be the unconditional mean returns until the last date before the decision time[30].

The VaR* and the evolution of the risky investment

We started by analyzing how risky investments, in terms of percentages of total wealth, develop subject to different portfolio evaluation frequencies and to distinct ways of assessing the cushion. According to Benartzi and Thaler (1995), loss-averse investors, who evaluate the performance of their portfolios once a year and employ linear value functions with conventional PT parameter values, give rise to a market evolution that can explain the equity return premium observed in practice. As in our framework, past risky performance, among other factors, affected perceptions, but we were also interested in how different ways of assessing the cushion contribute to determining the amount of wealth to be invested in risky versus risk-free assets, in particular at different evaluation frequencies. To this end, we applied two cushion definitions: myopic and dynamic cushions[31].

In calculating myopic cushions, we fixed the benchmark level of past performance to be identical to the last-period risky holdings $Z_t = S_{t-1}$, so that the myopic cushion expression yields S_t - S_{t-1}. The basis of the dynamic cushions was Equation 18 in Barberis et al. (2001), which assumes a more complicated benchmark formula, in particular $Z_t = \eta Z_{t-1} \overline{R} + (1 - \eta) S_t$. Hence, the dynamic cushion results in $\eta(S_t - Z_{t-1}\overline{R})$ where the parameter η measures how far in the past the investors' memory stretches[32]. In line with Barberis et al., we subsequently concentrated on the case where $\eta = 0.9$[33]. We moreover took the variable in the definition of the dynamic cushion as the mean gross return[34].

First, we determined the portfolio VaR in Equation 13 (see the Appendix) for gross returns of the risky portfolio that are either (standard) normally distributed or Student's *t* (with 5 degrees of freedom) distributed and for a significance level of 5%. We took the probabilities π_t, ψ_t, and ω_t from Equation 14 (see the Appendix) to be the empirical frequencies of the cases where $z_t \leq 1$ (i.e., past gains), $x_{t+1} + (1 - z_t)R_{ft} \geq 0 | z_t \leq 1$ (a premium acceptable under a history of gains), and $x_{t+1} \geq 0 | z_t > 1$ (a positive premium conditional on the cases with past losses) respectively. We derived VaR* according to Equation 6 using either myopic or dynamic cushions. We then plugged this value into Equation 1 to determine the optimal level B_t of borrowing ($B_t > 0$) or lending ($B_t < 0$), which depends on the degree of loss aversion of nonprofessional investors.

Table 2 indicates the averages of the wealth percentages S_t/W_t invested in the risky portfolio, for both myopic and dynamic cushions and normally distributed and Student's *t* distributed portfolio gross returns R_t, at different portfolio evaluation horizons τ up to 1 year. We derived the current value of the risky investment S_t using Equation 2.

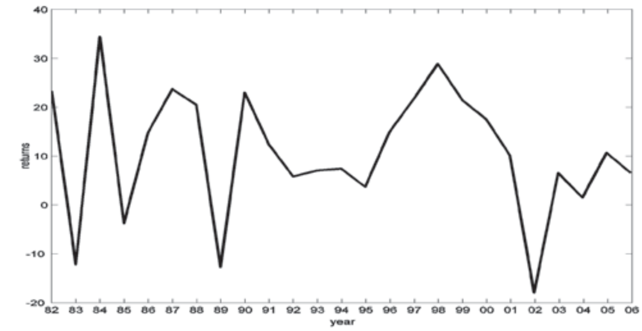
Table 2
Average Wealth Percentages Invested in S&P 500

	Myopic cushion		Dynamic cushion	
	Portfolio return		Portfolio return	
Evaluation frequency	Normal	Student's *t*	Normal	Student's *t*
1 year	34.51	25.79	30.50	24.48
6 months	20.23	15.67	19.92	16.08
4 months	16.96	13.23	16.30	13.16
3 months	13.42	10.55	13.00	10.52
1 month	7.70	6.21	7.69	6.29
1 week	3.85	3.13	3.85	3.15
1 day	1.90	1.55	1.90	1.56

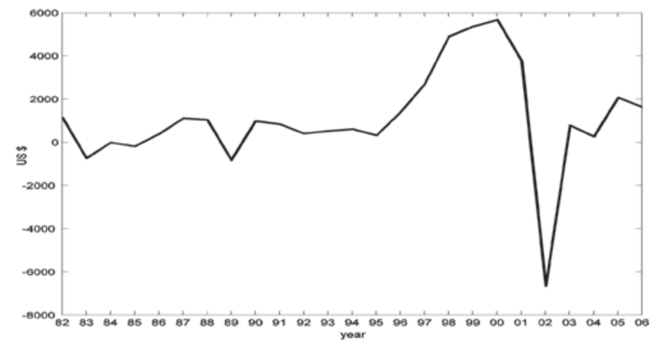
Note. The table presents the average wealth percentages S_t/W_t invested in the risky portfolio at different evaluation horizons τ up to 1 year for both myopic cushions $S_t - S_{t-1}$ and dynamic cushions $\eta(S_t - Z_{t-1}\overline{R})$ and standard normal and Student's *t* with 5 degrees of freedom distributed portfolio gross returns R_t. Other parameter values used included $\lambda = 2.25$, $k = 3$, $\eta = 0.9$, and $\overline{R} = mean\ (R_t)$.

Our nonprofessional investors allocated from almost no money to over 30% of their wealth to risky assets. The main cause of the substantial fluctuation of these percentages is the frequency at which investors evaluated risky performance. More frequent checks entailed lower investments in the risky portfolio, irrespective of the way in which the investors accounted for past performance (i.e., of the cushion type). This result is consistent with previous findings on mLA, such as Benartzi and Thaler (1995) and Barberis et al. (2001). Loss-averse investors, who perform high frequency evaluations and narrow-frame financial projects by overly focusing on a long series of past performances, become extremely loss averse.

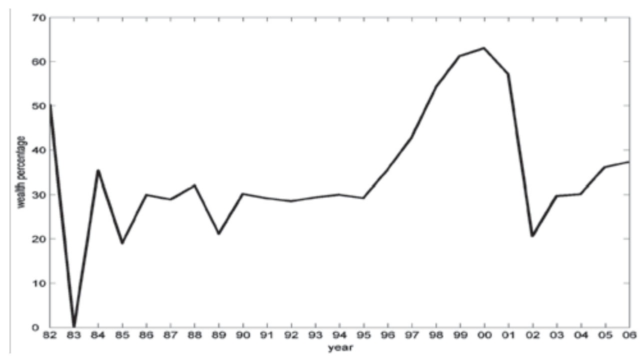
At the evaluation horizon of 1 year, nonprofessional investors who dynamically assessed cushions appeared to be more loss averse than did their myopic peers and allocated less money to the risky portfolio. However, the difference becomes negligible at higher evaluation frequencies. Moreover, independent of the cushion type, the investors' reluctance towards risky investments is higher for normally distributed than for Student's *t* distributed portfolio gross returns.



(A) Yearly S&P 500 Log Returns



(B) Yearly Myopic Cushions



(C) Yearly Wealth Percentages Invested in S&P 500

Figure 1. Evolution of risky returns, myopic cushions, and wealth percentages invested in the risky portfolio for yearly portfolio evaluations.

Note. The figure illustrates the annual log returns R_t of the index S&P 500, the corresponding yearly evolution (in US $) of the past performance encompassed by the myopic cushion $S_t - S_{t-1}$, and the resulting yearly wealth percentages S_t/W_t invested in the risky portfolio. We obtained the wealth percentages from Equation 1, where VaR^{ex} is replaced by the VaR* values from Equation 6 and the risky investment S_t results from Equation 2. We assumed $R_t \sim N(0,1)$, $E_t\ [R_{t+1}] = \underset{s=0,\ldots,t}{mean}\ [R_s]$, $\lambda = 2.25$, and $k = 3$. The sample included 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to March 1 of each year.

Because our VaR* is a VaR-type measure and because VaR is an adequate market-risk quantifier for normal distributions[35], we focused on the case with normally distributed returns. Next, let us concentrate on the magnitude of the cushion effect and its evolution over time. Due to lack of space, we will only present the results for myopic cushions[36]. To this end, we fixed the evaluation horizon at 1 year and plotted in Figure 1 the annual returns of the index S&P 500, the evolution of the myopic cushion generated by a series of past gains or losses, and the resulting yearly wealth percentages invested in the risky portfolio. Figure 1 illustrates a positive correlation of the three variables (returns, cushions, and risky investments).

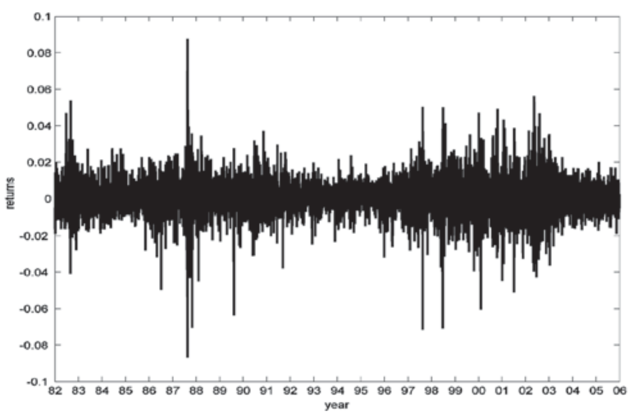
In line with the idea that loss aversion is sensitive to past performance, Panel C of Figure 1 indicates that the lower the cushions, the more loss averse investors become because they dispose of less backup for later contingent losses.

At this point, a further interesting empirical question arises: After how long does an investor performing frequent evaluations quit the risky market? Figure 2 shows the dramatic effect of high evaluation frequencies for investors who act upon myopic cushions (see Panel C). Specifically, nonprofessional investors who checked their portfolio performance every single day invested less than 5% of their wealth in risky assets. Each day can bring substantial changes to the perceived past performance. Therefore, although nonprofessional investors do not completely quit the risky market, they keep their risky holdings at very low levels during the entire trading interval.

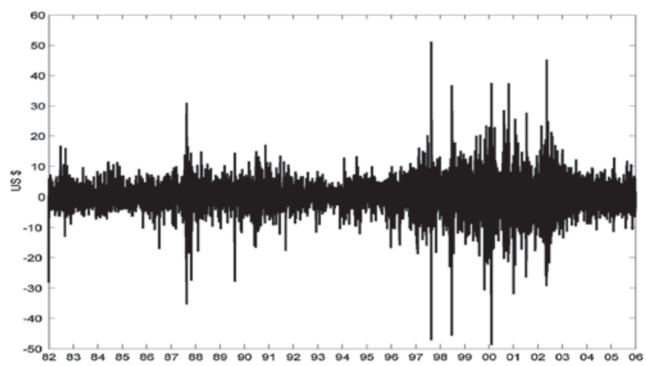
The evolution of the prospective value of risky investments

Next, our attention turns to the prospective value (i.e., the subjective utility ascribed by individual investors to the risky portfolio). The focus was on the influence of the evaluation frequency, which is twofold: direct (i.e., through the expected returns and thus the expected risk premium) and indirect (i.e., through other model parameters influenced by past returns), such as the cushion or the probabilities of past and current gains and losses. Thus, the prospective value sheds light on the total impact of the evaluation frequency on investors' behavior. Henceforth, we refer to the descriptions of variables that depend on the frequency at which investors check the risky performance as representations in the evaluation frequency domain.

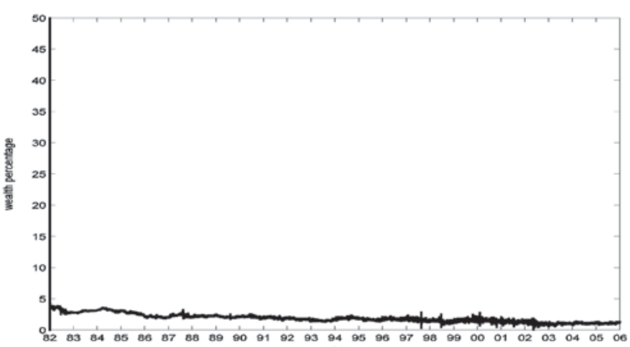
First, we briefly comment on the time evolution of the prospective value V and its two components, the cushion effect and the PT effect. Figure 3 illustrates these variables for myopic cushions and evaluation horizons of



(A) Daily S&P 500 Returns



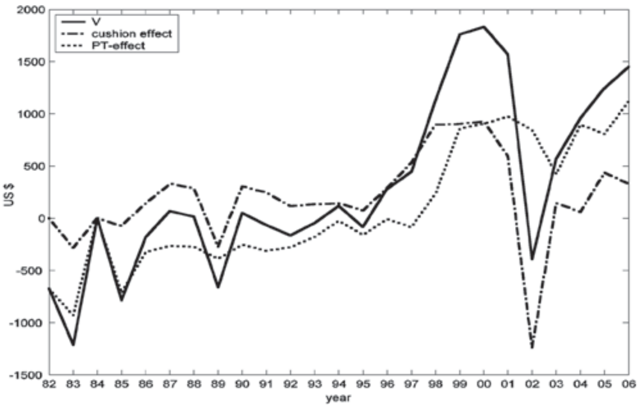
(B) Daily Myopic Cushions



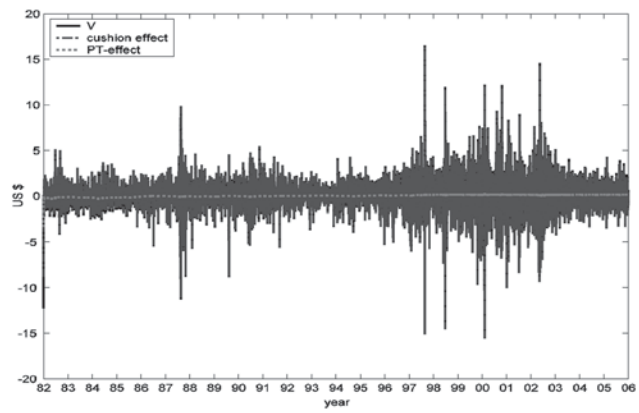
(C) Daily Percentage Investments in S&P 500

Figure 2. Evolution of risky returns, myopic cushions, and percentages invested in the risky portfolio for daily portfolio evaluations.

Note. The figure illustrates the annual log returns R_t of the index S&P 500, the corresponding daily past performance (in US $) encompassed by the myopic cushion $S_t - S_{t-1}$, and the resulting daily wealth percentages S_t/W_t invested in the risky portfolio. We obtained the wealth percentages from Equation 1, where VaR^{ex} is replaced by the VaR* values from Equation 6 and the risky investment S_t results from Equation 2. We assumed $R_t \sim N(0,1)$, $E_t[R_{t+1}] = \underset{s=0,\ldots,t}{mean}[R_s]$, $\lambda = 2.25$, and k = 3. The sample included 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to March 1 of each year.



(A) Yearly Evaluations



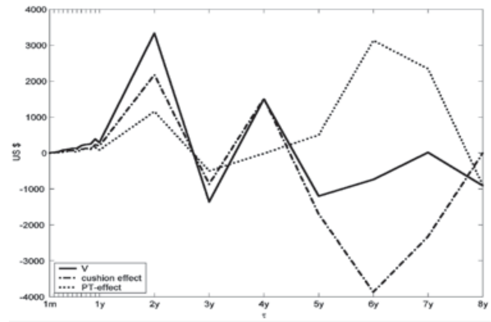
(B) Daily Evaluations

Figure 3. Prospective value evolution for yearly and daily evaluations.

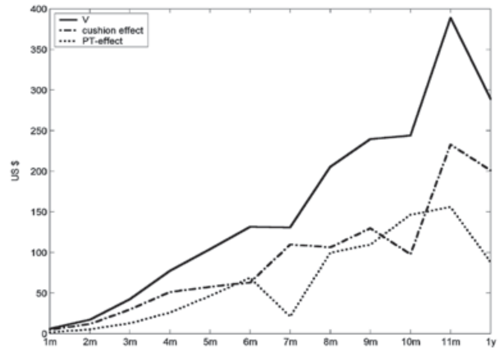
Note. The figure illustrates the yearly and daily evolution of the prospective value V (in US \$) from Equation 8 and, of its two components, the PT effect and the cushion effect, which correspond to the two terms in this equation. The PT effect corresponds to the representation of V in PT, without accounting for the influence of past performance, which is encompassed by the cushion effect. We assumed myopic cushion $S_t - S_{t-1}$, $R_t \sim N(0,1)$, $E_t\ [R_{t+1}] = \underset{s=0,\dots,t}{mean}\ [R_s]$,$\lambda = 2.25$, and k = 3. The sample included 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to March 1 of each year.

1 year and 1 day. At both frequencies, as long as cushions were sufficiently high in absolute value, the cushion effect dictated the shape of the prospective value. This lead role is even more evident for daily evaluations where the expected return premium was very small and hence the PT effect weak[37].

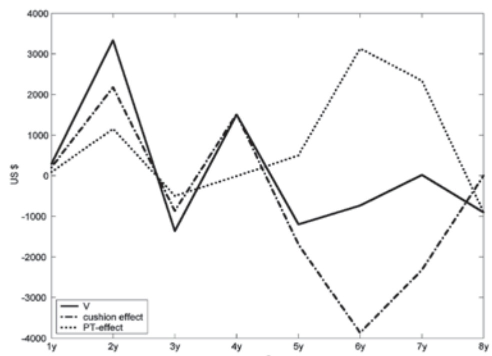
In Figure 4, Panel A, we plotted the prospective value and its two components again but now as functions of the



(A) All Frequencies



(B) $\tau \leq 1$ Year



(C) $\tau \geq 1$ Year

Figure 4. Prospective value evolution for different evaluation frequencies.

Note. The figure illustrates the prospective value *V* (in US \$) from Equation 8 and its two components, the PT effect and the cushion effect, as functions of the portfolio evaluation horizon τ. *V* reflects the perceived utility of risky investments and captures the total impact of τ on investor behavior (through expected returns and other model variables). The PT effect corresponds to the representation of *V* in PT, without accounting for the influence of past performance, which is encompassed by the cushion effect. Higher *V* values indicate an increased utility of risky investments as perceived by nonprofessional investors. Panel A depicts the evolution of *V* for all evaluation horizons, Panel B reflects a focus on horizons up to 1 year (in monthly increments), and Panel C exhibits horizons from 1 to 8 years (in yearly increments). We assumed myopic cushion $S_t - S_{t-1}$, $R_t \sim N(0,1)$, E_t, $[R_{t+1}] = \underset{s=0,\dots,t}{mean}\ [R_s]$, $\lambda = 2.25$, and k = 3. The sample included 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to March 1 of each year.

evaluation horizon τ. This horizon ranges from 1 month to 8 years, where we considered monthly increments of up to 1 year and yearly increments thereafter[38].

The findings show that up to 2 years, the perceived attractiveness of financial investments increases with the evaluation horizons. This tendency is consistent with mLA and characterizes the evolution of both the PT effect and the cushion effect at higher evaluation frequencies. In particular, the PT effect is upward sloping across all considered evaluation frequencies, which supports the coherency of mLA within the framework initially suggested in PT.

However, the prospective value yielded even negative values for higher evaluation horizons (such as 3, 5, or 6 years). The cause was the leading role of the cushion effect and the fact that for lower evaluation frequencies, the cushion values are negative and sufficiently high to counterbalance the PT effect and to reduce the perceived value of risky investments dramatically[39]. In short, checking risky performance less often than once every 1 or 2 years appears to deteriorate the perception of the utility of risky investments.

Indeed, as documented in Benartzi and Thaler (1995) a decade ago, in practice, investors used to perform yearly portfolio checks. Nowadays, due to the wealth of information available at almost no cost and due to the enhanced dynamics of market events, investors may reconsider financial decisions more often. However, 1 year remains as an important anchor in investors' minds given that various events (e.g., release of annual activity reports, taxes, etc.) take place with this frequency. In addition, nonprofessional investors may not be sufficiently impatient (perhaps because they do not dispose of time, financial resources, knowledge, experience, or a combination of these factors) to perform much more frequent portfolio checks. In our opinion, nonprofessional investor perceptions reasonably rely on evaluation horizons of 1 year and less.

Based on these ideas, we delimited two distinct segments of the prospective value in the evaluation horizon domain depicted in Figure 4. The segments meet at the critical frequency of 1 year and reflect different evolutions. We denoted the segment with evaluation horizons lower than 1 year as the left segment, and because we view this segment as the only one relevant in practice, we concentrated on it in our analysis. The part of the prospective value in the frequency domain encompassing evaluation horizons higher than 1 year is the right segment. Figure 4 (Panels B and C) illustrates the two segments separately, for myopic cushions.

In the left segment, the perceived risky value appears to increase on average with the evaluation horizon. In effect, describing the curve $V(\tau)$ in Panel B of Figure 4 as a polynomial of the first order would be accurate[40]. Accordingly, the subjectively perceived utility of the nonprofessional investors, captured by the prospective value, should be maximized at the highest frequency of this domain, which is once a year[41]. Henceforth, we designate 1 year as the optimal evaluation horizon with respect to minimizing loss aversion and therefore maximizing risky investments $V(\tau^* = 1 \text{ year}) = \max$.

In the same spirit, the lowest considered evaluation horizon of 1 day entails a minimal expected value of the risky portfolio, pushing investors to step out of the risky market and to allocate (almost) all their money to risk-free assets. In other words, loss-averse investors should check the performance of their risky investments as seldom as possible to maximize the corresponding prospective value of their investments. Under the practical informational constraints that govern financial markets nowadays, 1 year appears to be the most reasonable evaluation time that would increase the perceived returns of risky investments.

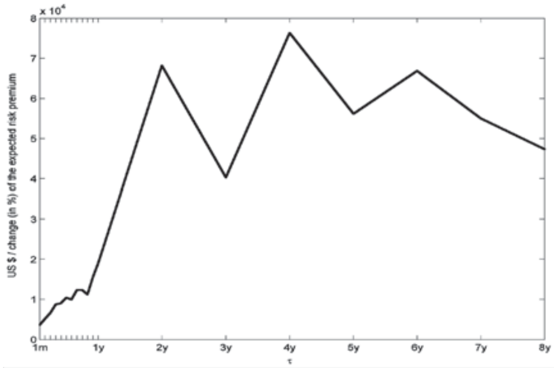
The evolution of the loss attitude

We extended the analysis in the frequency domain to our new measure of loss attitudes gRA. In doing so, we studied the indirect transmission mechanism of the evaluation frequency to capital allocation decisions. Because V in our model is linear in the expected risk premium, its first derivative gRA does not contain any direct influence of the evaluation frequency. The variation of gRA captures the indirect impact of τ on other model parameters.

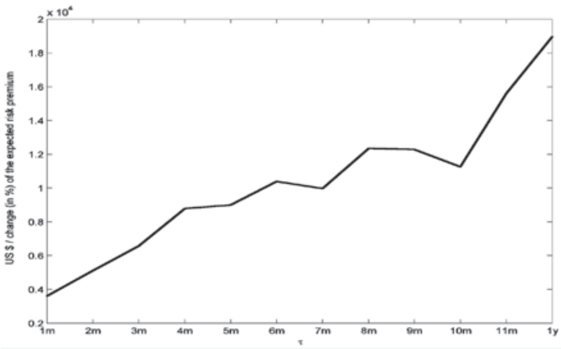
Panel A (see Figure 5) illustrates the gRA course for myopic cushions and evaluation frequencies ranging from 1 month to 8 years[42]. On average, gRA appears to increase with the evaluation horizon, pointing to a more relaxed attitude towards financial losses as investors check portfolio performance less often. Note that the result occurs at all frequencies and not only in the left segment, as was the case for the prospective value. Thus, while the impact of the evaluation frequency on the loss perception can be ambiguous in a context considering both direct and indirect transmission mechanisms, the indirect mechanism consistently supports the concept of mLA[43].

As with the prospective value, we considered a segmentation of gRA around 1 year (see Panels B and C in Figure 5). In the left segment (Panel B), simple lines appear to fit the data well[44]. Our measure gRA attained its maximum for the lowest frequency of this segment of once a year $gRA(\tau^* = 1 \text{ year}) = \max$[45].

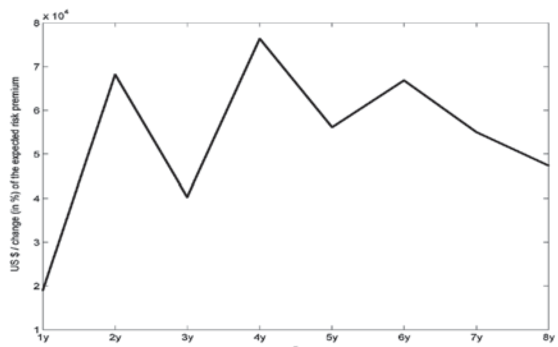
As mentioned, higher gRA values are the result of a more relaxed attitude towards financial losses. Thus, minimizing the loss aversion, as measured by gRA, requires again that investors should check portfolio performance as seldom as possible. For the left segment, this is consistent with the recommendation derived from the perception of risky investments as captured by the prospective value[46]. In sum, both the total and the indirect



(A) All Evaluation Frequencies



(B) $\tau \leq 1$ Year



(C) $\tau \geq 1$ Year

Figure 5. Evolution of the global first-order risk aversion for different evaluation frequencies.

Note. The figure illustrates the evolution of our measure of the loss attitude gRA (in US $ per percentage change of the expected risk premium) from Equation 9 as a function of the portfolio evaluation horizon τ. gRA reflects the sensitivity of the prospective value to the variation of expected returns. It captures merely the indirect impact of τ on investor behavior (i.e., through channels other than expected returns, such as the cushion, the probabilities of past gains and losses, etc.). Higher gRA values indicate a more relaxed loss attitude. Panel A depicts gRA for all evaluation horizons, Panel B reflects a focus on horizons up to 1 year (in monthly increments), and Panel C illustrates horizons from 1 to 8 years (in yearly increments). We assumed myopic cushion $S_t - S_{t-1}$, $R_t \sim N(0,1)$, E_t, $[R_{t+1}] = \underset{s=0,\ldots,t}{mean}\,[R_s]$, $\lambda = 2.25$, and k = 3. The sample included 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time axis corresponds to March 1 of each year.

mechanisms, by which the evaluation frequency impacts perceptions and decisions, indicate that, under practical information constraints, an improvement in investors' attitude towards risky holdings is possible for yearly performance checks.

A comparison with the "exogenous" portfolio optimization framework

Next, we suggested a way to translate the results obtained in our framework in terms of the portfolio optimization language of professional managers. Recall that our investors individually ascertain the maximum sustainable level of losses VaR* based on subjective behavioral parameters. In contrast, managers mostly standardize the risk definition (e.g., when using the VaR concept to measure risk) to specific confidence levels and time horizons. To provide a comparison of these two frameworks, termed as endogenous and exogenous respectively, we confronted the VaR* in our model with the standard VaR used by portfolio managers.

In particular, we performed twofold equivalence computations. First, we started from our VaR* estimates and derived equivalent significance levels α from the VaR formula. Second, we applied confidence levels commonly used (such as 1% and 10%) to the same VaR formula and obtained equivalent average coefficients of loss aversion and equivalent wealth percentages invested in the risky portfolio, on the basis of the corresponding formulas and estimates in our model.

VaR*-equivalent significance levels

Portfolio managers equate the risk level indicated by their clients VaR* with the lower quantile of the portfolio gross returns at a given (i.e., fixed) significance level that we denote by α^* (or confidence $1 - \alpha^*$). According to Equation 1, if the portfolio VaR at time *t* corresponds to $\alpha t > \alpha^*$ (or equivalently, to a confidence level $1 - \alpha_t < 1 - \alpha^*$), then too much risk would arise by investing the entire wealth in the risky portfolio. The portfolio manager would conclude that a percentage of the investors' wealth should be lent (i.e., invested in the risk-free asset) $Bt < 0$. On the contrary, if $\alpha t < \alpha^*$, then the portfolio risk met the individual risk requirements, being lower than the subjective risk threshold, and investors borrowed extra money $Bt > 0$ to increase their S&P 500 holdings. The time averages α^* that would deliver the VaR* values obtained from our market data appear in Table 3 for different portfolio evaluation frequencies, normally distributed and Student's t distributed gross returns, and myopic and dynamic cushions.

Table 3

Portfolio-Equivalent Significance Levels of the Estimated VaR*t+1 (α*)

Evaluation frequency	Myopic cushion: Portfolio return: Normal	Myopic cushion: Portfolio return: Student's *t*	Dynamic cushion: Portfolio return: Normal	Dynamic cushion: Portfolio return: Student's *t*
1 year	0.00	0.00	0.00	0.00
6 months	0.00	0.00	0.00	0.00
4 months	0.00	0.00	0.00	0.00
3 months	0.00	0.00	0.00	0.00
1 month	0.00	0.00	0.00	0.00
1 week	0.00	0.00	0.00	0.00
1 day	0.00	0.00	0.00	0.00

Note. The table presents the portfolio-equivalent significance levels α of the estimated VaR* from Equation 6 at different evaluation horizons τ up to 1 year for both myopic cushions $S_t - S_{t-1}$ and dynamic cushions $\eta(S_t - Z_{t-1}\overline{R})$ and standard normal and Student's *t* with 5 degrees of freedom distributed portfolio gross returns R_t. Other parameter values used included λ = 2.25, *k* = 3, η = 0.9, and $\overline{R} = \text{mean}[R_t]$.

The results are striking: The equivalent significance level α* lies below the commonly acceptable interval (being practically zero). Thus, the assumption of classical portfolio selection models based on the VaR concept that investors choose significance levels α in the interval [1,10]% appears to be at odds with the findings in our VaR* framework for any evaluation frequency higher than once a year. Even the lowest significance level of 1% used in standard portfolio models is not able to capture the loss aversion of nonprofessional investors acting according to our setting. In other words, investors may be substantially more risk averse in practice than actually considered in theory.

Portfolio-equivalent indices of loss aversion

We can also address the equivalence issue from the opposite viewpoint: determining λ^*_{t+1} and the average investment in risky assets that result from our VaR* formula in Equation 6 for usual risk levels, such as α of 1% and 10%[47]. Tables 4 and 5 show the results of this analysis for normally distributed and Student's *t* distributed portfolio returns and myopic cushions. As previously mentioned, the portfolio VaR in Equation 13 is estimated using a 5% significance considered the benchmark for the values in these tables (i.e., this significance level corresponds to 100% risky investments).

Table 4

Wealth Percentages Invested in S&P 500 and the Average λ*, for α = 1%

Evaluation frequency	Wealth %: Portfolio return: Normal	Wealth %: Portfolio return: Student's *t*	λ*: Portfolio return: Normal	λ*: Portfolio return: Student's *t*
1 year	60.99	36.48	1.02	1.01
6 months	59.72	34.63	0.91	0.90
4 months	59.40	34.17	1.00	0.82
3 months	59.30	34.01	1.43	1.67
1 month	59.04	33.65	0.90	1.62
1 week	58.82	33.34	0.80	0.58
1 day	58.70	33.20	1.00	1.02

Note. The table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ* from Equation 7 (last two columns) equivalent to a portfolio significance level α = 1% at different evaluation horizons τ up to 1 year. The 100% level for the wealth percentages invested in risky assets corresponds to α = 5%, and λ = 1 points to a loss-neutral attitude. We assumed myopic cushions St – St-1, standard normal, and Student's t with 5 degrees of freedom distributed portfolio gross returns Rt, λ = 2.25, and k = 3.

Table 5

Wealth Percentages Invested in S&P 500 and the Average λ*, for α = 10%

Evaluation frequency	Wealth %: Portfolio return: Normal	Wealth %: Portfolio return: Student's *t*	λ*: Portfolio return: Normal	λ*: Portfolio return: Student's *t*
1 year	120.80	125.37	1.20	1.02
6 months	121.47	126.11	0.88	1.06
4 months	121.64	126.29	1.00	1.00
3 months	121.70	126.50	1.00	1.00
1 month	121.84	126.50	1.00	1.00
1 week	121.96	126.63	1.00	1.00
1 day	122.00	126.67	1.00	1.00

Note. The table presents the average wealth percentages invested in the risky portfolio (first two columns) and the average loss-aversion coefficients λ* from Equation 7 (last two columns) equivalent to a portfolio significance level α = 10% at different evaluation horizons τ up to 1 year. The 100% level for the wealth percentages invested in risky assets corresponds to α = 5%, and λ = 1 points to a loss-neutral attitude. We assumed myopic cushions $S_t - S_{t-1}$, standard normal, and Student's *t* with 5 degrees of freedom distributed portfolio gross returns R_t, λ = 2.25, and *k* = 3.

The equivalent recommendations concerning the money to be invested in risky assets that result from the optimal portfolio allocation under VaR at 1% (10%) significance lie well below (above) the benchmark VaR at 5%. Thus, a higher (lower) loss aversion in our endogenous VaR* framework was evident—after restating it in terms of the exogenous VaR model—relative to the portfolio risk measured by VaR. A comparison of Tables 4 and 5 illustrates that the lower the significance (or the higher the confidence level), the more risk averse the nonprofessional investors become because the proportion of wealth invested in the risky portfolio is less than 100%. However, even the lowest percentages in Table 4 are still much higher than are those in Table 2, where VaR* is treated as endogenous, mainly for high frequency revisions.

Interestingly, the results for α = 1% are qualitatively consistent with our previous findings supporting mLA because the wealth percentages invested in risky assets decrease for higher evaluation frequencies. Their variation is however much weaker than for our VaR* approach from Table 2. In contrast, when the significance level increases to α = 10%, investors allocate slightly more money to the risky portfolio for more frequent performance evaluations. Because mLA is a widely documented phenomenon, we can conclude that the traditional portfolio optimization framework failed once more to capture the real investor behavior in a consistent way. The problem appears to become more acute for more relaxed assumptions regarding the risk attitude.

We can draw similar conclusions with respect to the equivalent loss-aversion coefficient λ^* derived for conventional significance levels. Its values in Tables 4 and 5 for myopic cushions are much lower than the empirical level of 2.25 estimated in the original PT and largely used in previous empirical research[48]. For the majority of the considered combinations of α values and evaluation frequencies, we obtained $\lambda^* \approx 1$, a level that indicates identical perception over gains and losses according to the value function from Equation 8 (and recalling that $k = 0$, meaning no influence of past losses). Actually, this neutral level of 1 is rarely exceeded for some evaluation frequencies for α = 1% and 10%, which reinforces our earlier claim that even assuming low significance levels (e.g., α = 1% as was common in previous portfolio optimization research) entails an underestimation of the loss attitude of real investors captured by the specific coefficient λ.

Summary and Conclusions

This paper reflects an investigation of the behavior of nonprofessional investors facing the problems of fixing a maximally acceptable level of financial losses and of optimally allocating wealth between a risk-free asset and a risky portfolio. We assumed that these investors are loss averse, narrowly frame financial investments, and perceive that past portfolio performance influences future portfolio returns.

Relying on the investors' perception of the risky investment in line with Barberis et al. (2001) and on the notion of myopic loss aversion introduced in Benartzi and Thaler (1995), we accounted for the formation of an individual loss level VaR*. In addition, we proposed a way in which nonprofessional investors can assess the utility of risky prospects (the prospective value) and introduced an extended measure, the gRA, to better capture the actual attitude towards financial losses of real investors. Incorporating the individual VaR* in the portfolio allocation model developed by Campbell et al. (2001), we can quantify wealth allocation decisions. Moreover, we investigated how the portfolio evaluation frequency affects the prospective value and gRA through different mechanisms and suggested a method to derive the evaluation frequency that maximizes risky investments.

Our original findings based on real market data (specifically, the S&P 500 and the U.S. 3-month T-bill returns) enriched the theoretical results. In summary, our nonprofessional investors behaved myopically loss averse by allocating the main part of their wealth to risk-free assets and smaller sums at higher evaluation frequencies to the risky assets. Financial wealth fluctuations determined by the success or failure of previous decisions (the cushions) exerted a significant impact on the current portfolio allocation. Once a year appears to be a critical evaluation horizon under practical market constraints. The horizon is also optimal from the viewpoint of maximizing risky holdings as the result of a more relaxed attitude (in particular, it maximizes both the prospective value and the gRA measure of loss aversion). Further estimates aimed at establishing an equivalence between the theoretical portfolio optimization under exogenous VaR constraints and our extended framework with individual VaR* (such as significance levels, loss-aversion coefficients, and investments in risky assets) indicate an underestimation of the attitude of nonprofessional investors towards financial losses.

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Footnotes

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3 See Bodie, Kane, and Marcus (2005) for example (p. 197).

4 For example, a nonprofessional investor can be a person who contributes to a pension fund and whose main decision is how to allocate his or her wealth between risky and risk-free assets, given budget and risk profile. Such a person fits our definition of a nonprofessional investor because his or her main occupation does not necessarily concern financial investments. He or she may lack the necessary knowledge, expertise, time, or any combination of these factors to make more sophisticated investment decisions and may rely on the help of professional portfolio managers in devising an optimal mix of risky assets (in this particular case, the pension fund and its portfolio managers).

5 For additional information on how investors decide what sum of money to invest in financial markets, refer to Brunnermeier and Nagel (2008), Gollier (2002), and Rengifo and Trifan (2009).

6 In practice, portfolio managers can use several tests and questionnaires to measure their clients' risk appetite.

7 The process described here is nothing more than the two-fund separation theorem of classical portfolio optimization literature.

8 The definition of VaR is the worst expected loss on an investment over a specified horizon given some confidence level.

9 This is equivalent to an investment in risk-free assets.

10 Specifically, managers interpret the client indication (a single number) in terms of the theoretical concept of VaR (i.e., of two elements: a confidence level and an investment horizon).

11 Further mathematical details and the exact expressions of w_t^{opt} and VaR_t appear in the Appendix.

12 Our allocation problem extended to incorporate not only the portfolio optimization in the strict sense, as performed by managers, but also the earlier decision of nonprofessional

investors with respect to the desired risk level.

13 Note that Tversky and Kahneman (1992) elaborate fully on the concepts on which we base our setting in the cumulative prospect theory (CPT). Because we are not particularly interested in the formal details and most of these concepts are already present in the original PT in Kahneman and Tversky (1979), we refer to both theories as PT.

14 We restate the term in the condition of Equation 15 by Barberis et al. (2001) as $R_{t+1} - z_t R_{ft} = x_{t+1} + (1+z_t)R_{ft}$.

15 The derivation of the expectation and the variance of the loss utility as well as the final expression of VaR* appear in the Appendix.

16 In other words, the loss aversion that equivalently results under the manager assumption of a fixed, exogenous risk level.

17 For the derivation, see the Appendix.

18 In contrast to Barberis et al. (2001), our investors derived utility merely from financial wealth fluctuations, not being concerned with consumption.

19 The exact expression is evident in the Appendix. We also applied a slightly different definition of the prospective value. Accordingly, gains continue to be considered as possible events and are hence weighted by the occurrence probability. Losses are instead assessed in what we can call a "worst-case scenario" (i.e., with maximum probability), which is equivalent to saying that losses are accounted for in the form of VaR*. The obtained results, available upon request, were qualitatively similar to applying Equation 8.

20 See the Appendix for the detailed mathematical derivation.

21 As the prospective value is the PT counterpart of the classic concept of investment utility, gRA is the pendent of a marginal utility with respect to the expected premium.

22 According to Barberis and Huang (2006), myopia refers strictly to annual evaluations of gains and losses, so the term *narrow framing* would be better suited to describe the underlying phenomenon. In a financial context, narrow framing illustrates the isolated evaluation of stock market risk (i.e., unrelated to the overall wealth risk). As underlined in Barberis and Huang (2004), this isolated evaluation entails an underestimation of the stock desirability, even though, viewed in a wide utility-risk frame, stocks represent a good diversification modality.

23 Numerous researchers conducting direct experimental tests, such as Thaler, Tversky, Kahneman, and Schwartz (1997a); Gneezy and Potters (1997); Gneezy, Kapteyn, and Potters (2003); and Haigh and List (2005), supported the occurrence of mLA.

24 In the same context, Gneezy and Potters (1997) suggested that managers could manipulate the evaluation period of prospective clients.

25 Several years passed before the financial reform became operative.

26 This method is appropriate for preserving some of the particularities of less probable market events, such as crashes, while at the same time allowing for circumvention of excessive impacts due to extreme outliers.

27 Thaler, Tversky, Kahneman, and Schwartz (1997b) made a similar assumption.

28 This assumption implies that the evaluation period is shorter than the lifetime of our loss-averse agents or, equivalently, that investors are long-lived beyond the VaR horizon. Basak and Shapiro (2001); Berkelaar, Kouwenberg, and Post (2004); and Berkelaar and Kouwenberg (2006) made identical assumptions.

29 We performed parallel simulations for all values $\lambda \in \{0.5;1;2.25;3\}$ and $k \in \{0;3;10;20\}$. The results are qualitatively similar and available upon request.

30 We also performed simulations for the cases where expected portfolio gross returns were computed as a zero-mean process, or as an AR(1) process. The results, available upon request, are qualitatively similar. Because unsophisticated investors (such as our nonprofessional traders) are more likely to rely on simple descriptive statistics from past data, we concentrate here on the case where expected returns are derived from average past returns.

31 We also considered other cushion definitions. For instance, cumulative cushions amass from the date zero of the trade, so that $Z_t = Z^1 = S_0$ (e.g., the purchase price). Moreover, we also defined new myopic cushions assuming $Z_t = Z_{t-1} R_t$. The corresponding results are available upon request.

32 See Barberis et al. (2001, p. 15). This parameter allows for adjustments of the benchmark, wherefrom the denomination of "dynamic" cushions. Specifically, lower η-values put increased weight on the current risky value St relative to past evolutions captured by $Z_{t-1}\overline{R}$, which corresponds to a more myopic view. In contrast, higher η-values denote a more pronounced conservativeness in assessing the past performance benchmark, as the current term St decreases in importance relative to the past-oriented $Z_{t-1}\overline{R}$.

33 In fact, we considered three values of η, namely 0.1, 0.5, and 0.9. The results are qualitatively similar and are available upon request.

34 Because no dividend data were available for our analysis, we could not apply the simultaneous estimation procedure of Barberis et al. (2001). Note also that because the mean and median of our return sample lie very close to each other, the results with $\overline{R}$ = mean $[R_t]$ and $\overline{R}$ = median $[R_t]$ are almost identical.

35 Although VaR is a popular measure of risk, critics emphasize that it does not satisfy one of the four properties for coherent risk measures, namely subadditivity (see Artzner, Delbaen, Eber, & Heath, 1999; Rockafellar & Uryasev, 2000; Szegö, 2002). However, according to Embrechts, McNeil, and Straumann (1999), VaR becomes subadditive, and hence coherent, for elliptic joint distributions, such as normal and Student's *t* with finite variance.

36 The results using dynamic cushions are qualitatively similar and available upon request.

37 Specifically, in this case, the prospective value (black) cannot be practically disentangled from the cushion effect (blue).

38 To obtain a suggestive graphic representation, we considered all frequencies from 1 to 12 months and discarded the observations for 1 day and 1 week. An evaluation frequency of 8

years implies that investors could only make three portfolio checks during our estimating sample. Therefore, a further increase of the evaluation time was senseless.

39 Intuitively, when risky performance is checked at longer time intervals, the decision flexibility is lower because current decisions fix the portfolio composition over the entire coming interval of several years. Thus, investors would be more wary of the possibility of registering current losses for lower evaluation frequencies. As the cushion effect accounts for the perception of possible current losses—a perception that varies depending on the past performance; see the cushion weight in Equation 8—it increases in absolute value for more seldom portfolio checks, but its sign is given by the sign of the cushion. Here, investors create negative cushions, which gives rise to the observed drop in the cushion effect and consequently in the prospective value.

40 Specifically, the adjusted R-squared yields 91.69% (77.44%) for myopic (dynamic) cushions. The basis of the estimations is polynomial regression fitting performed with the Matlab Curve Fitting Toolbox. All findings in this section are robust across different parameter specifications, such as the loss aversion coefficient, the sensitivity to past losses, the cushion, returns distribution, expected returns, etc. Further results are available upon request.

41 In fact, the prospective value in the left segment of Figure 4 attains its maximum at 11 months. Because this value lies closely to the predicted maximum point of 1 year and because 1 year is a much more noticeable value in investor perception, we consider 1 year a sufficiently good approximation for the optimum.

42 All findings in this section are robust across different parameter specifications. Further results are available upon request.

43 The ambiguity of the total transmission mechanism reported for the prospective value appears to be therefore given by its direct component (i.e., through expected returns). The cushion effect, which is highly dependent on returns, distorts the evolution of the prospective value for very seldom portfolio checks, making it extremely sensitive to past performance.

44 Specifically, the adjusted R-squared yields 90.5% (91.57%) for myopic (dynamic) cushions.

45 The statement is now consistent with both the data and the fitted curve.

46 In the right evaluation-frequency segment, the course of gRA is more complex, so second-order polynomials are necessary to describe the data acceptably. Specifically, the adjusted R-squared yields 49.61% (60.74%) for myopic (dynamic) cushions. The maximum of these parabolas is evident at an evaluation frequency of around 5 years, which might indicate this frequency as optimal in this segment. Specifically, this frequency yields 4.9859 (5.3178) for myopic (dynamic) cushions. Nevertheless, as stressed previously, we consider the right segment to be of less practical importance.

47 Results are qualitatively similar for the dynamic cushion case and are available upon request.

48 See Barberis et al. (2001) and Benartzi and Thaler (1995).

49 Clearly, $a_{i,t} = w_{i,t} (W^{\prime} + B_t) / p_{i,t}$.

50 See the comments concerning the two-fund separation below.

51 Note that VaRex is imposed by the client prior to the portfolio formation and enters the portfolio optimization problem in the form of a constraint. In contrast, the portfolio VaR is an output of this optimization and measures the actual maximum loss possible at time t at the confidence level 1 - α for the obtained optimal portfolio w_t^{opt}.

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Appendix

Optimal portfolio selection with "exogenous" VaR

The objective of the optimization problem in Campbell et al. (2001) is maximizing the next-period wealth W_{t+1}. This wealth results from the return expectation of the components of the risky portfolio and the risk-free assets. The risky portfolio consists of $i = 1,\dots,n$ financial assets with single time t, prices $p_{i,t}$, and portfolio weights $w_{i,t}$, such that $\sum_{i=1}^{n} w_{i,t} = 1$. Moreover, ai,t is the number of shares of the asset i contained in the portfolio at time t[49]. Formally, we can state the portfolio optimization problem as follows:

$$W_{t+1}(w_t) = (W_t + B_t)E_t[R_{t+1}(w_t)] - B_t R_f \xrightarrow[w_t]{} \max \tag{10}$$

$$W_t + B_t = \sum_{i=1}^{n} a_{i,t} p_{i,t} = a_t' p_t \qquad \text{(budget constraint)} \tag{11}$$

Such that,

$$P_t[W_{t+1}(w_t) \leq W_t - VaR^{ex}] \leq 1 - \alpha \quad \text{(risk constraint)},$$

where Rt+1(wt) stands for the portfolio gross returns at the next trade and $E_t[R_{t+1}(w_t)]$ for the corresponding expected returns. Henceforth, we refer to the gross returns of the risky portfolio as returns or portfolio returns.

In Equations 10 and 11, B_t denotes the risk-free investment, in other words the sum of money that can be borrowed ($B_t > 0$) or lent ($B_t < 0$) at the fixed risk-free gross return rate R_f. Note that the maximization in Equation 10 is carried over the weights of the risky portfolio wt but not over Bt. The risk-free investment results as a by-product of the optimization procedure[50]. Finally, P_t stands for the conditional probability given the information at time t, and $1 - \alpha$ stands for the chosen confidence level.

After some manipulations, Campbell et al. (2001) obtained the optimal weights of the risky portfolio as follows:

$$w_t^{opt} \equiv \arg\max_{w_t} \frac{E_t[R_{t+1}(w_t)] - R_f}{W_t R_f - W_t q_t(w_t, \alpha)}, \tag{12}$$

where $q_t(w_t,\alpha)$ represents the quantile of the distribution of portfolio gross returns $R_{t+1}(w_t)$ for the confidence level $1 - \alpha$ (or significance level α), that is, $P_t[R_{t+1}(w_t) \leq q_t(w_t, \alpha)] \leq 1 - \alpha$. Thus, the optimal mix of risky assets depends merely on the distribution of the portfolio gross returns and on the significance level α. Note that we do not elaborate further on the optimal weights from Equation 12 because we assume the details of wealth allocation among the different risky portfolio components to be the responsibility of portfolio managers.

Equation 12 shows that, similar to the traditional mean-variance framework, the two-fund separation theorem applies: Neither the nonprofessional investors' initial wealth nor the desired risk level VaR^{ex} affects the maximization procedure. In other words, investors first determine the optimal risky portfolio (i.e., the optimal allocation among different risky assets) and second decide upon the extra amount of money to be borrowed or lent (i.e., invested in risk-free assets). The latter reflects by how much the portfolio VaR, defined as

$$VaR_t = W_t\,(q_t\,(w_t^{opt}, \alpha) - 1, \tag{13}$$

varies according to the investor degree of loss aversion measured by the selected (desired) VaR^{ex} level[51]. The optimal investments in risk-free and risky assets follow Equations 1 and 2, respectively.

The loss distribution, the VaR*, and the equivalent loss-aversion coefficient

Henceforth, we use the following probability notations:

$$\pi_t = P_t\,(z_t \leq 1)$$

$$\omega_t = P_t\,(x_{t+1} \geq 0 \mid z_t > 1) \tag{14}$$

$$\psi_t = P_t\,(x_{t+1} + (1 - z_t)\,R_{ft} \geq 0 \mid z_t \leq 1),-$$

where π_t stands for the probability of past gains, and ω_t represents the probability of a positive premium given past losses. Finally, we can term ψt as the probability of obtaining a return premium x_{t+1}+$(1-z_t)\,R_{ft}$, higher than the risk premium x_{t+1}, that expresses increased expectations resulting from recurrent gains.
According to Equations 4 and 5, we can write the expected loss value as follows:

$$E_t\,[loss - value_t] = \pi_t\,(1 - \psi_t)\,(\lambda S_t E_t\,[x_{t+1}] + (\lambda - 1)(S_t - Z_t)R_{ft}) + (1 - \pi_t)\,(1 - \omega_t)\,(\lambda S_t E_t[x_{t+1}] - k(S_t - Z_t)\,E_t[x_{t+1}])$$

$$E_t\,[loss - value_t] = \lambda S_t E_t\,[x_{t+1}] + (\pi_t\,(1 - \psi_t)((\lambda - 1)\,R_{ft} + kE_t[x_{t+1}]) - kE_t[x_{t+1}])(S_t - Z_t)\,. \tag{15}$$

Thus, the expectation of the squared loss value and consequently the loss variance result in

$$E_t\,[loss - value^2_{t+1}] = \pi_t\,(1 - \psi_t)\,(\lambda S_t E_t\,[x_{t+1}] + (\lambda - 1)(S_t - Z_t)R_{ft})^2 + (1 - \pi_t)(1 - \omega_t)\,(\lambda S_t E_t[x_{t+1}] - k(S_t - Z_t)\,E_t[x_{t+1}])^2$$

$$E_t\,[loss - value^2_{t+1}] = (\lambda S_t E_t\,[x_{t+1}])^2 + (\pi_t\,(1 - \psi_t)((\lambda - 1)R_{ft} + kE_t[x_{t+1}] - kE_t[x_{t+1}]\,\lambda S_t E_t[x_{t+1}]\,(S_t - Z_t)$$

and

$$Var_t[loss - value_{t+1}] = E_t[loss - value^2_{t+1}] - E^2_t[loss - value^2_{t+1}]$$

$$Var_t[loss - value_{t+1}] = \pi_t\,(1 - \psi_t)\,(1 - \pi_t\,(1 - \psi_t))((\lambda - 1)R_{ft} + \mathrm{k}E_t[x_{t+1}])^2\,(S_t - Z_t). \tag{16}$$

The variance of the loss value is exclusively based on past performance, generated only by the cushion $S_t - Z_t$.
As explained in the main text, our individual loss level VaR* is a maximal value. We obtain this level, by setting the absolute probability of making a loss (i.e., independently of the prior performance in the value function to its maximum of 1, that is $\pi_t(1 - \psi_t) + (1 - \pi_t)\,(1 - \omega_t) = 1$). In line with the concept of VaR, we finally define VaR* as the quantile of the subjective loss distribution, yielding the following expression:

$$VaR^*_{t+1} = E_t\,[loss - value_{t+1}] - \sqrt{Var_t\,[loss - value_{t+1}]}$$

$$VaR^*_{t+1} = \lambda S_t E_t[x_{t+1}]\ - kE_t[x_{t+1}])(S_t - Z_t)\ldots$$

$$\ldots + \sqrt{\pi_t\,(1 - \psi_t)}\,\sqrt{\pi_t\,(1 - \psi_t)} - \varphi\,\sqrt{1 - \pi_t\,(1 - \psi_t)})\,((\lambda - 1)\,R_{ft} + \mathrm{kE_t}\,[x_{t+1}])\,(S_t - Z_t)$$

$$VaR^*_{t+1} = \lambda S_t E_t[x_{t+1}] + (\varsigma_t\,(\lambda - 1)\,R_{ft} + (\varsigma_t - 1)\,kE_t[x_{t+1}])(S_t - Z_t), \tag{17}$$

where $E_t[x_{t+1}] = E_t[R_{t+1}]$ - Rft denotes the expected risk premium, and the last expression results from the simplifying notation

$$\varsigma_t = \sqrt{\pi_t\,(1 - \psi_t)}(\sqrt{\pi_t\,(1 - \psi_t)} - \varphi\,\sqrt{1 - \pi_t\,(1 - \psi_t)}). \tag{18}$$

We distinguish two terms of the VaR* expression in Equation 6: The first term accounts for the expected risky return (relative to the risk-free rate) $S_t E_t[x_{t+1}]$, weighted by the loss-aversion coefficient λ. As it consequently resembles the prospective value according to the original PT, we denote the term as the PT term. The last term is responsible for the influence of previous performance represented by the cushion $S_t - Z_t$. For this reason, we denote it as the cushion term. The corresponding weight is a linear combination of the expected risky and the risk-free returns. The formula of λ* is then immediate from Equation 6, where k is taken to be zero (because λ^*_{t+1} depends on the fixed $\overline{VaR^*}$):

$$\lambda^*_{t+1} = \frac{\overline{VaR^*} + \varsigma_t R_{ft}(S_t - Z_t) + (1 - \varsigma_t)kE_t[x_{t+1}](S_t - Z_t)}{S_t E_t[x_{t+1}] + \varsigma_t R_{ft}(S_t - Z_t)} \tag{19}$$

$$\lambda^*_{t+1}\underset{k=0}{=}\frac{\overline{VaR^*} + \varsigma_t R_{ft}(S_t - Z_t)}{S_t E_t[x_{t+1}] + \varsigma_t R_{ft}(S_t - Z_t)}$$

The prospective value function

The prospective value from Equation 8 yields the following:

$$V_{t+1}= \pi_t[\psi_t S_t E_t\,[\mathrm{x}_{t+1}]+(1-\psi_t)(\lambda S_t E_t\,[\mathrm{x}_{t+1}]+(\lambda-1)(S_t-Z_t)R_{ft})](1-\pi_t)[\omega_t S_t E_t[\mathrm{x}_{t+1}]+(1-\omega_t)(\lambda S_t E_t[\mathrm{x}_{t+1}]+k(\mathrm{Z}_t-\mathrm{S}_t)\,E_t\,[\mathrm{x}_{t+1}])]$$

$$V_{t+1}= (\pi_t\psi_t+(1-\pi_t)\omega_t+(\pi_t(1-\psi_t)+(1-\pi_t)\,(1-\omega_t))\,\lambda)S_t E_t[\mathrm{x}_{t+1}](\pi_t(1-\psi_t)+(\lambda-1)R_{ft}-(1-\pi_t)(1-\pi_t)kE_t[\mathrm{x}_{t+1}](\mathrm{S}_t-\mathrm{Z}_t) \quad . (20)$$

As noted in the main text, a twofold effect is apparent in Equation 20: The first term of the last expression is the PT effect, which captures the expected risky value relative to the safe bank investment $S_t E_t[x_{t+1}]$. The corresponding probability weight can be rendered as the sum of the perceived gain and loss probabilities, laxly put as P_t(gain) + λP_t(loss). As in PT, losses loom larger than gains, additionally penalized by the loss-aversion coefficient λ.

The last term of the prospective value in Equation 20 is the cushion effect, which covers the cushion influence $S_t - Z_t$. The cushion weight is a combination of expected losses under the consideration of the performance history. Specifically, when current losses follow past gains, which occurs with the joint probability $\pi_t(1 - \psi_t)$, the past performance (given by the cushion) is valued at the risk-free rate R_{ft} and is amended by how much the loss-aversion coefficient λ exceeds the loss-neutral value of 1. Indeed, if risky investments were successful in the past, a current loss has value only compared to the alternative of having put the entire wealth in risk-free assets. When losses extend from past to present, where $(1 - \pi_t)$ $(1 - \omega_t)$ is the joint probability of current and past losses, the valuation involves a comparison of the risk-free rate to the risky performance $E_t[x_{t+1}]$ in view of the sensitivity to past losses k.

The global first-order risk aversion gRA

The expression of gRA from Equation 9 entails the following:

$$gRA_t = (\pi_t\psi_t + (1-\pi_t)\,w_t + (\pi_t\,(1-\psi_t) + (1-\pi_t)(1-\omega_t))\,\lambda)S_t - (1-\pi_t)(1-\omega_t)k(S_t-Z_t). \quad (21)$$

Due to the linearity of our prospective value in the expected risk premium $\mathrm{E}_t[\mathrm{x}_{t+1}]$, gRA is independent of this premium. The fact that its increase denotes a more relaxed loss attitude is formally evident in Equation 21: The first term increases with the sum invested in risky assets St, and the second is inversely proportional to the cushion $S_t - Z_t$. Note however that this second term accounts for the situation where current losses follow past losses, which occurs with the probability $(1 - \pi_t)(1 - \omega_t)$, and where, most probably, cushions are negative $S_t - Z_t \leq 0$. Thus, smaller negative cushions render the second term higher. In summary, gRA grows both when investors put more money into risky assets and when they manage to reduce recurrent losses.