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Abstract

We use the approach of Qu and Perron (2013) for the modeling and inference of volatility of a set of commodity prices in the presence of level shifts of unknown timing, magnitude and frequency. The model has two features: (i) it is a stochastic volatility model comprising both a level shift and a short-memory process where the first component is modeled as a compounded binomial process while the second one is an AR(1) process; (ii) the model is estimated using Bayesian techniques in order to obtain posterior distributions of the parameters and the two latent components. We use six commodity series: agriculture, livestock, gold, oil, industrial metals and a general commodity index. All series cover the period from January 1983 until December 2013 in daily frequency. The results show that although the occurrence of a level shift is rare (about once every 1.5 or 1.8 years), this component clearly contributes most to the variation in the volatility. The half-life of a typical shock from the AR(1) component is short, on average 13 days. Furthermore, isolating the level shift component from the overall volatility indicates a stronger relationship between volatility and Peruvian business cycle movements.

JEL Classification: C22, C52, G12.

Keywords: Stochastic Volatility, State-Space Models, Bayesian Inference, Structural Change, Commodity Prices.

Resumen

En este documento usamos el enfoque de Qu y Perron (2013) para la modelación, estimación e inferencia acerca de la volatilidad de un grupo de precios de commodities en la presencia de cambios de nivel de fecha, magnitud y frecuencia desconocidas. El modelo tiene dos rasgos: (i) es un modelo de volatilidad estocástica que comprende tanto un proceso de cambios de nivel como un proceso de corta memoria. El primer componente es modelado como un proceso mixto gobernado por una variable Binomial mientras que el segundo proceso es modelado como un proceso AR(1); (ii) el modelo se estima utilizando técnicas Bayesianas con el fin de obtener distribuciones posteriores de los parámetros y de los dos componentes latentes. Utilizamos seis series de commodities: agricultura, ganadería, oro, petróleo, metales industriales y un índice de commodities en general. Todas las series cubren el período de Enero de 1983 hasta Diciembre de 2013 con frecuencia diaria. Los resultados muestran que a pesar que la ocurrencia de un cambio de nivel es rara (aproximadamente una vez cada 1.5 o 1.8 años), este componente contribuye claramente más a la variación en la volatilidad. La vida media de un choque típico de la especificación AR(1) es corta, en un promedio de 13 días. Además, aislando el componente de cambio de nivel de la volatilidad global indica una relación más fuerte entre los movimientos de la volatilidad y el ciclo económico peruano.

Clasificación JEL: C22, C52, G12.

Palabras Claves: Volatilidad Estocástica, Modelos en Forma Espacio-Estado, Inferencia Bayesiana, Cambio Estructural, Precios de Commodities.

Modelling the Volatility of Commodities Prices using a Stochastic Volatility Model with Random Level Shifts[1]

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1 Introduction

The volatility of commodity prices such as oil or minerals is an important issue for small and open economies that depend on raw materials. For example, in many Latin American countries, the volatility of commodities can affect the operating costs or investment schedules of businesses in the primary sector. At the macroeconomic level, high volatility can produce changes in the current account and in capital inflows, or, on the side of importers and exporters, increase uncertainty regarding production costs and inflation. Therefore, modeling volatility of commodity prices would be useful for private agents and policy makers. For the former, it gives valuable information for better options contracts that allow hedging under great uncertainty, while for the latter, it can aid in a better understanding of business cycles given the correlation between mineral price fluctuations, capital inflows, and investment expectations.

In this paper we focus on modeling the volatility of the overall commodities market and some sectors that themselves have huge repercussions on the global economy (e.g. industrial metals, oil, gold). To this end, we study commodity market indexes, in particular the Standard & Poors Goldman Sachs Commodity Index (hereinafter S&P GSCI). As documented in Indices (2014), the S&P GSCI is a benchmark for investment in the commodity markets and a measure of commodity market performance over time. It is also a tradable index that is readily accessible to market participants, so we take this index as the best approximation of commodity market performance. The composition of this index is dominated by energy commodities, where oil accounts for 66% of the total index. Other commodities make up far less of the total; for instance, industrial metal and precious metals represent only 7% and 3% of the index, respectively. For this reason, in Section 2 we analyze the volatility of the commodity index as a whole, and of certain indexes that compose it, such as the gold, oil, industrial metals, agriculture, and livestock index.

The evolution of commodity prices are studied just like any other financial series in the literature. What is more, there exist commodity stocks markets and commodity future markets where a high degree of speculations mixes with fundamentals. The pioneering work of Brennan and Schwartz (1985) analyzes the stochastic nature of natural resources prices and applies stochastic optimal

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control to the valuation of investment projects. Fama and French (1987) evaluates commodity-futures prices using the theory of storage[3] and as a forecast of a future spot price with a risk premium. Fama and French (1988) focuses on metal-futures prices, analyzing them using the theory of storage and their relationship with stages in the business cycles. Gibson and Schwartz (1990) proposes a two factor model to analyze the pricing of oil contingent claims based on the convenience yield, and Schwartz (1997) analyzes the behavior of commodity prices under several factor models of stochastic basis, finding typical features of financial series such as mean reversion. A theoretical foundation on the stochastic nature of the uncertainty in investment can be found in Ingersoll and Ross (1992). Other works analyzing the role of certain commodities in portfolio investment include Jaffe (1989), which highlights the role of gold or precious metals in diversified portfolios, and Ankrim and Hensel (1993) which focuses on the similarities between commodity and real estate investment as inflation hedges. Moreover, Gorton and Rouwenhorst (2006) describes the financial properties of commodity-based financial instruments such as futures and finds similar behavior to equity risk premium and a negative correlation with other instruments. All these financial studies evaluate volatility dynamics as crucial for their results. Also, evaluation of volatility behavior alone is found in different studies such as that of Askari and Krichene (2008), who find that oil is very volatile and sensitive to small shocks even though assumptions about market fundamentals hold. Brunetti and Gilbert (1995) studies the volatility of industrial metals from 1972 to 1995 and find that volatility does not increase during that period, which runs contrary to common opinion. These findings suggest that commodity prices evolve quite similarly to other financial series. Therefore, it is relevant to talk about the returns of commodity prices and their associated volatility.

For this reason, we apply stock return volatility models to commodities-price series. In this field the literature is vast, and the different models proposed can be grouped into two categories: GARCH models and stochastic volatility (SV) models. For a complete survey of theses approaches, see Engle (1995) and Shephard (1995), respectively. The principal characteristic of GARCH models is that they explicitly model the conditional variance of returns given past returns; specifically, volatility is predicted one-step-ahead. Meanwhile, in the SV model the predictive distribution of returns is specified indirectly via the structure of the model, rather than explicitly. The main advantage of SV models is that they have strong theoretical support, primarily from Taylor (1986) and Taylor (1994). Also, there are many possible filtering techniques to estimate the volatility as a latent variable.

The SV models are difficult to estimate in the sense that volatility is an unobserved variable. SV models have error terms in the mean and also in the variance equation, making the likelihood function difficult to evaluate. The Method of Moments was suggested as a possible option to estimate SV models, and was developed by Taylor (1986) among others, but is subject to efficiency problems. Also, quasi-maximum likelihood estimators can be found using the Kalman filter, as in works like Harvey, Ruiz, and Shephard (1994). Finally, Bayesian procedures have been the most popular method of evaluating SV models since Jacquier, Polson, and Rossi (1994), who found that this estimation procedure outperforms the others. In this field, Kim, Shephard, and Chib (1998) provide quite an extensive discussion of various alternative methods for the actual implementation of Markov Chain Monte Carlo (MCMC) algorithms in order to simulate posterior distributions. The issue of sampling to simulate posterior distributions is relevant in most Bayesian analyses and makes a very large difference in the computational efficiency of the methods. Kim, Shephard,

[3] Theory of storage explains the spread between spot and futures prices based on the convenience yield on inventory.

and Chib (1998) proposes an improved MCMC algorithm based on an offset mixture of normal distributions for the error term. Finally, filtering follows as a special case of Pitt and Shephard (1999) and results are better than GARCH models.

Volatility in financial series has some important features, such as clustering, leverage effects, and long memory. This study addresses the phenomenon of long memory in the volatility of commodity return series. Long memory can be described as having a slow rate of decay in the autocorrelation function (hereafter ACF) of a particular time series. These processes have been termed in the literature as fractional integrated processes $I(d)$ since Adenstedt (1974) and Granger (1980). This phenomena can be found in any time series, but is a particular feature that has been observed in the financial volatility of returns, and as such much work has been trying to model this characteristic. For example, Baillie (1996) is a complete survey of econometric developments related to long memory and its applications in economic and financial series. Also, Baillie, Bollerslev, and Mikkelsen (1996) develops a fractionally integrated model for volatility in the family of ARCH models (FIGARCH).

The other topic of interest in this work is structural change, which is an important feature in macroeconomic time series. The seminal paper of Perron (1989) shows that the presence of a unit root may be confused with structural breaks in the series. This idea has been generalized to the context of volatility in which structural breaks may lead to the false identification of long memory. The confusion of long-memory processes with the presence of level shifts has been studied since Diebold and Inoue (2001), but that work presents evidence against long-memory processes on the basis of level shift tests that are biased. Perron and Qu (2007) find theoretical results regarding the behavior of a short-memory process affected by level shifts, focusing on the time and spectral domain. They observe that the periodogram of the above-mentioned process follows a similar pattern to a long-memory process, so it is possible to confuse these processes in empirical applications. Also, Perron and Qu (2010) analyze the properties of the ACF, the periodogram, and the log periodogram estimate of the memory parameter of a short-memory process with level shift explained by a mixture model, and find behavior similar to long-memory processes. By analyzing data from various indices of stock markets, they identify similarities between the estimated statistics and their theoretical results.

Qu and Perron (2013) and Lu and Perron (2010) estimate two different kinds of random level-shift (RLS) models of volatility. Lu and Perron (2010) estimates their model using an extension of the Kalman filter, and the model proposed can be transformed directly in state-space form by assuming a linear combination of a short-memory process and a random level-shift component to explain the log of the absolute returns as a proxy of volatility. They find that the remaining component, if accounting for level shifts, is a short-memory process. Qu and Perron (2013) proposes a stochastic volatility model affected by random level shifts. Hence, Bayesian estimation follows procedures based on Kim, Shephard, and Chib (1998) for sampling of the posterior distributions by taking into account the random level-shift term. The distribution of the probability of shifts follows a Bernoulli distribution, so the probability changes in time. They apply this model to the Nasdaq and S&P 500 time series from 1980 to 2012 and for different priors in order to address a sensitivity analysis. Also, they get better results on the interaction of the volatility with indicators from the business cycle in the United States.

Our work is based on the final model of Qu and Perron (2013) and seeks to determine whether long memory exists in commodity-return volatility series, or whether a short-memory process with structural breaks applies. Commodity prices and volatilities affect portfolio decisions and business cycles, but little work has been done on modelling financial series of interest to Peru using econometric techniques. An initial approach by Humala and Rodríguez (2013) studies exchange rate and Lima Stock Exchange returns, and concludes that this series has same statistical properties as any other financial series in a developed market. More recently, Alanya and Rodríguez (2014) have used a SV model following Kim, Shephard, and Chib (1998) to track Peruvian stock market and exchange rate volatilities. Our study constitutes an attempt to fill a gap in this line of research by analyzing commodity volatilities.

The remainder of this document is structured as follows: Section 2 contains some features of the commodities volatility; Section 3 describes the applied methodology; Section 4 contains the overall results and for each kind of commodity, as well as a brief analysis of business cycle comovements. Finally, Section 5 presents the conclusions.

2 Features of Commodity Volatility

In this paper we focus on commodity-price volatility because this variable is relevant to private and public agents in Latin American countries. However, before estimating this volatility, it is worth analyzing some features of the series and justifying the method that would fit best fit the volatility component of these series. First of all, we use the S&P GSCI as the approximation of commodity market performance. This index includes all eligible contracts that represent transactions of a physical commodity[4], and is built from the weighted-sum of contracts of different commodities. Table 1 shows the component of the S&P GSCI. Clearly, oil is the commodity with the greatest contribution to the index (67.2%), followed by agriculture subindexes (15.3%). We have chosen to analyze the entire commodity market and its components given possible differences between markets that may influence volatility. Thus, we study the commodity index, industrial metals, oil, gold, the agriculture index, and the livestock index[5].

In Figure 1 we can see the evolution of daily returns of commodities from January 1983 to December 2013. A first feature of all series is their volatility, which grows in certain periods. These periods of high volatility may be common to all series, as occurred between 2008-2009, which was associated with the international financial crisis; or to a particular commodity, as in late 1990 and early 1991, which was marked by high volatility in oil prices associated with the Gulf War. In general, we observe that the series behaves similarly to any given high-frequency financial asset, such as stock returns. Therefore, it is valid to use financial modeling techniques to analyze the volatility of commodity markets.

A second feature, also linked to the volatility of the series, is the difference in behavior between markets. For example, variations in returns are larger in oil and industrial metals than in agricultural goods or livestock. In addition, these goods have different paths of volatility. For example, gold underwent a period of volatility during late 2000 and early 2001, possibly associated with the dot-com crisis in the United States; industrial metals were subject to a period of high volatility between 2005 and 2008, which was probably caused by high demand in developing countries such as China; while agricultural goods witnessed a high volatility period in the late '90s due to the fall of the Soviet block, which was a major crop producer in the world market. Each of these periods of high volatility for a certain commodity have not been replicated by other markets. Therefore,

[4] For more details of S&P GSCI methodology, see Indices (2014).

[5] We separate oil and gold from their respective subindexes due to the individual importance of these commotities to the global economy.

while analysis of a set of commodities is useful at the aggregate level, it is important to analyze each market separately given the intrinsic characteristics that influence their level of volatility.

Since it is plausible to analyze the returns of commodity prices as if they were financial series, it is worth noting two of the most important features of this type of time series. First, as has been already seen, the presence of volatility clustering; and second, the volatility persistence, or long-memory. The latter characteristic has taken on increased relevance in the literature on volatility. A simple way to detect whether the volatility of a series has long memory is by estimating the ACF of the logarithm of its squared returns. If long memory exists, then the ACF will slowly decay to zero. As shown in Figure 2, commodities decay slowly to zero after 1500 days, on average. Moreover, after reaching zero, the ACF fluctuates around zero up to the maximum number of lags. Similar behavior in the ACF is reported by Perron and Qu (2010) in their analysis of the S&P 500 index of the New York Stock Exchange, in which they argue that this behavior is a stylized fact of financial series that are suspected to have long memory[6].

As mentioned above, the assumption of long memory must be carefully analyzed. The empirical evidence (see, for example, Perron and Qu (2010)) suggests that the long-memory phenomenon can be confused with a process that has rare discrete level changes which is alters the levels of volatility in the long run. A first approach to assessing whether a process has long memory is by estimating the parameter d using the log-periodogram, as proposed by Geweke and Porter-Hudak (1983). The results of this estimation are shown in Figure 3. Each frame shows the estimation of the parameter memory, d, for each commodity, which is on the y axis, while the frequency of the data is on the x axis. If the process is long memory then the parameter d should be the same for all sizes of frequency. However, the parameter memory tends to decay the higher the frequency is. The vertical lines crossing each of the figures represent the $T^{1/3}$, $T^{1/2}$ and $T^{2/3}$ frequencies for a sample of $T = 7818$. Thus, for low frequencies (between $T^{1/3}$ and $T^{1/2}$) the parameter d is greater than 0.5, on average, while higher frequencies tend to decline, which continues even for frequencies greater than $T^{2/3}$.

The results found in the log-periodogram are similar to those found by Perron and Qu (2010), who analyze the volatility of S&P 500. According to these authors, the fall in the long-memory parameter with increasing frequency is due to the existence of two components of volatility: a first component, short-run, present throughout the entire series; and another component, level shifts, that cause jumps in volatility levels that resemble long-memory processes[7]. The latter component is dominant at low frequencies, but as the number of frequencies increases, the short-term component is dominant and hence the parameter d tends to decline.

A second approach to assess long-memory processes is to rule whether or not they are spurious. For this, we use the test of Qu (2011), whereby, under the null hypothesis, the process has long memory, while under the alternative hypothesis, the process is one of short memory with level shifts. The results of the test applied to the volatility of commodities are presented in Table 2. The first column shows the estimated d for $T = 0.7$; that is, to a frequency which is slightly right of $T^{2/3}$. It has none of the estimated d exceeding 0.5, which is consistent with the literature.

[6] According to Qu and Perron (2013) a process has long memory if $\gamma_z(\tau) = g(\tau)\,\tau^{2d-1}$ as $\tau \to \infty$, where z_t is a stationary time series, $\gamma_z(\tau)$ its autocorrelation function, $d > 0$ and $g(\tau)$ is a slowly varying function as $\tau \to \infty$. The ACF decreases to zero at a hyperbolic rate, in contrast to the fast geometric rate observed for short-memory processes with $d \in (0, 1/2)$.

[7] As noted by Perron (1989), a time series with the presence of breaks or level shifts resembles the behavior of a non-stationary time series, which is equivalent to a very persistent process.

On the other hand, the next two columns show the test statistics for two types of trimming. All volatilities of commodity returns reject the null hypothesis of long memory with a significance level of 1%. This would indicate that commodity volatilities present discrete steps that can be interpreted as structural changes or strong shocks that permanently alter the level of volatility, simulating apparent long memory.

In summary, after analyzing the series of commodity prices, we observed: i) the high volatility of the series, accompanied by volatility clustering and high persistence, similar to that found in financial series; ii) certain differences between the commodities markets, suggesting a separate analysis for each series; and iii) the apparent long memory of the series is actually caused by discrete jumps in volatility, the occurrence of which is relatively low. In view of this evidence, it is reasonable to model the volatility of commodity returns using an econometric model of volatility including the possibility of level shifts.

In the econometric literature, the SV models have been improved to include level changes; for example, in the work done by Qu and Perron (2013). One advantage of this model is that volatility can be easily represented as the aggregation of two latent variables, one short term and one long term, the latter with level jumps, and both components can be estimated. GARCH type models also include level jumps, as in Stărică and Granger (2005). Another type of GARCH model, but applied to the volatility of oil prices, is developed by Charles and Darné (2014). Both models concur that level jumps are relevant in explaining the series with high persistence, but the jumps are exogenous to the model. In the present study we choose to follow the proposal of Qu and Perron (2013) and apply a SV model with random level shifts to model the volatility of commodity prices. The methodology used is described below.

3 Methodology

The SV model with random level shifts follows the estimation method and inference using Bayesian analysis of Qu and Perron (2013). The objective of the paper is to model volatilities of the returns of principal commodities exported by Peru with a short-memory component and random level shifts.

3.1 The Model

First, the process of the returns is mean corrected and is expressed by

$$x_t = \exp(\mu_t/2 + h_t/2)\epsilon_t, \tag{1}$$

where the error term ϵ_t is an *i.i.d.* standard Normal random variable. The term h_t gives us the stochastic volatility, while the second term μ_t expresses the random level-shifts component. The volatility h_t is explained by a stationary $AR(1)$ process with v_t as a Normal standardized error term:

$$h_t = \phi h_{t-1} + \sigma_v v_t. \tag{2}$$

On the other hand, the level-shifts component is given by the random Bernoulli variable δ_t that takes value 1 with probability p. Also, the size of the shift is stochastic and is given by the Normal standardized random variable η_t:

$$\mu_t = \mu_{t-1} + \delta_{t-1}\sigma_\eta \eta_t. \tag{3}$$

The random variables $\epsilon_i, v_j, \delta_k, \eta_l$ are mutually independent for all $1 \leqslant i, j, k, l \leqslant n$. The level-shifts component allows us to have different sized random shifts. Allowing for this characteristic of the process, we can determine the component h_t as a short-memory process for the variables analyzed.

Our proxy for volatility is given by the log-squared mean-corrected returns $\log x_t^2$, so our model can be expressed by the following form:

$$\log x_t^2 = h_t + \mu_t + \log \epsilon_t^2, \tag{4}$$

$$h_{t+1} = \phi h_t + \sigma_v v_t, \tag{5}$$

$$\mu_{t+1} = \mu_t + \delta_t \sigma_\eta \eta_t. \tag{6}$$

Because ϵ_t is Normally distributed, the model is a partial non-Gaussian state space model. The way of addressing this problem is by filtering, as in Kim, Shephard, and Chib (1998) with approximation of the term $\log \epsilon_t^2$ by a mixture of Normals. A new error process is defined by ϵ_t^* as $\epsilon_t^* = \log \epsilon_t^2 - E(\log \epsilon_t^2)$.

Following Kim, Shephard, and Chib (1998), we approximate the distribution of this new process using the mixture of Normals: $\epsilon_t^* \sim \sum_{i=1}^{K} q_i N(m_i, \sigma_i^2)$,where the parameters K, q_i, m_i, σ_i^2 that describe the distribution can be found in the work mentioned. We identify $w_t = j$, where w_t is assigned that value if ϵ_t^* is a realization of the j^{th} component of the mixture of Normals. This way of threatening the nonlinearity of $\log \epsilon_t^2$ allow us to puts all the models in a Gaussian state-space model conditioned on the mixture.

Finally, to complete the specification of the model we address the problem of return values close to zero that distorts the results of the estimations. We define another variable y_t by $y_t = \log(x_t^2 + c) - E(\log \epsilon_t^2)$, where c is a small number that renders the number inside the logarithm far away from the value of zero. This specification was first used by Fuller (1996) on the literature on stochastic volatility. The "offset" value c is 0.001, as in Qu and Perron (2013). At last, we have the model expressed by:

$$y_t = h_t + \mu_t + \epsilon_t^*, \tag{7}$$

$$h_{t+1} = \phi h_t + \sigma_v v_t, \tag{8}$$

$$\mu_{t+1} = \mu_t + \delta_t \sigma_\eta \eta_t, \tag{9}$$

with initial conditions $(h_0, \mu_o) = 0$ and $(h_1, \mu_1)' \sim N(0, P)$.

3.2 Sampling Procedure

We express variables and parameters in vector notations following Qu and Perron (2013). Let $\alpha_1 = (h_1, \mu_1)$, $R = \left\{(v_1, \eta_1)', ..., (v_T, \eta_T)'\right\}$, $\delta = (\delta_1, ..., \delta_T)$, $\omega = (\omega_1, ..., \omega_T)$, $\theta = (\phi, \sigma_v, \sigma_\eta, p)$ and $y = (y_1, ..., y_T)$. The location of shifts is related to the variable δ, whereas δ, R and α_1 jointly give the stochastic volatility process. Sampling from the joint posterior distribution $f(\theta, \alpha_1, R, \delta, \omega | y)$ is equivalent to sampling from the following four blocks: (i) $f\left(\theta_{(-p)}, \alpha_1, R | p, \delta, \omega, y\right)$, where $\theta_{(-p)}$ denotes the vector of parameters excluding p; (ii) $f(\delta | \theta, \alpha_1, R, \omega, y)$; (iii) $f\left(p | \theta_{(-p)}, \alpha_1, R, \delta, \omega, y\right)$; and (iv) $f(\omega | \theta, \alpha_1, R, \delta, y)$. Each of these blocks generates draws using the Gibbs sampling procedure.

3.3 Specification of Priors

We use the prior distribution of Kim, Shephard, and Chib (1998). For ϕ, we have $\pi(\phi) \propto \{\frac{1+\phi}{2}\}^{\phi^{(1)}-1}\{\frac{1-\phi}{2}\}^{\phi^{(2)}-1}$ with $\phi^{(1)}$, $\phi^{(2)} > \frac{1}{2}$. We set $\phi^{(1)} = 20$ and $\phi^{(2)} = 1.5$, implying a prior mean of 0.86. For the σ_{υ}, we use the Inverse-Gamma distribution so $\sigma_{\upsilon}^2 \sim \mathcal{IG}(\sigma_r/2, S_{\sigma}/2)$ with $\sigma_r = 5$ and $S_{\sigma} = 0.01 \times S_{\sigma}$. In the case of p and σ_{η}, we use the prior distribution of Qu and Perron (2013) which are the Beta and the Inverse-Gamma. For $p \sim Beta(\gamma_1, \gamma_2)$ with $\gamma_1 = 1$ and $\gamma_2 = 40$, which implies a prior mean of 1/41 or a shift each 41 days. For $\sigma_{\eta} \sim \mathcal{IG}(\sigma_r^*/2, S_{\sigma}^*/2)$ with $\sigma_r^* = 20$ and $S_{\sigma}^* = 60$, which implies a prior mean of 3.33 and a variance of 1.39. For the initial conditional state, we use diffuse priors with $(h_1, \mu_1) \sim N(0, P)$ with $P = diag\left(1 \times 10^6, 1 \times 10^6\right)$.

3.4 Filtering

In the filtering process we seek to recursively obtain a sample of draws from $(\alpha_t | X_t, \theta)$ for $t = 1, ..., T$. Then we use a particle filter like that of Kim, Shepard and Chib (1998), which, for a given sample of M, $\alpha_t^{(j)}$ $(j = 1, ..., M)$ is drawn from the distribution of $(\alpha_t | X_t, \theta)$, a sample from $f(\alpha_{t+1} | X_{t+1}, \theta)$ is obtained by drawing from $f[\alpha_{t+1} | \alpha_t^{(j)}, X_{t+1}, \theta]$, and they are reweighted using $f[\alpha_{t+1} | \alpha_{t+1}^{(j)}, X_{t+1}, \theta]$. The distribution $\frac{f(\alpha_{t+1}|\alpha_t^{(j)}, X_t, \theta)}{f(x_{t+1}|X_t, \theta)}$ depends on whether a shift occurs at time t and is given by $\alpha_{t+1} | [\alpha_t^{(j)}, X_t, \theta] \sim \delta_t W_{1t}^{(j)} + (1 - \delta_t) W_{2t}^{(j)}$ with

$$W_{1t}^{(j)} \sim N\left(\begin{bmatrix} \phi & 0 \\ 0 & 1 \end{bmatrix} \alpha_t^{(j)}, \begin{bmatrix} \sigma_{\upsilon}^2 & 0 \\ 0 & \sigma_{\eta}^2 \end{bmatrix}\right) \text{ and } W_{2t}^{(j)} \sim N\left(\begin{bmatrix} \phi & 0 \\ 0 & 1 \end{bmatrix} \alpha_t^{(j)}, \begin{bmatrix} \sigma_{\upsilon}^2 & 0 \\ 0 & 0 \end{bmatrix}\right).$$

The associated weights are given by $\omega_{t+1}^{(j)} = \frac{f[x_{t+1}|\alpha_{t+1}^{(j)}, X_t, \theta]}{\sum_{j=1}^{M} f[x_{t+1}|\alpha_{t+1}^{(j)}, X_t, \theta]}$, where $f[x_{t+1} | \alpha_{t+1}^{(j)}, X_t, \theta] \sim N[0, \exp(h_{t+1}^{(j)} + \mu_{t+1}^{(j)})]$.

4 Results

We apply this methodology to six indexes of SPGS: agriculture, livestock, gold, oil, industrial metals, and a general commodity index. The data are daily frequency over the period of January 1983 to December 2013. The products analyzed are the most representative of the Peruvian trade balance and, in many cases, their behavior has a big impact on the business cycles of the real economy. The analysis of commodity volatility is useful for both private and public agents. For the former, commodity volatility gives insights about risk management, while for the latter, it provides a better understanding of business cycles. We intend to describe the results in a comprehensive way, starting with a description of the posterior distribution results of each commodity, followed by an analysis of the contributions of the level-shift component over all volatility, and, finally, an analysis of the possible comovements between volatility and several indicators related to the Peruvian business cycle.

4.1 Posterior Distributions Results

The estimates of volatility parameters are shown in Table 3. A first interesting result is the level of probability of level shifts. Usually this probability is small; without taking into account gold, a break occurs between 300 to 1000 days. Another interesting result is the big differences between the variance of jumps σ^2_η and the variance of the short-memory component σ^2_ν. These findings are consistent with the theoretical proposal of Qu and Perron (2013) that jumps are uncommon events caused by structural breaks or big shocks that change the level of volatility abruptly and explain most of it. With respect to the size of the persistence of volatility, measured by the ϕ parameter, for most commodities the value of ϕ is between 0.93 and 0.98, which indicates that volatility shocks on average have a half-life from 9 days to 30 days, depending on the market analyzed. These findings are consistent with Qu and Perron (2013), who obtains similar stock index results when level shifts are counted, and runs counter to studies that hold long-memory assumptions; for example Vivian and Wohar (2012) estimate a half-life of between 90 and 300 days for commodity volatility shocks.

4.1.1 Commodity Index

As proposed above, we estimate a SV model for commodity prices as a whole. The model captures major shifts associated with huge shocks in the commodity markets. Panel b) of Figure 4 shows the level-shift component, the line with discrete changes, and the log volatility, with an overall measure of volatility that fluctuates around the level shifts. Some major jumps are associated with important events in commodities markets. For example, the jump that occurred in the beginning of 1986 was related to a crash in the oil markets. This crisis was a consequence of the "oil glut" in the first half of the 1980s. After a large expansion in oil production and a resultant surplus, oil prices fell by over 50% in 1986. The next major jump occurred during the Gulf War in response to fears of drastic cutbacks in oil production, from 1990 to 1991. The sequence of events and the evolution of volatility can be seen in the table below. First, the posterior mean of the level-shift variable μ_t is held low even during periods of tension, such as that between Iraq and Kuwait from July 15 to August 1, 1990, but when Iraq attacked Kuwait, the log volatility jumped significantly from -0.45 to 1.87[8]. Volatility remained high until the US-led coalition force attacked on January 17, 1991, then it decreased progressively to reach 1.42 at the end of January, when Iraq forces withdrew from Kuwait. After that, the level shift component fell to its previous levels of -0.44. After the war, the level of μ_t decreased and remained at these magnitudes for about three years. This result is consistent with the findings of Jacks, O'Rourke, and Williamson (2011) that point to high volatility periods in commodities during wars.

Date	07/15/1990-08/01/1990	**08/02/1990-01/16/1991**	01/17/1991-01/31/1991	02/01/1991-02/28/1991
Event	Tensions	**Attack by Iraq**	Attack by coalition	End of Gulf War
μ_t	-0.45	**1.87 to 1.89**	1.89 to 1.42	-0.44

On the other hand, two important increases in volatility were reported before the financial crisis of 2008: one at the beginning of 1996, and another in 2000. both were linked to US economic performance, but in opposing ways. The first was associated with a fall in the gold price due to

[8] These numbers theorically represent the level shift component of the volatility μ_t of commodity price returns. For example, according to (1), a value of $\mu_t = 0$ (as h_t component has mean zero and short variance) implies that commodity returns tend to a standard Normal distribution at any $t \geq 0$. For positive μ_t we have a commodity returns distribution with fat tails (more probability of extreme values), while for negative μ_t we have a distribution with a mass concentration around zero (less probability of extreme values). Thus, we are very interested in how the μ_t parameter evolves.

a strong dollar, while the second one owed to the *do-tcom* crash and the subsequent recession in the United States. Finally, the international financial crisis of 2008 caused a jump in volatility in several markets (oil, gold, and industrial metals). However, the jump in volatility occurred two months before the crash in September 2008, and the level stayed high for much longer than in previous crises. As we can see in the table below, log volatility increased progressively from 1.33 in April 2008 to 1.65 in July 2008, and then jumped very slightly to 1.72 and remained at that level for nine months. For this phenomenon, we venture some explanations. First, the increase in volatility was progressive and anticipated the crash due to a bad news sequence[9]. Therefore, the crash did not represent a great jump in volatility. Some studies, such as Cashin and McDermott (2002) and Vivian and Wohar (2012), highlight that commodity markets are always volatile and the last financial crisis did not necessarily represent a large increase in volatility over historical records. This fact is supported by our estimations; for example the level of volatility was higher during the Gulf War. However, our study provides new evidence of the duration of periods of high volatility, whereby volatility during the 2008 crisis remained high for a long time (nine months), more than in previous crises. Thereby, the magnitude of a crisis could be an important source of both the magnitude and the duration of volatility.

Date	01/02/08-04/24/08	04/25/08-07/01/08	**07/02/08-03/25/09**	03/26/09-11/09/09	11/10/09-12/31/09
Event	Pre-crash	Bad news	**Crash**	Post-crash	Recovery
μ_t	0.65	1.33 to 1.65	**1.72**	1.69 to 1.22	0.71 to 0.51

The method applied reproduces level shifts that are coherent with commodity market evolution and have permanent effects on the level of volatility[10]. An interesting result of the estimation is that shifts are uncommon. According to the posterior distributions reported in Figure 5 (see also Table 3), the probability of level shifts, p, has a posterior mean of 0.00149, which implies that a jump occurs each 671 days, roughly every 2.8 years. This makes a lot of of sense if we see jumps as being caused by rare and unexpected events with a big impact on commodity markets, such as wars, market crashes, recessions, or financial turmoil. Following with the parameters shown in Figure 5, we obtain the posterior density of the short-memory parameter ϕ with a mean of 0.948 and a 95% confidence interval of $(0.913, 0.971)$. This value indicates a persistence of the log volatility that is consistent with the theory, but it is less than in the long-memory process that reports autoregressive coefficients very close to 1. With respect to variances in volatility components, we find that level-shift variance has a posterior mean of 1.649, while the short-memory component has a posterior mean of 0.145. That is, perturbations on the permanent component, despite being rare, have a major impact on the volatility of the series. As we will analyze in the next section, this component is key to explaining changes in the volatility of commodities. The remaining panels in Figure 5 show the correlograms of the parameters. In general, these figures indicate that the Bayesian estimation have no problems related to autocorrelations, and therefore that the estimation is correct.

The estimation of parameters is robust to different priors. In Table 4 we report the posterior

[9] For example, the closure of the IndyMac Bank, the rescue of Fannie Mae and Freddie Mac, and several negative announcements about housing markets and financial indicators.

[10] We say permanent in the sense that the level shift holds until another structural change or big shock causes another jump.

means of commodity volatility under different priors. For example, we choose a range of prior of p from 0.0167 to 0.001, which implies level shifts of between 60 and 960 days. The results are not sensitive to this specification; the posterior mean of p are between 0.0013 and 0.0021, or a time occurrence of level shifts between 462 and 763 days that is consistent with our estimation of 671 days. The rest of the parameters remain unchanged. For example, the short-memory component has $\phi = 0.95$, while the variance of the level-shifts component is at least ten times higher than the variance of the short-memory component. We also change the prior of the variance of the level-shift component with very similar results. We repeat this exercise for the remaining commodities and find that posterior means and volatility components are not sensitive to prior specification. However, prior distributions do affect the level of autocorrelation of posterior distributions.

4.1.2 Industrial Metals

Now we turn our attention to the industrial metal index which includes copper, aluminium, lead, nickel and zinc. Copper, lead, and zinc are Peru's main exports; above all copper, which accounted for 23% of all exports from the country in 2013. The filtered volatility series and the shift levels are found in Figure 6 (see panels (b) and (c)). We can analyze whether the model identifies shifts that coincide with special events for this index. Specifically, the model identifies relevant positive shifts for 1987, 2006 and 2008. The table below shows the evolution of the level-shift component during 1987 and over the following three years. In the first four months of 1987, the level shift component was -0.79 on average, which was related to a slightly increase in the index price of 0.63% per month. Then, on April 20, the model was subject to a level shift that increased volatility and held it for six months. This period of high volatility coincided with a sharp increase in prices at a rate of 5% per month. The major shift occurred on October 20, a day after *"Black Monday"*[11], when volatility jumped from 0.55 to 2.28. Prices remained very volatile for the next six months, increased 70% in the first three months, to fall again to previous levels just two months later. After the crash, volatility dropped progressively and by the end of January 1991 a new level shift pushed down volatility to -0.72. This period coincided with the end of the Gulf War.

Date	01/02/87-04/16/87	04/20/87-10/19/87	**10/20/87-04/14/88**	04/15/88-01/23/91	01/24/91-03/23/91
Event	Price stability	Price rises	*Black Monday*	Post-crash	Price stability
μ_t	-0.79	0.55	**2.28**	1.78 to 0.90	-0.72

It is important to highlight that the volatility of industrial metals in 1987-1991 is explained mainly by supply and demand fundamentals. Even during the stock crash, the demand side would have been the channel of the impact of volatility on expectations, i.e. expectations of the agents or uncertainty about American economy. This argument is in the line with Brunetti and Gilbert (1995) where the high volatility in 1987-1990 is associated with tight demand. According to these authors, it was not until 1994 that industrial metals attracted hedge funds and investment institutions. They argue also that the participation of financial institutions in the metals market did not increase volatility relative to historically levels. This argument is examined in the next table, where we display the level-shift volatility component from 2006 to 2009, a period of huge financial speculation in commodity markets and interrupted by a financial crisis. The level shifts stayed low for more

[11] The S&P 500 fell by about 20% in a single day.

than ten years, from 1991 to mid-2006. However, on February 2006, a major shift occurred (the μ_t component jumped from 0.36 to 1.32). In this case the period of high volatility is explained by a mix of fundamentals and speculation. A commodity boom was caused mainly by high demand in developing countries, especially China, but market speculation contributed to a price rise of 50% in just six months. After this period, a new plateau was reached in which volatility fluctuated around one. Then, during the financial crash, volatility jumped to 1.51 and increased progressively for three months, coinciding with a collapse of 50% in price levels. Both periods, though highly volatile, did not reach the levels reported in 1988. This behavior is also highlighted by Vivian and Wohar (2012), but in the case of copper they do not find a significant difference between high volatility in recent years versus volatility in the 1980s.

Date	02/08/06	**02/09/06-08/11/06**	08/14/06-08/15/2008	**08/18/08-11/03/08**	11/04/08-09/02/09
Event	Pre-boom	**Market Boom**	Plateau	**Crash**	Post-crash
μ_t	0.36	**1.32 to 1.63**	0.97	**1.51 to 1.91**	1.90 to 1.30

Posteriors distributions and correlograms of the draws are found in Figure 7 (see also Table 3). The probability p has a posterior mean of 0.00292 which is higher than the value of p for the commodity index. This value of p implies that we have a shift occurring every 342 days, and this is still higher than our initial prior of every 41 days. The parameter ϕ has a mean value of 0.932, which implies a half-life cycle of 10 days, a very short-memory process. With respect to variances in volatility components, such as in the previous case, the level-shift component has a variance ten times that of the short-memory component. Jumps in volatility are caused by unusually big shocks, whereas small and regular shocks determine the stationary dynamic of volatility in the short term. In panel f), g), h), and i) we report the ACF for the posterior draws. The ACF decays around zero between the period 100 and 200, while the ACF is slightly out of the confidence bands for the parameters ϕ and σ_v.

4.1.3 Gold

Gold volatility has some characteristics that are different from the other commodities. First, it has averaged more **jumps** than other commodities, which can be clearly seen in Figure 8 (see panels (b) and (c)). Second, many of the periods identified as level shifts are not necessarily common to all commodities, such as breaks in the mid-90s, early 2000s, or late 2011. Third, if we look at the posterior distributions in Figure 9, the autoregressive component is about 0.1; i.e. very quickly converges to the average. Fourth, the difference between the size of the variance of the long-term and the short-term component is less than in other commodities. This would indicate that the volatility in gold has a very short memory, and the past **has little to do with** this volatility. The long-term impacts are not very large and the frequency is relatively higher. This finding is consistent with studies by Hammoudeh and Yuan (2008) and Batten, Ciner, and Lucey (2010) which show that gold is susceptible to various shocks such as economic crises, wars, changes in interest rates, or supply shocks and is generally more volatile than other metals. Another feature of gold is its dual role as a financial instrument and as a hedge against inflationary periods. This means that during periods of uncertainty, gold volatility can increase sharply, as in systems with high inflation expectations. The Table below shows this behavior through the presence of **jumps from level** to the mid-90s. First, from April 1993 until September of that year, an increase occurred in the component-level jumps in volatility due to inflation expectations for the US economy. Later, after interest rates increased throughout 1994, volatility fell instead of rising because officials had already adjusted interest rates. A similar phenomenon occurred prior to the crisis dot-com crisis in 2000; uncertainty about a possible bubble led to greater demand for gold among investors seeking a safe-haven asset. This entailed a rapid increase in volatility months before the crisis, and when the crisis erupted, the volatility of gold dropped instead of increasing, as most agents already had positions in this asset.

Date	04/1993-09/1993	02/1994-11/1994	**09/1999-10/1999**	03/2000-12/2000
Event	Inflation expectations	Interest rates up	**Uncertainty**	dot-com crash
μ_t	-1.27 to 0.00	-0.89 to -1.92	**-1.35 to 0.5**	0.05 to -0.92

From the above, it appears that gold level jumps seem to anticipate periods of crisis, in contrast to the volatility of other commodities which react primarily during periods of crisis. This idea is reinforced in the following table, for the periods prior to the 2008 financial crisis and the European debt crisis that intensified in 2012. Higher volatility is observed during the periods preceding these crises. This would indicate that the largely private operators, while not anticipating the crisis, did perceive a scenario of high-risk to their financial positions and therefore chose to use gold as a safe-haven, causing a sudden increase in its price and thus in the level of volatility. This pattern is repeated in the three crisis periods analyzed; that is, the level-shifts component anticipates periods of crisis. A study of this component as a predictor of the business cycle is beyond the scope of this research, but an interesting advantage of the method used is that it enables better analysis of the changes in volatility in relation to periods of crisis.

Date	**10/2007-08/2008**	09/2008-02/2009	**08/2011-10/2011**	01/2012-10/2012
Event	**Uncertainty**	Financial crisis	**Uncertainty**	European debt-crisis
μ_t	**-0.20 to 1.22**	1.24 to 1.11	**-0.47 to 1.10**	0.17 to -0.71

In Figure 9 (see also Table 3), we find posterior distributions and the correlograms for the draws. This index has a particular result in the parameter ϕ because its posterior mean is 0.078. This is the lowest value for the parameter ϕ and is close to zero, so the short-memory component has no persistence at all. Also, the volatility of the gold index has the largest probability of shifts of our six indexes. Posterior mean of p is 0.00684 or in terms of duration of the shift, it occurs every 146 days; this is the reason why we found so many shifts in this series. Another important result is that related to the parameter σ_ν that has the posterior mean value of 0.822, very high compared to the rest which have maximums of 0.15. This parameter gives us the variance of the shock to the short-memory component, so it implies that this component is very volatile for gold. In Figure 8, we find that gold undergoes many shifts during our period of analysis. Also, we report the ACF of posterior draws, where it can be seen that almost all parameters do not have autocorrelation problems with the exception of p, which falls to zero very slowly. We find that the ACF of p is sensitive to prior specification. For example, we explore a sensitivity analysis for the gold index, similar to that reported in Table 4 for the commodity index, and for some prior values the ACF converges rapidly to zero, while for others it does not.

4.1.4 Oil

In Panel a) of Figure 10 we show the series of oil price returns, and the level-shift component and the log volatility are represented in Panel b). The results are close to the ones obtained for the commodity index, which is to be expected because oil is the main component of the general index. There have been three major shifts in the evolution of oil volatility: first, the jump in volatility due to the "oil glut" of 1985 to 1986; second, the jump related to the Gulf War of 1990 to 1991; and finally, the period of high volatility during the international financial crisis of 2008. Just as we reviewed the impact of the Gulf War period in our analysis of the commodity index, now we will to look at the oil glut of the mid-1980s as well as the last financial turmoil. As regards the former, the table below shows the behavior of the level-shift component μ_t from 1985 to 1986. Almost right throughout 1985, the level of volatility remained low (around 0.16). In parallel, many negotiations between OPEC members were carried out in order to regulate overproduction. However, these negotiations failed and in December of 1985 a price war began, causing prices to fall by more than 50% over the next three months. High volatility was exacerbated by the Iraq-Iran war, and continued until August 1986 when OPEC finally came to an agreement.

Date Event	04/26/85-12/03/1985 Negotiations	**12/04/85-01/22/1986** **Price war**	**01/23/86-08/13/1986** **Price war**	08/14/86-10/01/1986 OPEC Agreement
μ_t	0.16	**1.88**	**2.13 to 2.70**	0.84

In our above examination of the commodity index during the last financial crisis, the probabilities of jumps was under 0.5, and in that case we argue that a possible explanation for this may be the mix of commodities with different volatility paths. This is also the case of oil, where, as opposed to gold, level shifts occurred on differences dates, and as we can see in Panel c) of Figure 10 they have a strong probability of occurring at the beginning and the end of a crisis. The level of volatility prior to crisis is estimated at 1.23, and was relative stable from the beginning of the 2000s. But it underwent a big jump a few weeks before the Lehman Brothers bankruptcy and stayed high six months after the crash (see Table below). This "long" period of high volatility was consistent with other commodities and with the estimations of Qu and Perron (2013) for the S&P 500 Index, and reflects the magnitude of the last financial crisis in comparison to previous crises.

4.1.5 Agriculture

In Figure 11 (see also Table 3) we show the posterior distributions of parameters. The posterior mean of probability p has a value of 0.00178, which implies a shift every 562 days. That is to say, level shifts are rare events, but when it happens, its variance σ_η^2 is ten times higher than the variance of the short-memory volatility component σ_v^2. Another important feature is the autoregressive estimator, which is 0.942, implying a half-life cycle of 12 days, very close to the cycle of industrial metals. As with other commodities, persistence of volatility is manifested through high values of ϕ, but lower than 1. These findings are the opposite of Vivian and Wohar (2012) who assume a long-memory process, but in accordance with Charles and Darné (2014) who include structural changes in the behavior of volatility. The ACF reported in panels f) to i) present some autocorrelation problems. Similarly to the case of gold, the ACF is sensitive to prior specification, but this does not affect the estimation of volatility. The agriculture index is constructed with information on the following commodities: wheat, corn, soybeans, coffee, sugar, cocoa, and cotton. The majority of these commodities are import products for Peru with the remarkable exception of coffee, which is an important export for that country.

In Figure 12, we can observe that shifts are rare and the model identifies three major shifts that increased volatility, which coincides with the specific context of agriculture commodities. In 1988, the volatility of the index increased dramatically between May and August of that year. This volatile period was related to the drought conditions in the United States, which caused an increase in the prices of wheat, corn and soybeans produced in that country. The increases in volatility are identified by the shift component of the model, which rose from -0.52 to 1.18 in May of 1988 and stayed there for three months before dropping to -0.46 at the end of August of that year, as observed in the Table below.

Date	03/09/1988-05/12/1988	**05/13/1988-08/30/1988**	08/31/1988
Event	Low volatility	**Drought**	Low volatility
μ_t	-0.52	**1.18**	-0.46

In 2007, the model identified two major shifts coinciding with the world food-price crisis, marked by prices increases of these commodities for different reasons, such as financial speculation and the use of food for fuel. On March 30, 2007, the level-shift component rose from 0.03 to 0.46 and remained at that level until May 18, when other high shifts increased that component to 0.92. After that period, the model identified a regime where the level-shift component stayed at high levels of between 0.93 to 0.73 from May 2007 to October 2012. However, our model shows that the long period of high volatility in food prices came to an end in October 2012, which ushered in two major downward shifts that saw the level-shift component move to 0.40 and -0.02, respectively.

Date	07/28/06-03/29/2007	**03/30/07-05/17/2007**	05/18/07-10/01/2012	10/02/2012	10/22/2012
Event	Low volatility	**Speculation**			Low volatility
μ_t	0.03	**0.46**	0.92 to 0.73	0.40	-0.02

In Figure 13 (see also Table 3), we can find the posteriors distributions and correlograms of the draws for the 4 parameters. The probability p has a posterior mean of 0.00099 which is very different from the prior of 1/41 and indicates that the probability of shifts is very low. This implies that on average a shift occurs every 1010 days. Also, we find that parameter ϕ is 0.973 where the implicitly half-life cycle of short-memory component is 25 days, doubling the size of industrial or oil volatilities. As to the variances of volatility components, we find a posterior mean of σ_η equal to 1.65, and for σ_ν the posterior mean is 0.12. Similarly to other indexes, the variance of the level-shift component is ten times higher than the variance of the short-memory component. In general, estimators behave according to expectations, as do draws of posterior distributions do not present problems of serial correlation. As shown in panels f) to i), the ACF decays to a maximum of zero in 50 periods for all parameters.

4.1.6 Livestock

Finally, the analysis of Livestock volatility will not be so exhaustive because it is not of central importance to the external trade of Peru. The results can be found in Figure 14, where it is observed that livestock volatility stays constant in perfectly identified regimes of volatility. Livestock volatility exhibits the lowest number of shifts in volatility. The posterior parameters can be found in Figure 15 and in Table 3 and reinforce the results observed in the evolution of the series. We find that the posterior mean of p is 0.00081, which is the lowest value for all of the probabilities of shifts in our series. What is more, the lowest value of the confidence interval of probabilities p is 0.00016, which is very close to zero. On average, a shift is expected every 1234 days, so shifts are very rare. Also, the parameter ϕ has a value of 0.977; thus we have more persistence for the short-memory component of the volatility than for other commodities, with an implicit half-life cycle of 30 days. In this case, the short-memory component has the lowest variance ($\sigma_v = 0.076$) in comparison with other indexes, while the variance of the level-shift component is twenty times higher. Although level shifts are very uncommon events, they impregnate high variation in volatility. As regards the serial correlation of draws, only the ACF of σ_η holds in bandwidths.

4.2 Contributions to the Overall Variation in Volatility

The model has the particular feature of splitting the global volatility in two components: a level shifts and a short-memory component. If we contend that this model can replicate empirical features of the data, we must analyze whether this decomposition is significant. To this end, we divide the contributions to overall volatility following Qu and Perron (2013): $s_t = \mu_t + h_t$ with s_t being the overall volatility, μ_t and h_t are the level shifts and the short-memory components, respectively. If we denote the sample means of the correspondent processes by $\overline{s}$, $\overline{\mu}$ and $\overline{h}$, then we obtain $(s_t - \overline{s}) = (\mu_t - \overline{\mu}) + (h_t - \overline{h})$, so the following ratios

$$\frac{\sum_{i=1}^{n}(\mu_t - \overline{\mu})^2}{\sum_{i=1}^{n}(s_t - \overline{s})^2} \quad \text{and} \quad \frac{\sum_{i=1}^{n}(h_t - \overline{h})^2}{\sum_{i=1}^{n}(s_t - \overline{s})^2},$$

give us the contributions of μ_t and h_t to the global variation in volatility of our indicators. Qu and Perron (2013) find that the level-shifts component is more important than the short-memory component in explaining the variations in volatility of the S&P 500 and Nasdaq daily returns.

Table 5 outlines our results for the six indexes and finds similar results to Qu and Perron (2013) for all cases except for livestock volatility. The level-shifts component goes a long way to explaining the variation in volatility. The maximum contribution of the level-shift component to volatility is 0.84, and corresponds to industrial metals. The gold index and commodities index level-shifts components closely follows Industrial metals in contribution to the overall variation in volatility. Those volatilities have different evolutions as observed in Section 4.1, but what they have in common is that accounting for level-shift components is relevant for volatility modelling. The agriculture level-shifts component accounts for 54 percent of the variation in volatility, which is significant but less than the others. This is similar to what is observed for livestock volatility, and these are the cases where the level-shift component explains the lower variation in volatility. However, as seen before, it explains more than 50% percent of volatility with not many shifts. Finally, the oil index has the same results as the commodities index because it is the main component thereof and has many shifts, though less than gold.

With this measurement, we can conclude that variation in volatility can be better predicted with the level-shifts component; this is less volatile than the short-memory component, which is a noisy process. Therefore, commodities volatilities can be better predicted and analyzed with a level-shifts framework instead of a long-memory analysis.

4.3 Business Cycle Comovements

An important aspect of commodities index volatility is the presence of comovements with business cycle indicators in small and commodity-exporting economies like Peru. We estimate the correlation between components of commodity-return volatility and some indicators of the Peruvian economy using common regressions. The indicators used are cement consumption, electricity production, expectations of the economy,[12] and money supply, because these are observed constantly by private and government analysts in Peru. Also, we measure the correlations obtained between volatility of commodities with some indicators of production: total and sectorial gross domestic product (GDP), where the sectors analyzed are agriculture, mining, construction, and manufacture.

The data is obtained from the Central Bank of Peru in monthly frequencies. Thus, we adapt our results of the level-shifts component, short-memory component, and the overall volatility of the series to monthly data, with monthly averages. After the transformation of frequency, we get the correlations with the interannual variation of the business cycle indicators. The results are presented in Table 6. First, all commodity-price volatilities are correlated with business cycle indicators, but not in the same direction. Industrial minerals and oil volatility present a positive correlation, and gold a negative one. This may be explained by the correlation between financial markets and gold volatility, while some periods of high volatility in industrial minerals or oil have been linked to the commodities boom. Second, only gold is a significant variable in explaining expectations, which suggests the relevance of gold volatility as an indicator of financial stability and therefore of outcome performance in the future.

It is to be expected that industrial metals and oil will be highly correlated to business cycle indicators, and this is the case for the indicators of cement consumption of cement consumption and electricity production. Also, we get some spurious correlations of the agriculture and livestock indexes volatilities with those indicators because they are not expected to affect or to be affected by the Peruvian business cycle.

In addition, we obtain some correlations with GDP indicators. Agriculture volatility is correlated positively with total GDP and agriculture GDP. Moreover, Industrial metals and oil volatilities are highly correlated with total, manufacturing, and construction GDP, while the correlations with mining GDP are not high but still significant. Gold volatility does not present a correlation with total and mining GDP. Finally, the volatility of the index of all commodities shows correlations with total and all-sector GDPs because it is mainly composed of oil and industrial metal indexes.

When we analyze the correlations for all the components, we find that the short-memory component h_t has no correlation at all with the indicators of the business cycle. The level-shifts component

[12] Expectations indicator constructed the Central Bank of Peru.

accounts for all the correlation that the volatility of commodities index has with economic activity indicators. These could be interpreted as meaning that the level-shifts component captures macroeconomic drivers behind volatility, while the short-memory component accounts for the noise of daily activity in commodities markets.

4.4 Analysis of Residuals

One way of observing whether the model fits our analysis of the data well is by studying the behavior of residuals. From Equation (1) we find that $x_t = \exp(\mu_t/2 + h_t/2)\epsilon_t$ and the series x_t, h_t and μ_t are outputs from the estimation and filtering. Hence, $\widehat{\epsilon_t}$ could be extracted directly from our results as an estimation of ϵ_t. The assumptions are that ϵ_t is *i.i.d.* with Normal distribution. Therefore, we can observe whether the standardized estimated residuals $\widehat{\epsilon_t}$ behave as Gaussian and are independent by applying some well known graphical analysis.

The QQ plots are used to ensure that our residuals approximate a random variable with Normal distribution. To analyze independence in estimated residuals we can study the ACF of residuals and squared residuals. However, as the returns do not exhibit autocorrelations, we only need to determine whether our measurements of volatility of the residuals present autocorrelations. The results presented include the Figures of the ACFs obtained from the log-squared residuals ϵ_t.

Figures 16 and 17 present the results of the residual analyses of Commodity, Industrial metals, gold, oil, agriculture, and livestock indexes, respectively. All of the series have the characteristic that their estimated residuals $\widehat{\epsilon_t}$ do not exhibit significant autocorrelation in their log-squared and absolute values[13]. The values of the autocorrelation are in general less than 0.05, and are inside the Bartlett windows.

On the other hand (see Figure 17), each of the series do not have the same QQ plot results. Estimated standardized residuals for Agriculture and Livestock present the best QQ-plots results in the sense that their estimated distribution approximates the standard Normal distribution more. However, Gold index residuals do not have the same behavior. They exhibit large fat tails that indicate the presence of large shocks, even though we include the level-shifts component. This is not in fact surprising, because the gold index is the most volatile of all the six indices analyzed. Finally, industrial metal, oil, and overall commodities indices exhibit reasonable QQ-plots results.

5 Conclusions

This study models the volatility of the commodities indexes of the S&P GSCI following the methodology of Qu and Perron (2013), which includes random level shifts in the SV model of Kim, Shephard, and Chib (1998).

The main results seem to confirm the relevance of shifts in the volatility of the studied series. After considering these breaks, the alleged long-memory disappears and volatility converges to its mean in a short period of time. The persistence of the short-memory component is lower than one so the average life of a shock reduces compared to standard SV models. However, the exception is the livestock index, which presents extremely rare shifts, and these shifts do not explain variations in volatility. Moreover, the persistence of its noise component is close to one. Despite these results,

[13] The ACF of absolute value of residuals has also been performed but not reported; the results indicate no problems of serial correlation. They are available upon request.

the livestock index is not so important to Peruvian trade. Likewise, the gold index has different results because it exhibits so many shifts and the parameter ϕ is close to zero.

Shifts are rare in volatilities but they account for most of their variation for all commodity indexes. It is not important that gold has more shifts than industrial metals or oil more than agriculture; in all cases, the level-shift component is significant in volatility modelling.

The analysis of residuals shows that autocorrelation in the log-squared and absolute-value of standardized residuals disappears. This means that the model captures all of the second-moment autocorrelations of the series. The QQ plot gives us similar results, with the standardized residuals being close to the Normal distribution as assumed by the model, with the exception of the gold index which has fat tails.

Finally, we find that the components of level shifts in the volatility of commodity prices are strongly correlated with indicators of the Peruvian economic cycle, such as capital goods imports, expectations of the economy, electricity production, and internal cement consumption. However, Livestock index and Agriculture index are the exception, as they do not account for much of the international trade of Peru. Not only that, if we include indicators of sectorial gross domestic product, the volatility is still highly correlated with interannual variations of these indicators.

With the new estimated parameters, we can construct better measurements of risk for the commodities prices to help private companies, or to create special government funds in order to avoid being affected by highly volatile prices of traded commodities.

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Table 1. Composition of SP GSCI

	Weight	Included Commodities
Energy	69.8%	-
- Oil	67.2%	-
- Natural Gas	2.6%	-
Industrial Metals	6.7%	-
- Copper	3.2%	-
- Others	3.5%	Aluminum, Lead, Nickel and Zinc
Precious Metals	3.3%	-
- Gold	2.8%	-
- Silver	0.5%	-
Agriculture	15.3%	Wheat, Corn, Soybeans, Coffee, Sugar, Cocoa, Cotton
Livestock	4.9%	Lean Hogs, Live Cattle, Feeder Cattle

Source: S&P GSCI Methodology, 2014.

Table 2. Test Against Spurious Long Memory

	$\widetilde{d}$ (local Whittle)	W ($\varepsilon = 0.02$)	W ($\varepsilon = 0.05$)
Commodity Index	0.37	2.14**	2.14**
Copper	0.41	2.03**	1.63**
Gold	0.37	1.56**	1.37*
Oil	0.34	2.03**	2.03**
Agriculture	0.34	1.84**	1.84**
Livestock	0.24	2.12**	1.79**

H_0: series is a stationary long-memory process, H_1: series is affected by regime change or a smoothly varying trend.

Table 3. Posterior Estimates for Commodities Indexes volatilities

Index	Parameters			
	p		ϕ	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
Commodity Index	0.00149	[0.00068, 0.00258]	0.948	[0.913, 0.971]
Agriculture	0.00099	[0.00035, 0.00198]	0.973	[0.960, 0.983]
Livestock	0.00081	[0.00016, 0.00177]	0.977	[0.959, 0.992]
Industrial Metals	0.00292	[0.00166, 0.00451]	0.932	[0.902, 0.960]
Oil	0.00178	[0.00079, 0.00319]	0.942	[0.914, 0.964]
Gold	0.00684	[0.00461, 0.00949]	0.078	[0.012, 0.177]

Index	Parameters			
	σ_ν		σ_η	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
Commodity Index	0.145	[0.107, 0.195]	1.649	[1.273, 2.157]
Agriculture	0.123	[0.101, 0.147]	1.650	[1.260, 2.187]
Livestock	0.076	[0.056, 0.104]	1.645	[1.245, 2.213]
Industrial Metals	0.152	[0.118, 0.189]	1.479	[1.177, 1.891]
Oil	0.168	[0.133, 0.206]	1.652	[1.271, 2.147]
Gold	0.822	[0.773, 0.878]	1.267	[1.064, 1.531]

Table 4. Posterior Means for Commodity Index Volatility Under Different Priors

(a) Vary γ_1

Parameter	$\gamma_1 = 0.25$		$\gamma_1 = 4$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00146	[0.00068, 0.00265]	0.00216	[0.00108, 0.00352]
ϕ	0.945	[0.915, 0.967]	0.948	[0.916, 0.970]
σ_ν	0.147	[0.113, 0.186]	0.140	[0.108, 0.181]
σ_η	1.626	[1.264, 2.143]	1.610	[1.242, 2.112]

(b) Vary γ_2

Parameter	$\gamma_2 = 60$		$\gamma_2 = 960$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00180	[0.00089, 0.00304]	0.00129	[0.00060, 0.00222]
ϕ	0.933	[0.887, 0.963]	0.952	[0.929, 0.970]
σ_ν	0.158	[0.117, 0.212]	0.144	[0.114, 0.179]
σ_η	1.566	[1.226, 2.018]	1.628	[1.266, 2.134]

(c) Vary σ_r^*

Parameter	$\sigma_r^* = 10$		$\sigma_r^* = 40$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00138	[0.00063, 0.00236]	0.00170	[0.00076, 0.00302]
ϕ	0.936	[0.895, 0.965]	0.942	[0.911, 0.968]
σ_ν	0.159	[0.115, 0.210]	0.149	[0.111, 0.197]
σ_η	2.026	[1.487, 2.794]	1.217	[1.000, 1.491]

(d) Vary S_σ^*

Index	$S_\sigma^* = 30$		$S_\sigma^* = 120$	
	Posterior Mean	95% credible set	Posterior Mean	95% credible set
p	0.00162	[0.00072, 0.00287]	0.00131	[0.00060, 0.00224]
ϕ	0.951	[0.917, 0.975]	0.949	[0.917, 0.970]
σ_ν	0.140	[0.105, 0.191]	0.145	[0.113, 0.190]
σ_η	1.271	[0.969, 1.664]	2.195	[1.702, 2.870]

Table 5. Contributions to overall volatility

Index	Component: Level Shift	Component: Stationary
Commodities	0.81	0.14
Agriculture	0.54	0.35
Livestock	0.52	0.36
Industrial Metals	0.84	0.10
Oil	0.70	0.23
Gold	0.80	0.17

Note: The contributions are obtained from the decomposition $s_t = \mu_t + h_t$ where μ_t corresponds to the level shifts component while h_t is the stationary component. The contributions to the overall volatilities are obtained from the following: $\frac{\sum_{i=1}^{n}(\mu_t - \overline{\mu})^2}{\sum_{i=1}^{n}(s_t - \overline{s})^2}$ and $\frac{\sum_{i=1}^{n}(h_t - \overline{h})^2}{\sum_{i=1}^{n}(s_t - \overline{s})^2}$

Table 6. Comovements between Volatility Components and Business Cycle Indicators

Panel (a). Agriculture Index						
	μ_t		h_t		μ_t+h_t	
Variables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	10.64	0.16	-0.67	0.00	5.89	0.09
	(6.70)		(-0.29)		(4.68)	
Production of Electricity	3.82	0.06	0.21	0.00	2.26	0.04
	(3.72)		(0.15)		(2.88)	
Expectations of the economy	4.18	0.01	-22.16	0.09	-2.51	0.00
	(1.04)		(-3.47)		(-0.77)	
Money Supply	-2.17	0.00	0.28	0.00	-1.13	0.00
	(-0.63)		(0.06)		(-0.44)	
GDP	7.37	0.45	2.09	0.01	5.34	0.36
	(9.95)		(1.24)		(8.21)	
Agriculture production	4.59	0.16	3.56	0.03	3.84	0.17
	(4.71)		(2.02)		(4.89)	

Table 6 (continued). Comovements between Volatility Components and Business Cycle Indicators

Panel (b). Industrial Metals Index

Variables	μ_t Coefficient (t-stat)	μ_t R^2	h_t Coefficient (t-stat)	h_t R^2	μ_t+h_t Coefficient (t-stat)	μ_t+h_t R^2
Consumption of Cement	9.35	0.25	4.26	0.01	8.33	0.23
	(8.63)		(1.16)		(8.27)	
Production of Electricity	4.42	0.15	2.01	0.00	3.94	0.14
	(6.32)		(0.90)		(6.09)	
Expectations of the economy	0.63	0.00	-14.11	0.01	-0.27	0.00
	(0.23)		(-1.33)		(-0.10)	
Money Supply	5.97	0.02	6.52	0.00	5.73	0.02
	(2.39)		(0.90)		(2.49)	
GDP	5.62	0.56	-1.13	0.00	4.91	0.48
	(12.29)		(-0.42)		(10.52)	
Mining GDP	1.69	0.05	4.25	0.02	1.75	0.06
	(2.50)		(1.59)		(2.75)	
Manufacturing GDP	6.30	0.31	-2.88	0.00	5.40	0.26
	(7.37)		(-0.72)		(6.48)	
Construction GDP	10.91	0.44	1.31	0.00	9.73	0.39
	(9.60)		(0.22)		(8.76)	

Table 6 (continued). Comovements between Volatility Components and Business Cycle Indicators

Panel (c).Gold Index						
	μ_t		h_t		μ_t+h_t	
Variables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	-1.82	0.02	7.33	0.00	-1.71	0.02
	(-2.00)		(0.76)		(-1.90)	
Production of Electricity	-2.42	0.08	7.99	0.01	-2.29	0.08
	(-4.54)		(1.37)		(-4.34)	
Expectations of the economy	-20.75	0.17	15.56	0.00	-19.16	0.15
	(-5.12)		(0.46)		(-4.84)	
Money Supply	-14.37	0.28	1.01	0.00	-14.02	0.27
	(-9.77)		(0.05)		(-9.60)	
GDP	0.67	0.00	-0.43	0.00	0.62	0.00
	(0.60)		(-0.05)		(0.58)	
Mining production	2.45	0.04	7.16	0.01	2.39	0.04
	(2.23)		(0.78)		(2.25)	

Table 6 (continued). Comovements between Volatility Components and Business Cycle Indicators

Panel (d). Oil Index						
	μ_t		h_t		μ_t+h_t	
Variables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	4.71	0.19	-7.32	0.02	4.01	0.15
	(7.37)		(-2.34)		(6.36)	
Production of Electricity	4.72	0.53	-1.68	0.00	4.28	0.47
	(15.85)		(-0.88)		(14.13)	
Expectations of the economy	-0.82	0.00	-18.78	0.03	-1.55	0.00
	(-0.41)		(-1.98)		(-0.80)	
Money Supply	12.58	0.30	-4.17	0.00	11.25	0.26
	(10.35)		(-0.70)		(9.45)	
GDP	4.40	0.60	-3.26	0.02	4.04	0.54
	(13.48)		(-1.35)		(11.74)	
Mining production	2.13	0.14	0.21	0.00	2.03	0.13
	(4.39)		(0.09)		(4.28)	
Manufacturing production	5.14	0.37	-7.65	0.04	4.56	0.31
	(8.35)		(-2.14)		(7.24)	
Construction production	7.93	0.41	-10.49	0.03	7.09	0.34
	(9.04)		(-1.99)		(7.87)	

Table 6 (continued). Comovements between Volatility Components and Business Cycle Indicators

Panel (e).Commodities Index						
	μ_t		h_t		μ_t+h_t	
Covariables	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2	Coefficient (t-stat)	R^2
Consumption of Cement	5.62	0.09	-6.35	0.02	3.62	0.05
	(4.59)		(-1.98)		(3.28)	
Production of Electricity	5.27	0.20	-3.57	0.01	3.69	0.13
	(7.61)		(-1.83)		(5.75)	
Expectations of the economy	-1.19	0.00	-35.84	0.11	-4.10	0.02
	(-0.38)		(-3.96)		(-1.46)	
Money Supply	-1.67	0.00	-0.63	0.00	-1.43	0.00
	(-0.68)		(-0.10)		(-0.65)	
GDP	5.53	0.40	-0.73	0.00	4.38	0.31
	(8.84)		(-0.30)		(7.30)	
Agricultural production	3.56	0.15	0.16	0.00	2.87	0.12
	(4.53)		(0.06)		(4.01)	
Mining production	2.58	0.08	4.13	0.02	2.43	0.09
	(3.32)		(1.70)		(3.51)	
Manufacturing production	6.20	0.22	-5.71	0.02	4.48	0.14
	(5.84)		(-1.59)		(4.49)	
Construction production	10.41	0.29	-5.93	0.01	7.84	0.21
	(6.99)		(-1.12)		(5.55)	

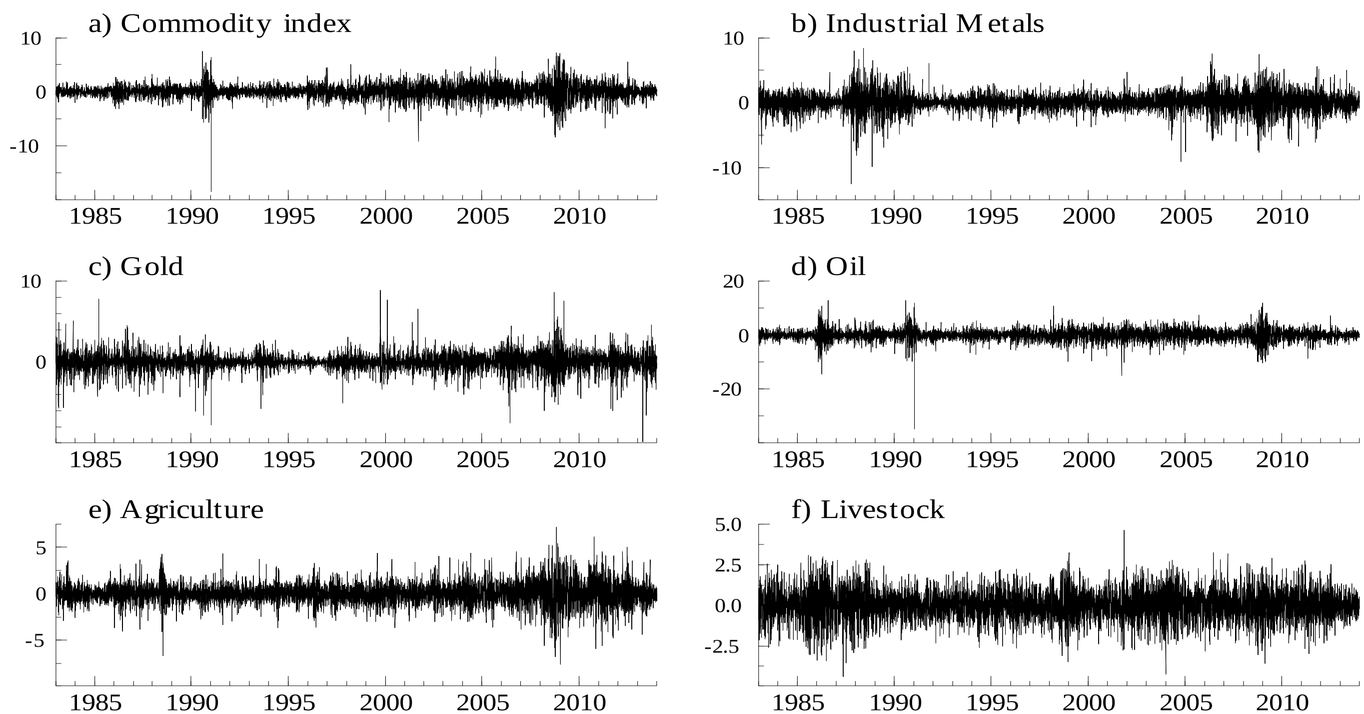

Figure 1. Returns of Commodities

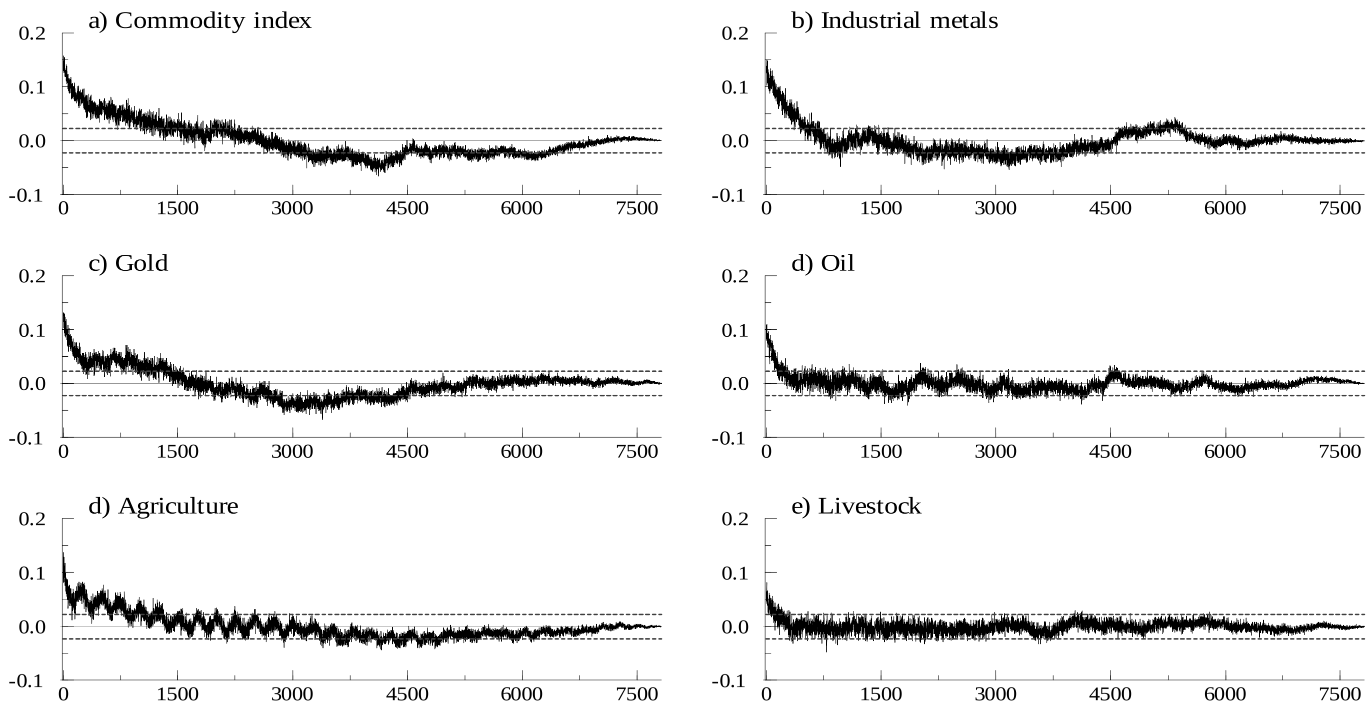

Figure 2. The autocorrelation function of commodities log-squared returns

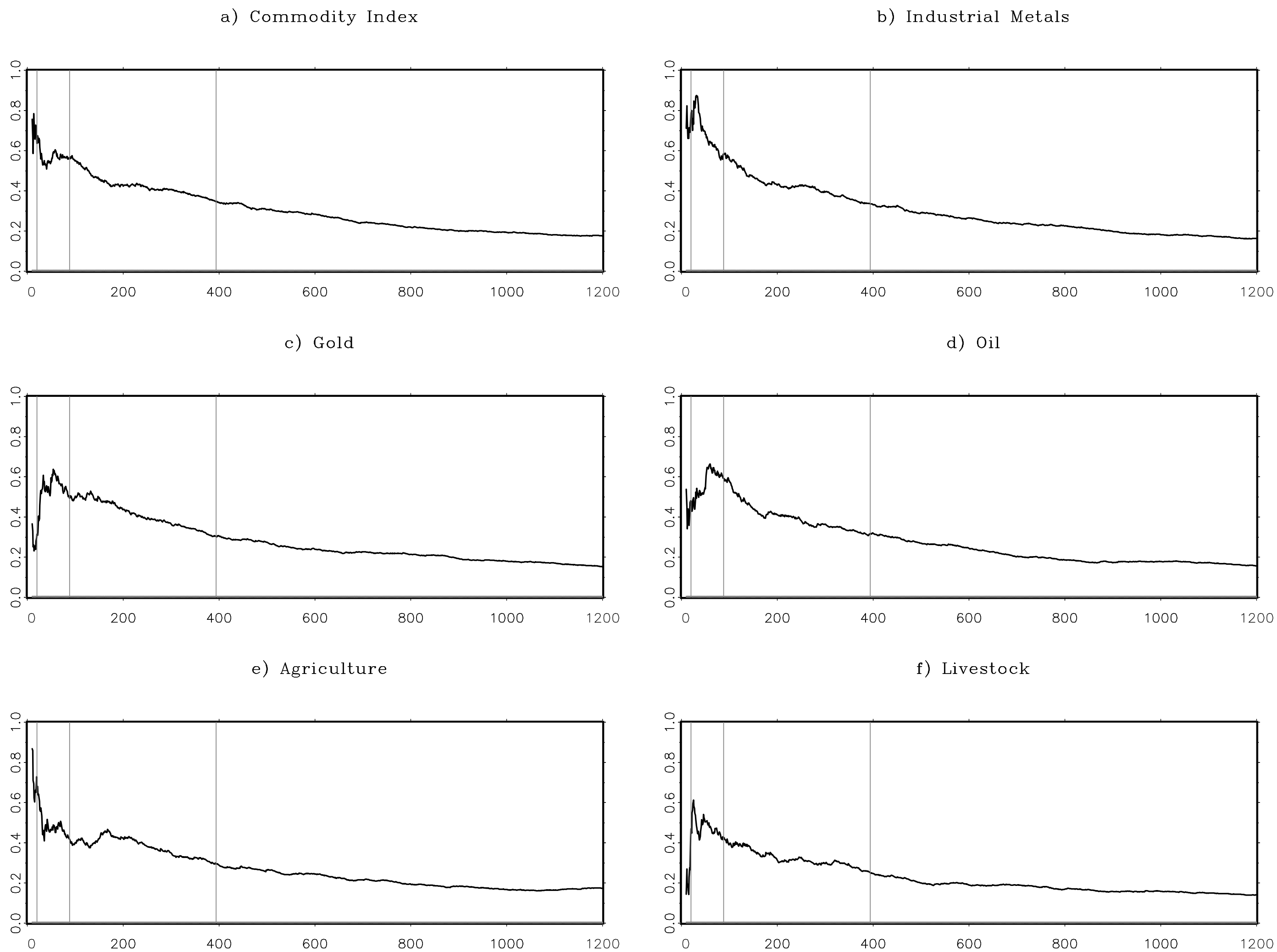

Figure 3. The Log Periodogram estimate of Fractional d as a function of m

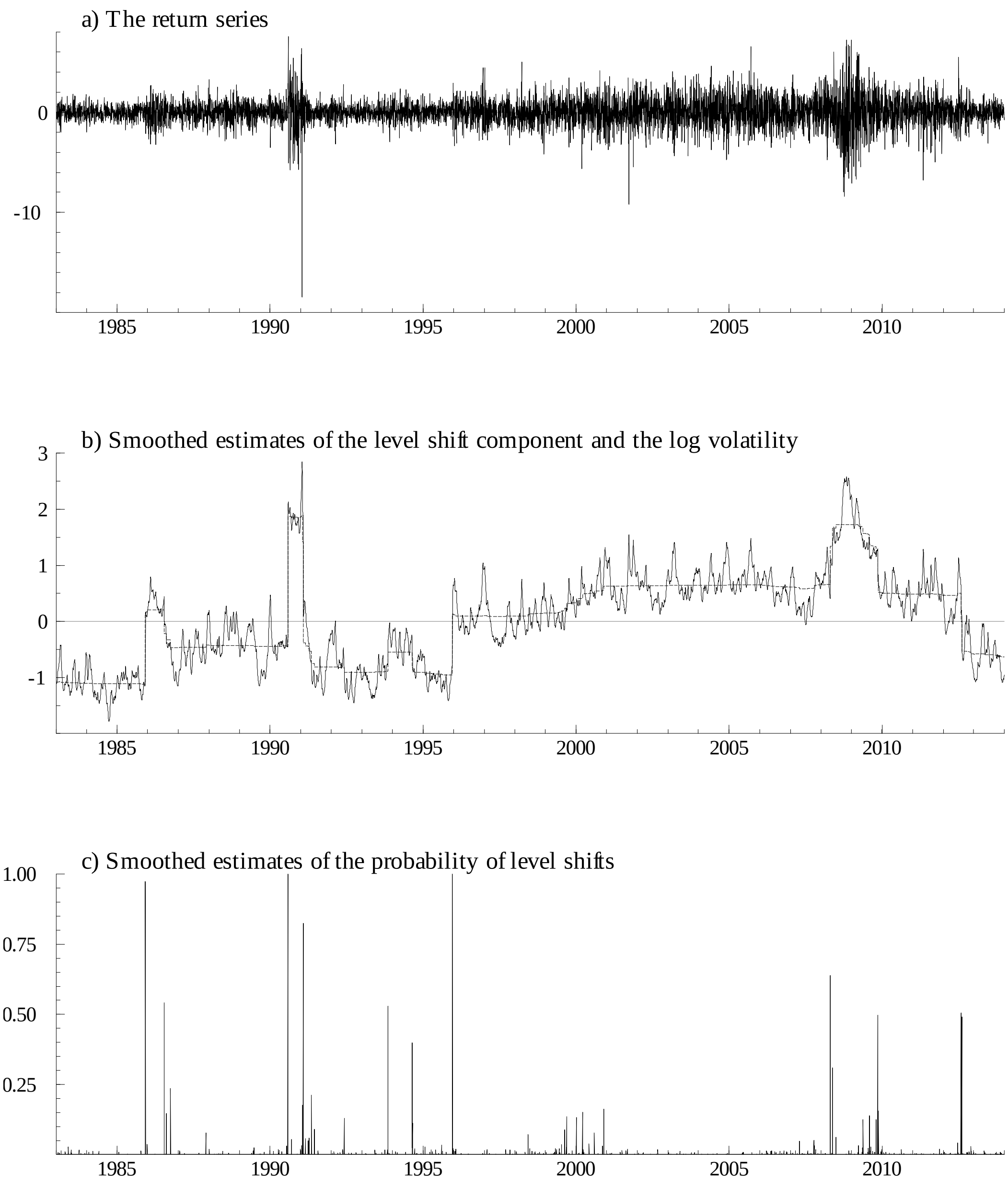

Figure 4. Results for Commodity Index Volatility

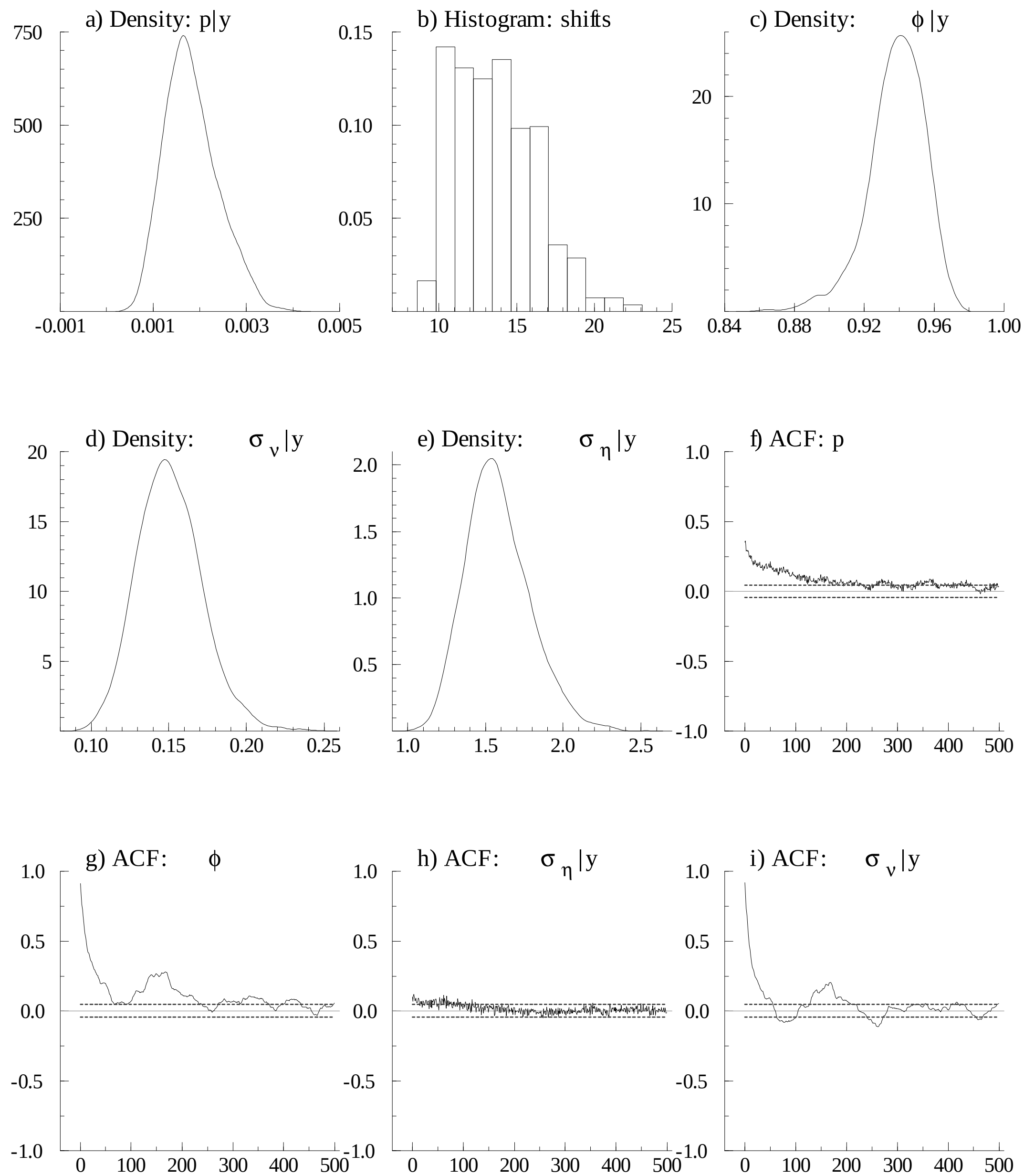

Figure 5. Posterior Estimates for Commodity Index Volatility

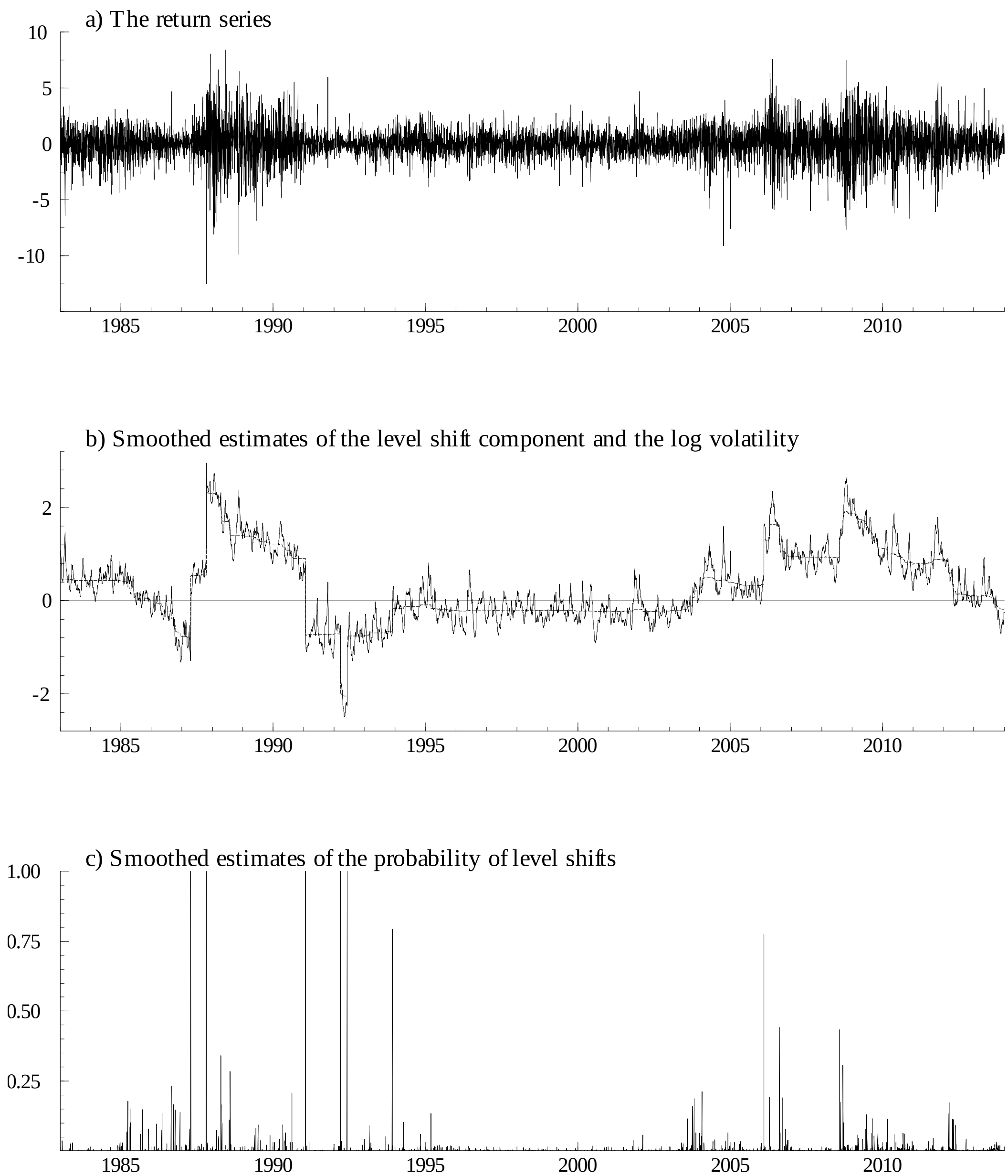

Figure 6. Results for Industrial Metals Index Volatility

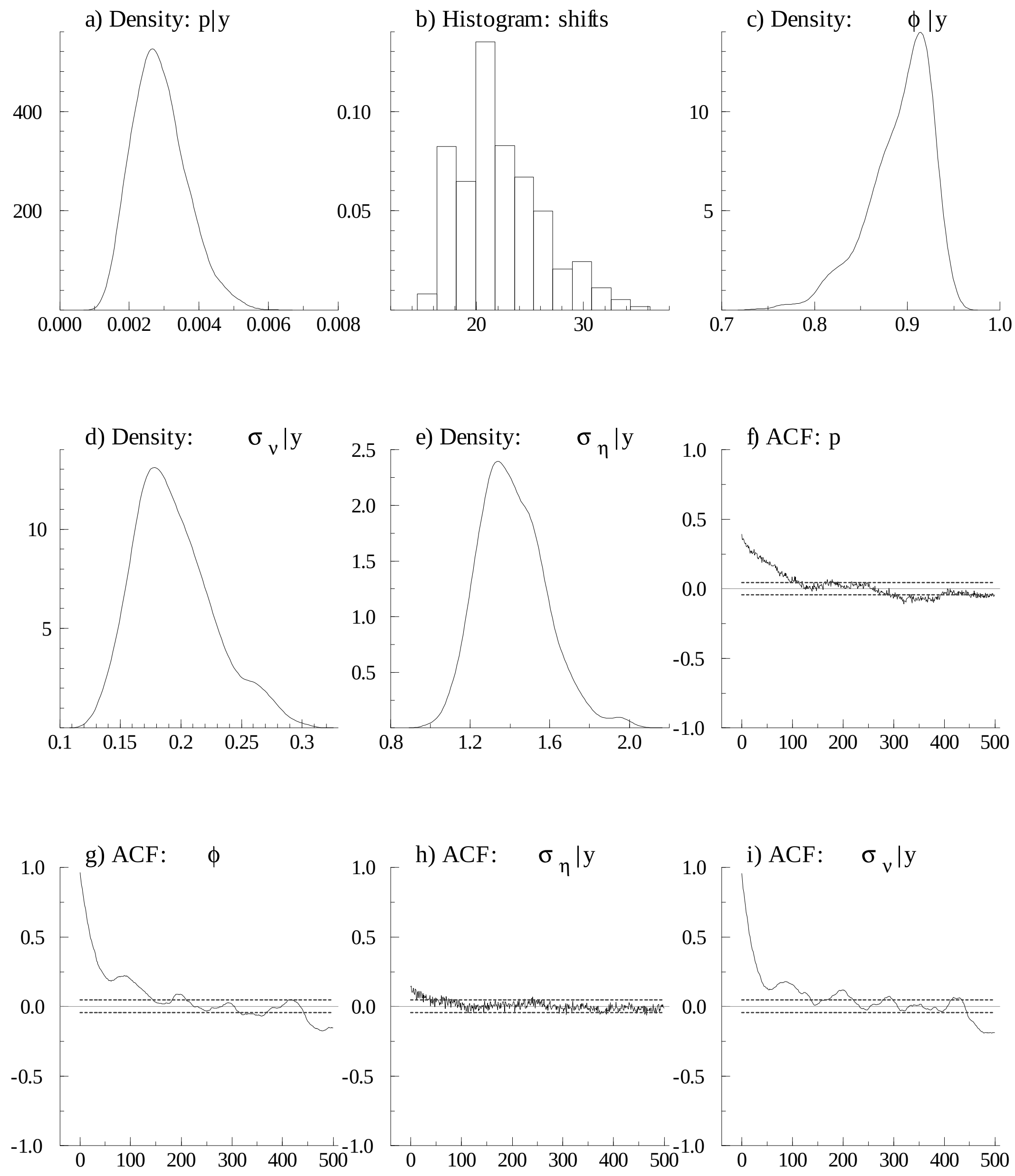

Figure 7. Posterior Estimates for Industrial Metals Index Volatility

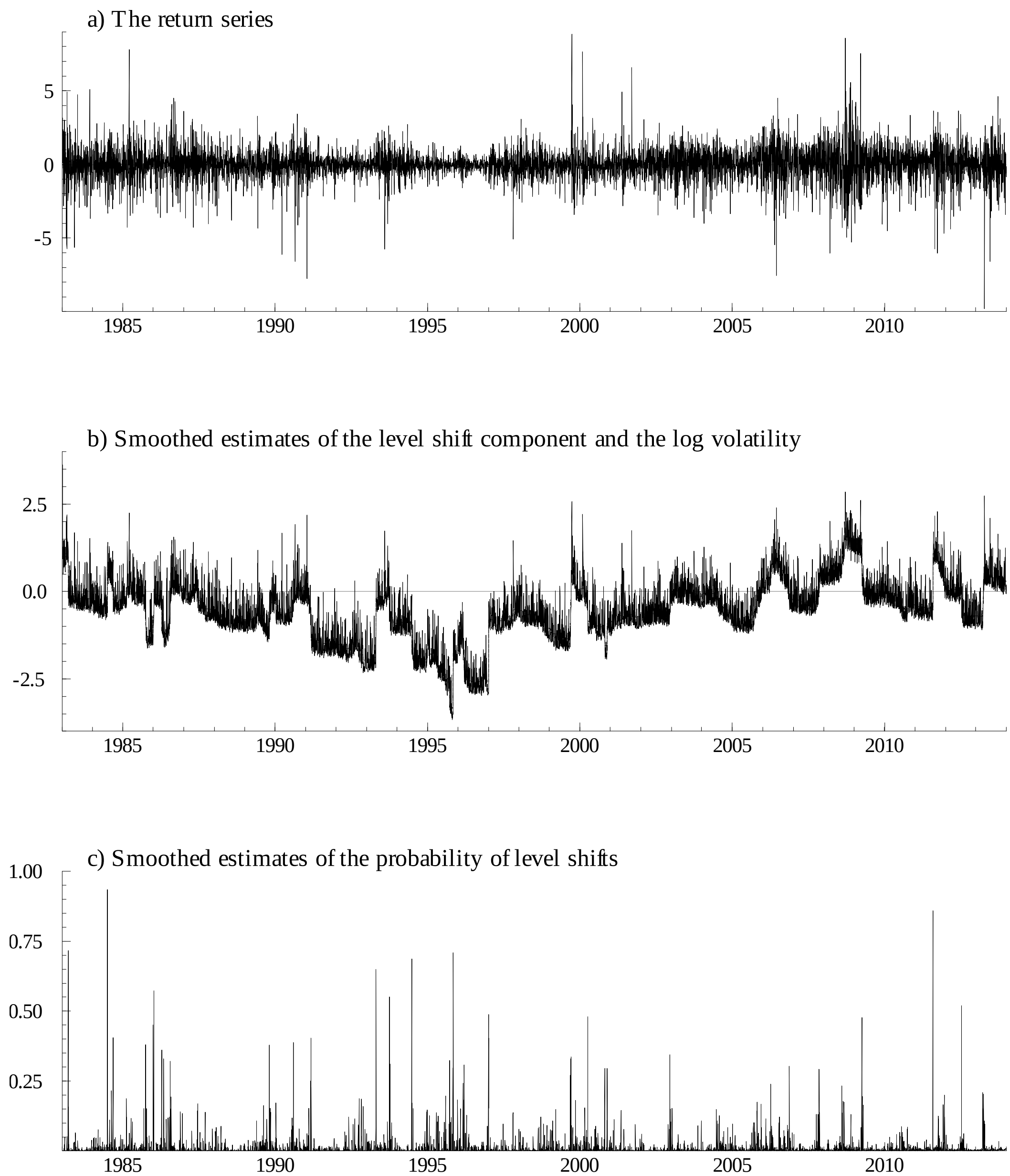

Figure 8. Results for Gold Index Volatility

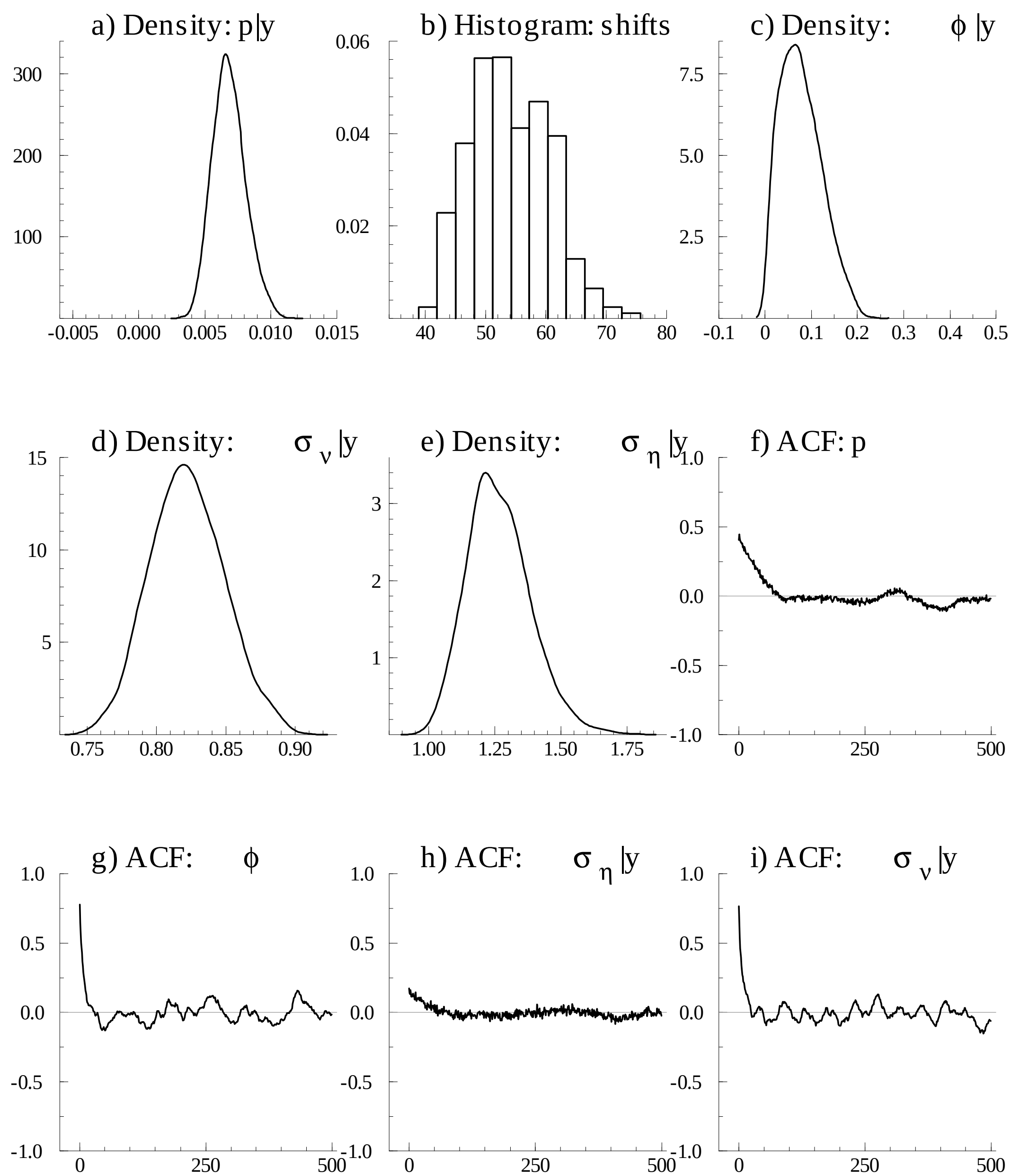

Figure 9. Posterior Estimates for Gold Index Volatility

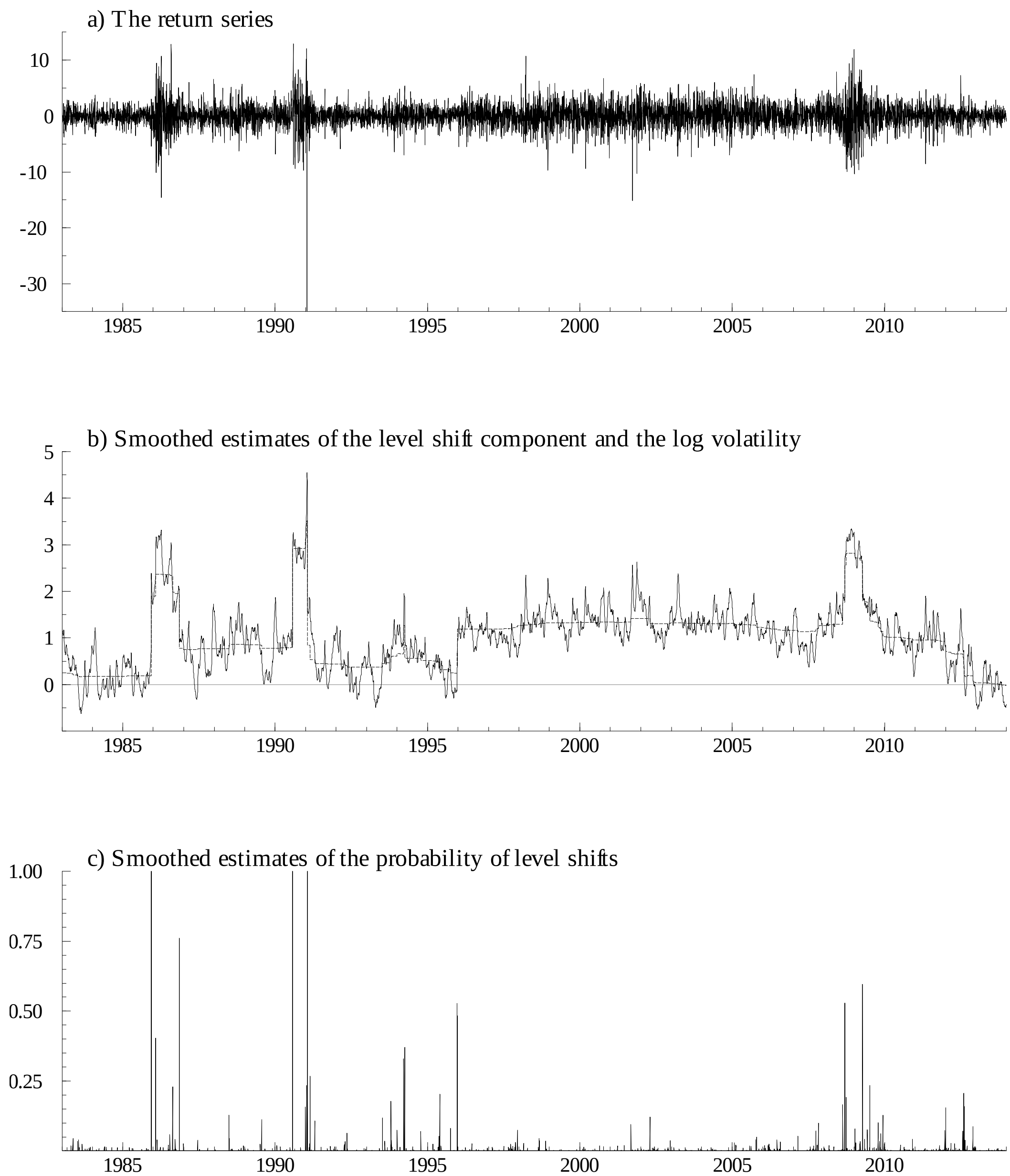

Figure 10. Results for Oil Index Volatility

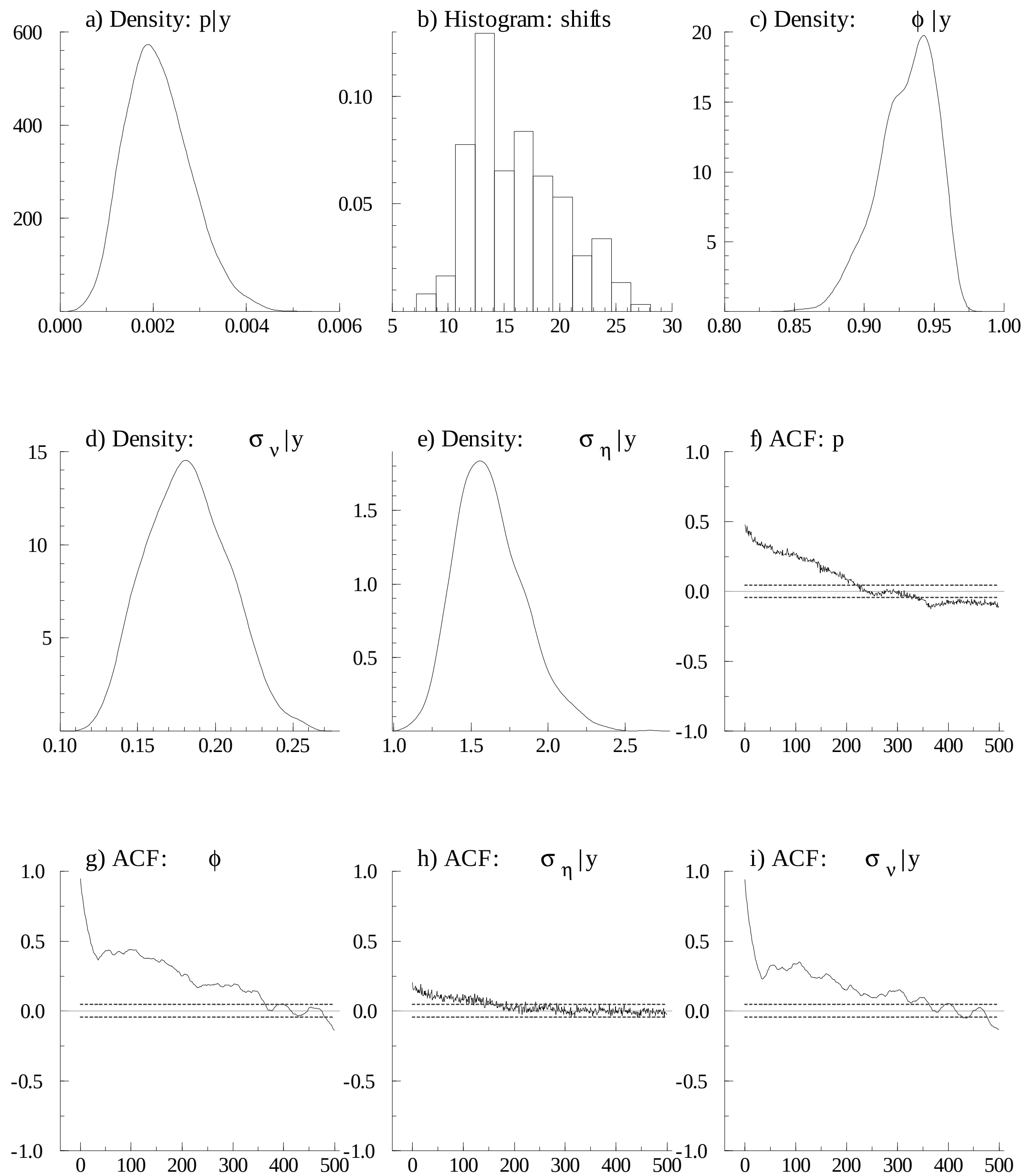

Figure 11. Posterior Estimates for Oil Index Volatility

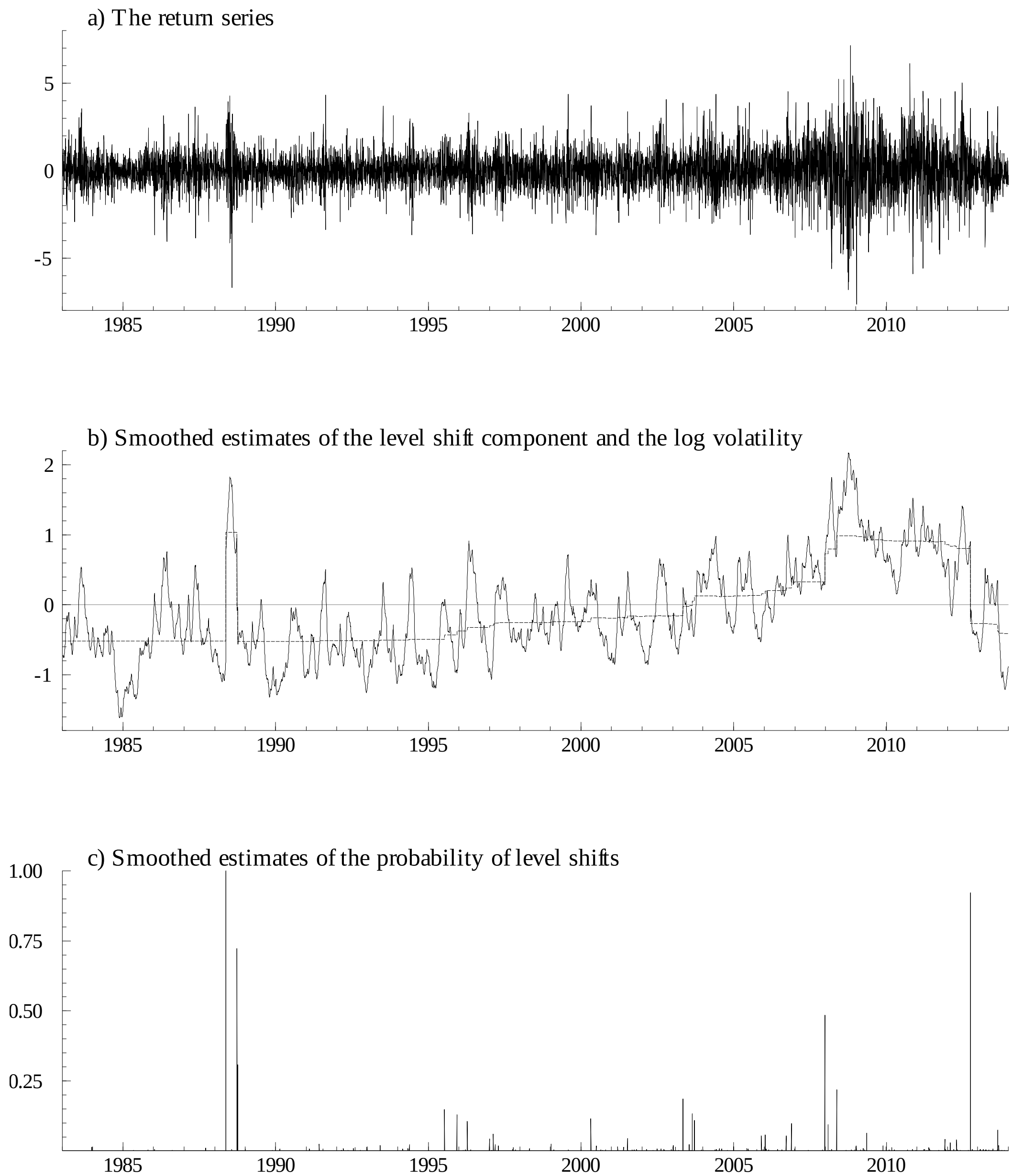

Figure 12. Results for Agriculture Index Volatility

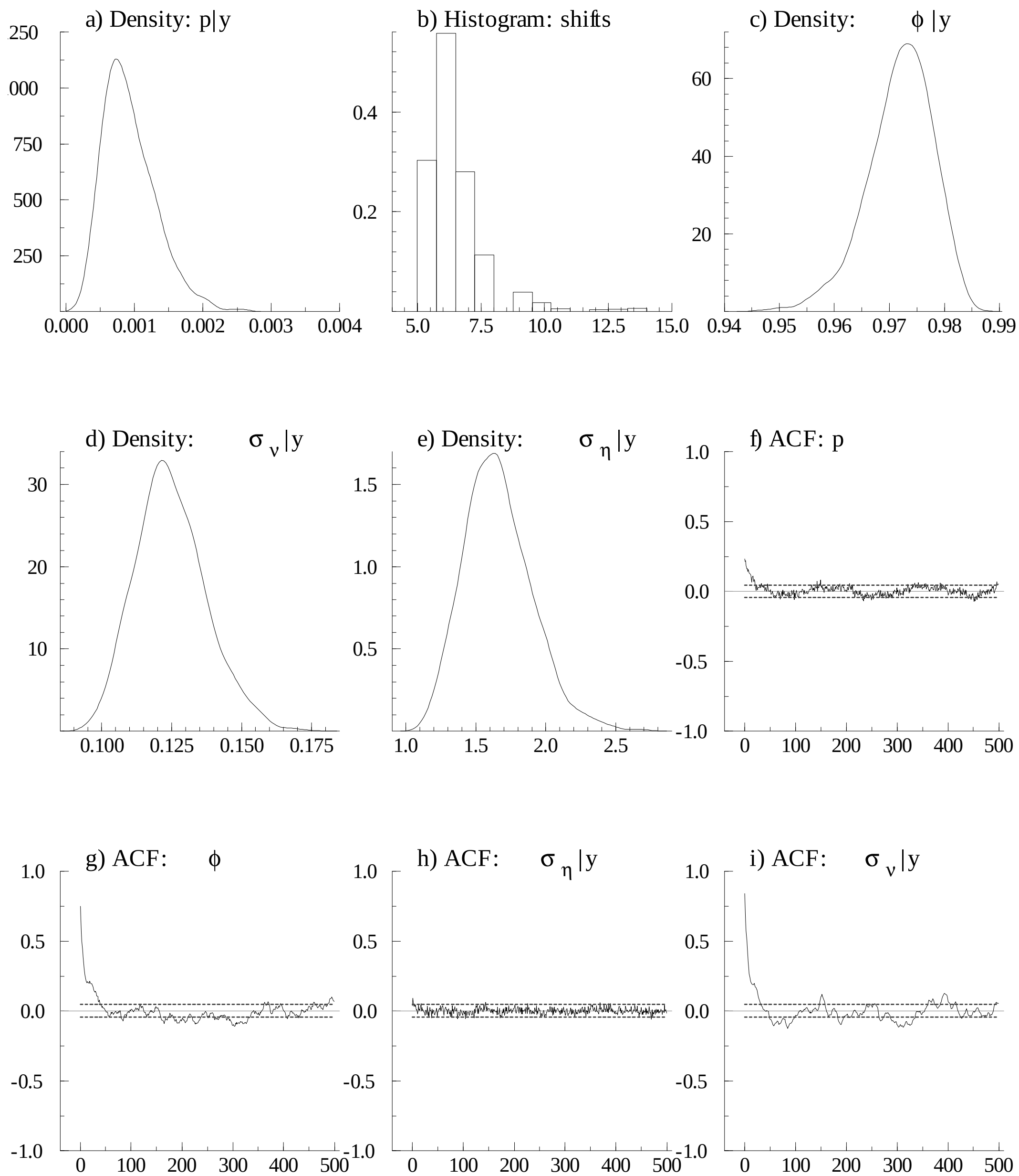

Figure 13. Posterior Estimates for Agriculture Index Volatility

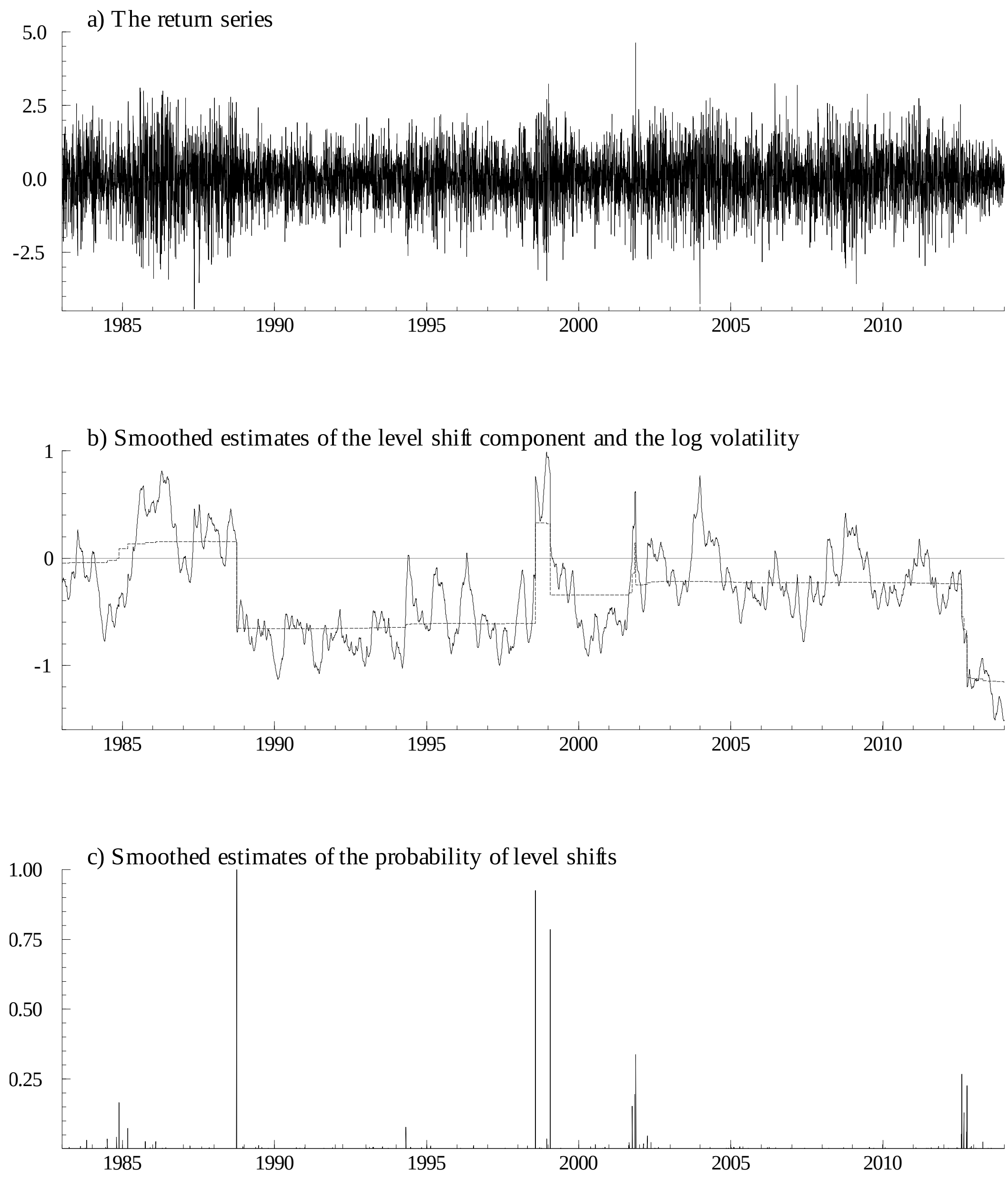

Figure 14. Results for Livestock Index Volatility

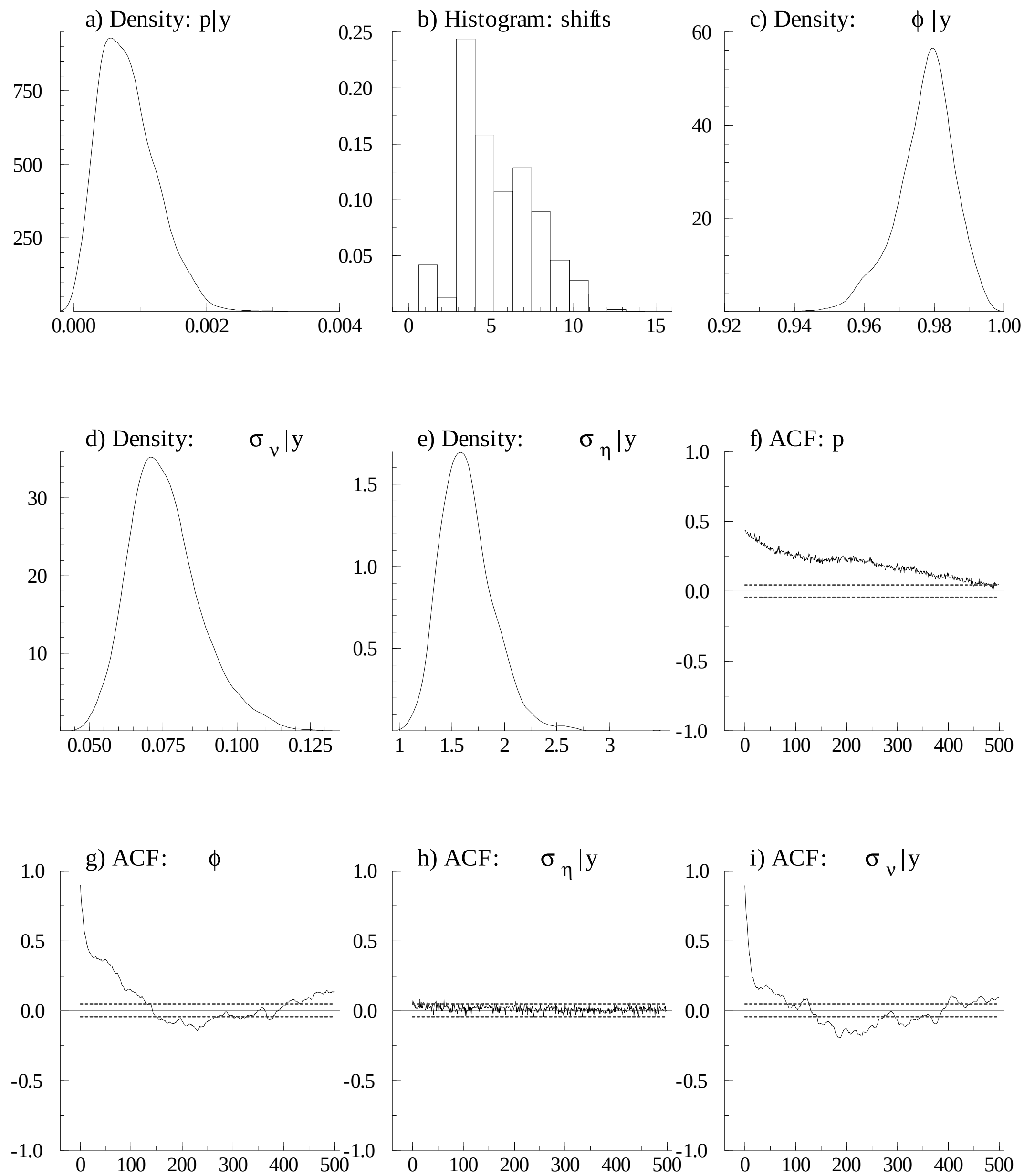

Figure 15. Posterior Estimates for Livestock Index Volatility

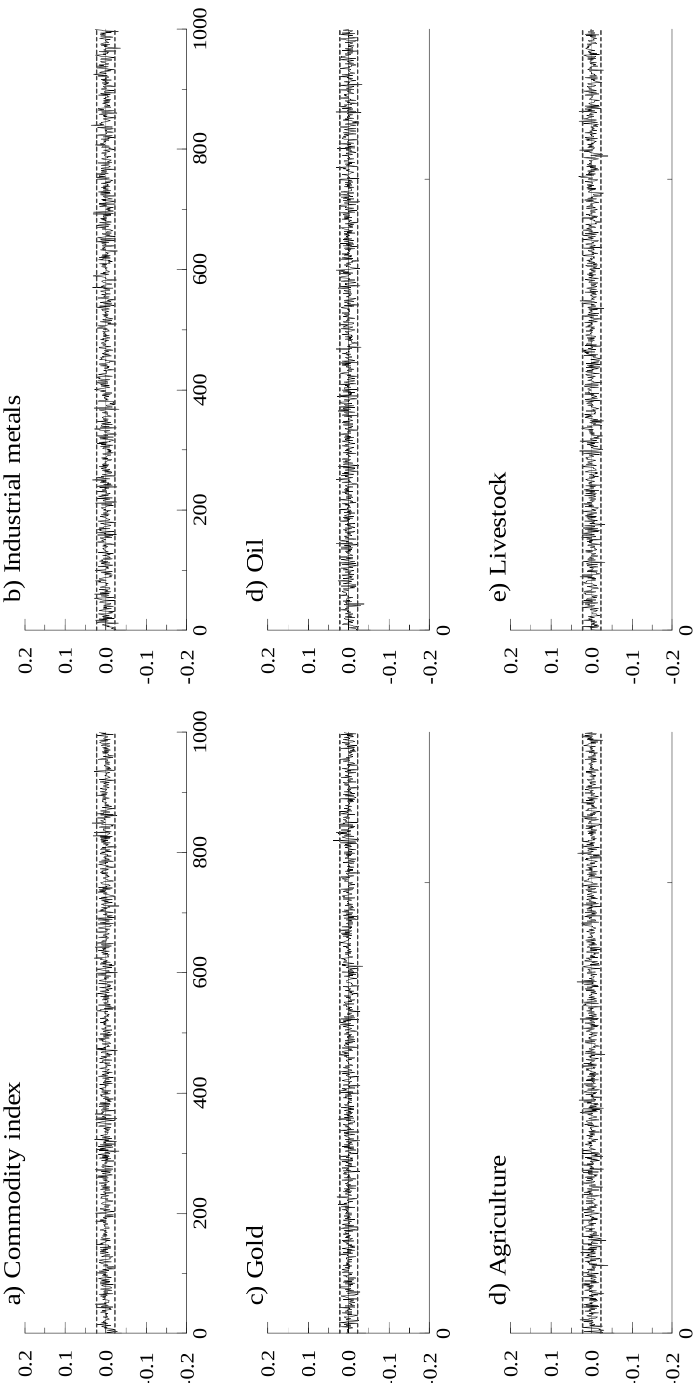

Figure 16. Autocorrelations of Log Squares

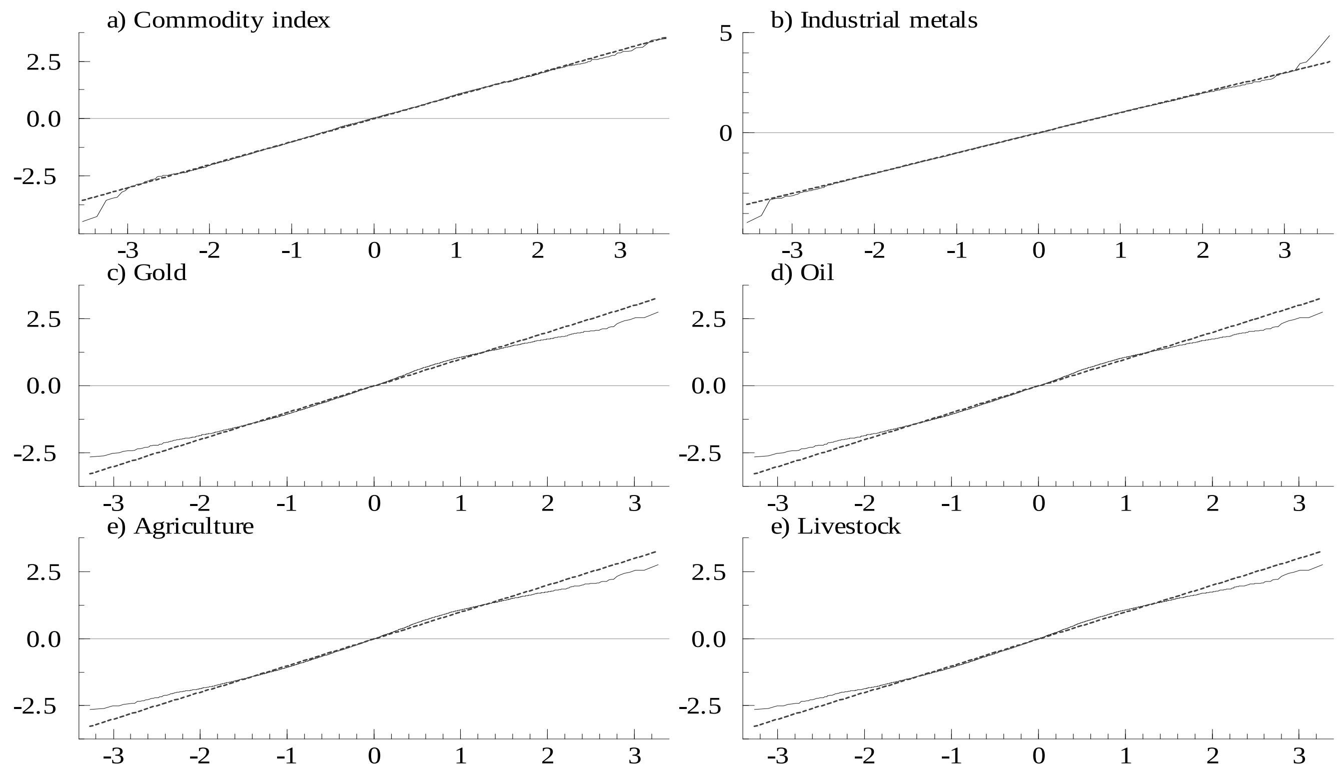

Figure 17. Normal QQ Plots

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