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Stochastic Volatility in Mean: Empirical Evidence from Stock Latin American Markets[1]

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Abstract

Using a Stochastic Volatility in Mean (SVM) model, we perform an empirical study of five Latin American indexes in order to see the impact of the volatility in the mean of the returns. We use MCMC Hamiltonian dynamics. The results indicate that volatility has a negative impact on returns suggesting that the volatility feedback effect is stronger than the effect related to the expected volatility. This result is clear and opposite to the finding of Koopman and Uspensky (2002). The other countries present negative values but the upper tail of the intervals are near to the zero value.

JEL Codes: C11, C15, C22, C51, C52, C58, G12.

Keywords: Stock Latin American Markets, Stochastic Volatility in Mean, Feed-Back Effect, Hamiltonian Monte Carlo, Markov Chain Monte Carlo, Riemannian Manifold Hamiltonian Monte Carlo, Non Linear State Space Models.

Resumen

Utilizando un modelo de volatilidad estocástica en la media (SVM) junto con la dinámica Hamiltoniana MCMC, realizamos un estudio empírico de cinco índices latinoamericanos para analizar el impacto de la volatilidad en la media de los rendimientos. Los resultados indican que la volatilidad tiene un impacto negativo en los rendimientos, lo que sugiere que el efecto de retroalimentación de la volatilidad es más fuerte que el efecto relacionado con la volatilidad esperada. Este resultado es claro y opuesto al hallazgo de Koopman y Uspensky (2002). Los otros países presentan valores negativos, pero la cola superior de los intervalos está cerca del valor cero.

Clasificación JEL: C11, C15, C22, C51, C52, C58, G12

Palabras Claves: Stock Latin American Markets, Stochastic Volatility in Mean, Feed-Back Effect, Hamiltonian Monte Carlo, Markov Chain Monte Carlo, Riemannian Manifold Hamiltonian Monte Carlo, Non Linear State Space Models.

[1] The views expressed in this paper are those of the authors and do not reflect necessarily the position of the Fiscal Council of Peru. Any remaining errors are our responsibility.

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1 Introduction

In recent years, stochastic volatility (SV) models have been considered as a useful tool for modeling time-varying variances, mainly in financial applications where policymakers or stockholders are constantly facing decision problems that usually depend on measures of volatility and risk[1] An attractive feature of the SV model is its close relationship to financial economic theories (Melino and Turnbull, 1990) and its ability to capture the main empirical properties often observed in daily series of financial returns in a more appropriate way (Carnero et al., 2004).

The daily asymmetrical relation between equity market returns and volatility has received unprecedented attention in the financial literature (Black, 1976; Campbell and Entchel, 2000; Bekaert and Wu, 2000). Asymmetric equity market volatility is important for al least three reasons. First, it is an important characteristic of the market volatility dynamics, has asset pricing implications and is a characteristic of priced risk factors. Second, it plays an important role in risk prediction, hedging and option pricing. Finally, asymmetric volatility implies negatively skewed returns distributions, i.e. it may help to explain some of the probability market value losses. The relation between expected returns and expected volatility have been extensively examined in recent years. Overall, there appears to be stronger evidence of a negative relationship between unexpected returns and innovations to the volatility process, which French et al. (1987) interpreted as indirect evidence of a positive correlation between the expected risk premium and *ex ante* volatility. If expected volatility and expected returns are positively related and future cash flows are unaffected, the current stock index price should fall. Conversely, small shocks to the return process lead to an increase in contemporaneous stock index prices. This theory, known as the volatility feedback theory hinges on two assumptions: first, the existence of a positive relation between the expected components of the return and volatility process and second, volatility persistence. An alternative explanation for asymmetric volatility where causality runs in the opposite direction is the leverage effect put forward by Black (1976), who asserted that a negative (positive) return shock leads to an increase (decrease) in the firm's financial leverage ratio, which has an upward (downward) effect on the volatility of its stock returns. However, French et al.

[1] The other important branch of related models is GARCH models where time-varying variance is modeled as a deterministic function of past squared perturbations and lagged conditional variances. Details and explanations of the extensive GARCH literature may be found in (Bollerslev et al., 1992, 1994; Diebold and Lopes, 1995). SV models are reviewed in (Taylor, 1994; Ghysels et al., 1994; Shephard, 1996).

(1987) and Schwert (1989) argued that leverage alone cannot account for the magnitude of the negative relationship. For example, Campbell and Entchel (2000) found evidence of both volatility feedback and leverage effects, whereas Bekaert and Wu (2000) presented results suggesting that the volatility feedback effect dominates the leverage effect empirically.

Frequently, the volatility of daily stock returns has been estimated with SV models, but the results have relied on a extensive pre-modeling of these series to avoid the problem of simultaneous estimation of the mean and variance. Koopman and Uspensky (2002) introduced the SV in mean (SVM) model to deal with this problem and the unobserved volatility is incorporated as an explanatory variable in the mean equation of the returns under the normality assumption of the innovations. They derived an exact maximum likelihood estimation based on Monte Carlo simulation methods. Recently, Abanto-Valle et al. (2012) extended the SVM model to the class of scale mixture of Normal distributions and developed an Markov chain Monte Carlo algorithm to sample parameters and the log-volatilities from a Bayesian viewpoint.

One contribution of this article is to apply the Hamiltonian Monte Carlo (HMC) and Riemann Manifold HMC (RMHMC) methods within the Markov chain Monte Carlo (MCMC) algorithm to update the log-volatilities and parameters of the SVM model, respectively. Our MCMC simulation employs a HMC algorithm (Duane et al., 1987; Neal, 2011) for updating the log-volatilities at once and RMHC (Girolami and Calderhead, 2011; Nugroho and Morimoto, 2015) for parameters from the mean and volatility equations at once in two blocks.

Time-varying volatility for financial variables of developed economies have been studied extensively (Liesenfeld and Jung, 2000; Jacquier et al., 2004; Abanto-Valle et al., 2010); however, empirical studies of the volatility characteristics of the financial markets in Latin America are very scarce and are far from being thoroughly analyzed despite their growth in recent years(Abanto-Valle et al., 2011; Rodríguez, 2017; Lengua Lafosse and Rodríguez, 2018). For this reason, we perform a detailed empirical study of five Latin American indexes: MERVAL (Argentina), IBOVESPA (Brazil), IPSA (Chile), MEXBOL (Mexico) and IGBVL (Peru) in the context of the SVM model. We also include the S&P 500 returns in order to perform some comparisons. We found empircally, that the coeficient estimate, which measures both the ex ante relationship between returns and volatility and the volatility feedback effect, was found to be negative and significant for all the indexes considered here with the exception of the IGBVL.

The remainder of this paper is organized as follows. Section 2 gives a brief introduction about HMC and RMHMC based methods in a general context. Section 3 outlines SVM model as well as the Bayesian estimation procedure using HMC and RMHMC methods. Section 4 outlines model comparison criteria. Section 5 is devoted to application using five Latin American indexes and the S&P 500. Finally, some concluding remarks and suggestions for future developments are given in Section 6. An Appendix outlines details of the HMC and RHMC algorithms, and the likelihood approximation using hidden Markov models.

2 MCMC using Hamiltonan Dynamics

2.1 Hamiltonian Monte Carlo (HMC)

HMC method proposes a new state by computing a trajectory obeying Hamiltonian dynamics (Neal, 2011). The trajectory is guided by a first-order gradient of the log of the posterior by applying time discretization in the Hamiltonian dynamics. This gradient information supports the HMC trajectories in the direction of high probabilities, resulting in a high-acceptance rate and ensuring that the accepted draws are not highly correlated (Marwala, 2012).

Let $\boldsymbol{\theta} \in \mathbb{R}^p$ be a $p-$dimensional random variable with density $\pi(\boldsymbol{\theta})$. In HMC sampling an independent auxiliary variable $\boldsymbol{\omega} \in \mathbb{R}^d$, such that $\boldsymbol{\omega} \sim \mathcal{N}_d(\mathbf{0}, \mathbf{M})$ is introduced. The negative joint log-probability is

$$H(\boldsymbol{\theta}, \boldsymbol{\omega}) = -\mathcal{L}(\boldsymbol{\theta}) + \frac{1}{2}\log\{(2\pi)^p|\mathbf{M}|\} + \frac{1}{2}\boldsymbol{\omega}'\mathbf{M}^{-1}\boldsymbol{\omega},$$

where $\mathcal{L}(\boldsymbol{\theta}) = \log \pi(\boldsymbol{\theta})$. In physical analogy, if $\boldsymbol{\theta}$ is interpreted as a position of a particle, $-\mathcal{L}(\boldsymbol{\theta})$ describes its potential energy, while the auxiliary variable $\boldsymbol{\omega}$ is interpreted as the momentum with kinetic energy $\frac{1}{2}\boldsymbol{\omega}'\mathbf{M}^{-1}\boldsymbol{\omega}$ and the covariance matrix $\mathbf{M}$ denotes a mass matrix. Then the total energy of a closed system is the Hamiltonian function $H(\boldsymbol{\theta}, \boldsymbol{\omega})$ (Duane et al., 1987; Leimkuhler and Reich, 2004).

For continuous time τ, the deterministic evolution of a particle that keeps the total energy constant is given by the Hamiltonian dynamics equations:

$$\begin{aligned} \frac{\partial \boldsymbol{\theta}}{\partial \tau} &= \frac{\partial H(\boldsymbol{\theta}, \boldsymbol{\omega})}{\partial \boldsymbol{\omega}} = \mathbf{M}^{-1}\boldsymbol{\omega} \\ \frac{\partial \boldsymbol{\omega}}{\partial \tau} &= -\frac{\partial H(\boldsymbol{\theta}, \boldsymbol{\omega})}{\partial \boldsymbol{\theta}} = \nabla_{\boldsymbol{\theta}}\mathcal{L}(\boldsymbol{\theta}), \end{aligned}$$

where, $\nabla_{\boldsymbol{\theta}}\mathcal{L}(\boldsymbol{\theta})$ is the gradient of $\mathcal{L}(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$. For practical applications of interest, these differential equations cannot be solved analytically and numerical methods are required. It is common to use the Stormer-Verlet leapfrog integrator (Leimkuhler and Reich, 2004), that retains the reversibility and volume preservation properties required to obtain an exact sampler, and computes the updates as the following expressions:

$$\begin{aligned}
\boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})} &= \boldsymbol{\omega}^{(\tau)} + \frac{\epsilon}{2}\nabla_{\boldsymbol{\theta}}\mathcal{L}(\boldsymbol{\theta}),\\
\boldsymbol{\theta}^{(\tau+\epsilon)} &= \boldsymbol{\theta}^{(\tau)} + \epsilon\mathbf{M}^{-1}\boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})},\\
\boldsymbol{\omega}^{(\tau+\epsilon)} &= \boldsymbol{\theta}^{(\tau)} + \frac{\epsilon}{2}\nabla_{\boldsymbol{\theta}}\mathcal{L}(\boldsymbol{\theta}^{(\tau)}),
\end{aligned}$$

for some user-specified small step-size $\epsilon > 0$. Starting with the current state $(\boldsymbol{\theta}, \boldsymbol{\omega})$ and after a given number of time steps this results in a proposal $(\boldsymbol{\theta}^*, \boldsymbol{\omega}^*)$. As total energy is only approximately conserved with the Stormer-Verlet integrator then a corresponding bias is introduced into the joint density which can be corrected by an accept-reject step. The proposal is accepted as the next state of Markov chain with probability given by:

$$\alpha(\boldsymbol{\theta}, \boldsymbol{\omega}; \boldsymbol{\theta}^*, \boldsymbol{\omega}^*) = \min\{1, \exp(-H(\boldsymbol{\theta}^*, \boldsymbol{\omega}^*) + H(\boldsymbol{\theta}, \boldsymbol{\omega})\}. \tag{1}$$

2.2 Riemann Manifold Hamiltonian Monte Carlo (RMHMC)

Girolami and Calderhead (2011) proposed a new HMC method called Riemmanian Manifold HMC (RMHMC) for improving the convergence and mixing of the chain. The RMHMC provides an adaptation mechanism for HMC by exploiting the Riemannian geometry of the parameters space. RMHMC accounts by adapting the covariance matrix $\mathbf{M}$ used in HMC. The idea is to redefine the Hamiltonian function as

$$H(\boldsymbol{\theta}, \boldsymbol{\omega}) = -\mathcal{L}(\boldsymbol{\theta}) + \frac{1}{2}\log\left\{(2\pi)^p|\mathbf{M}(\boldsymbol{\theta})|\right\} + \frac{1}{2}\boldsymbol{\omega}'\mathbf{M}(\boldsymbol{\theta})^{-1}\boldsymbol{\omega},$$

where $M(\boldsymbol{\theta}) = -E(\frac{\partial^2\mathcal{L}()}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'})$ is the expected Fisher information matrix plus negative Hessian of the log prior. Therefore, the Hamiltonian equations for the momentum and position variables, respectively are now defined by:

$$\begin{aligned}
\frac{\partial\boldsymbol{\theta}}{\partial\tau} &= \frac{\partial H(\boldsymbol{\theta}, \boldsymbol{\omega})}{\partial\boldsymbol{\omega}} = \mathbf{M}(\boldsymbol{\theta})^{-1}\boldsymbol{\omega},\\
\frac{\partial\boldsymbol{\omega}_i}{\partial\tau} &= -\frac{\partial H(\boldsymbol{\theta}, \boldsymbol{\omega})}{\partial\theta_i} = \nabla_{\theta_i}\mathcal{L}(\boldsymbol{\theta}) - \frac{1}{2}\mathrm{tr}\left[\mathbf{M}(\boldsymbol{\theta})^{-1}\frac{\partial\mathbf{M}(\boldsymbol{\theta})}{\partial\theta_i}\right] + \frac{1}{2}\boldsymbol{\omega}'\mathbf{G}(\boldsymbol{\theta})^{-1}\frac{\partial\mathbf{M}(\boldsymbol{\theta})}{\partial\theta_i}\mathbf{G}(\boldsymbol{\theta})^{-1}\boldsymbol{\omega}.
\end{aligned}$$

In order to simulate values in discrete time, we adopt the generalized Stormer-Verlet solution (Leimkuhler and Reich, 2004), which is described as follows:

$$\begin{aligned}
\boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})} &= \boldsymbol{\omega}^{(\tau)} - \frac{\epsilon}{2}\nabla_{\boldsymbol{\theta}} H(\boldsymbol{\theta}^{(\tau)}, \boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})}), \\
\boldsymbol{\theta}^{(\tau+\epsilon)} &= \boldsymbol{\theta}^{(\tau)} + \frac{\epsilon}{2}\Big[\nabla_{\boldsymbol{\omega}} H(\boldsymbol{\theta}^{(\tau)}, \boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})} + \nabla_{\boldsymbol{\omega}} H(\boldsymbol{\theta}^{(\tau+\epsilon)}, \boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})}\Big], \\
\boldsymbol{\omega}^{(\tau+\epsilon)} &= \boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})} - \frac{\epsilon}{2}\nabla_{\boldsymbol{\theta}} H(\boldsymbol{\theta}^{(\tau+\epsilon)}, \boldsymbol{\omega}^{(\tau+\frac{\epsilon}{2})}).
\end{aligned}$$

At each iteration, repeated application of these steps provides a proposal $(\boldsymbol{\theta}^*, \boldsymbol{\omega}^*)$ which is accepted with probability $\alpha(\boldsymbol{\theta}, \boldsymbol{\omega}; \boldsymbol{\theta}^*, \boldsymbol{\omega}^*)$ given by equation (1).

3 The Stochastic Volatility in Mean (SVM) Model

The SVM model is defined by

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 e^{h_t} + e^{\frac{h_t}{2}}\epsilon_t, \tag{2a}$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma\eta_t, \tag{2b}$$

where y_t and h_t are, respectively, the compounded return and the log-volatility at time t. We assume that $|\phi| < 1$, i.e., that the log-volatility process is stationary and that the initial value $h_1 \sim \mathcal{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$. The innovations ϵ_t and η_t are assumed to be mutually independent and Normally distributed with mean zero and unit variance. The SVM model incorporartes the unobserved volatility as an explanatory variable in the mean equation. The aim of the SVM model is to estimate the ex-ante relation between returns and volatility and the volatility feedback effect (parameter β_2). Under a Bayesian paradigm, we use MCMC methods to conduct the posterior analysis in the next subsection.

3.1 Parameter Estimation via MCMC

Let $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \mu, \phi, \sigma^2)'$ be the full parameter vector of the SVM model, $\mathbf{h}_{1:T} = (h_1, \ldots, h_T)'$ be the vector of the log volatilities and $\mathbf{y}_{0:T} = (y_0, \ldots, y_T)'$ be the information available up to time T. The Bayesian approach to estimate the parameters in the SVM model uses the data augmentation principle (Tanner and Wong, 1987), which considers $\mathbf{h}_{1:T}$ as latent parameters. The joint posterior density of parameters and latent unobservable variables can be written as:

$$\begin{aligned} p(\mathbf{h}_{1:T}, \boldsymbol{\theta} \mid \mathbf{y}_{0:T}) &\propto p(\mathbf{y}_{1:T} \mid y_0, \boldsymbol{\theta}, \boldsymbol{\lambda}_{1:T}, \mathbf{h}_{0:T}) p(\mathbf{h}_{1:T} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}) \\ &= \prod_{t=1}^{T} \Big[p(y_t \mid y_{t-1}, \beta_0, \beta_1, \beta_2, h_t) \Big] p(h_1 \mid \mu, \phi, \sigma^2) \prod_{t=1}^{T-1} \Big[p(h_{t+1} \mid h_t, \mu, \phi, \sigma^2) \Big] p(\boldsymbol{\theta}), \end{aligned} \tag{3}$$

where $p(y_t \mid y_{t-1}, \beta_0, \beta_1, \beta_2, h_t)$ and $p(h_{t+1} \mid h_t, \mu, \phi, \sigma^2)$ are defined for equations (2a) and (2b), and $p(\boldsymbol{\theta})$ denotes the prior distribution. We assume $p(\boldsymbol{\theta})$ is prior independent. The priors distributions of parameters in the SVM model are set as: $\beta_0 \sim \mathcal{N}(\bar{\beta}_0, \sigma^2_{\beta_0})$, $\frac{\beta_1+1}{2} \sim \mathcal{B}e(a_{\beta_1}, b_{\beta_1})$, $\beta_2 \sim \mathcal{N}(\bar{\beta}_2, \sigma^2_{\beta_2})$, $\mu \sim \mathcal{N}(\mu_0, \sigma^2_\mu)$, $\frac{\phi+1}{2} \sim \mathcal{B}e(a_\phi, b_\phi)$ and $\sigma^2 \sim \mathcal{IG}(a_\sigma, b_\sigma)$, where $\mathcal{N}(.,.)$, $\mathcal{B}e(.,.)$ and $\mathcal{IG}(.,.)$ denote the Normal, Beta and Inverse Gamma distributions, respectively. This choice of priors ensures that all parameters have the right support; in particular the Beta prior on β_1 and ϕ ensures that $-1 < \beta_1, \phi < 1$.

Since $p(\mathbf{h}_{1:T}, \boldsymbol{\theta} \mid \mathbf{y}_{0:T})$ does not have closed form, we sample from using the Gibbs sampling as described in Algorithm 1.

Algorithm 1

(1) Set $i = 0$ and get starting values for the parameters $\boldsymbol{\theta}^{(i)}$ and the latent quantities $\mathbf{h}^{(i)}_{1:T}$.

(2) Generate $(\mu, \phi, \sigma)^{(i+1)} \sim p(\mu, \phi, \sigma \mid \mathbf{y}_{1:T}, \mathbf{h}^{(i)}_{1:T})$

(3) Draw $(\beta_0, \beta_1, \beta_2)^{(i+1)} \sim p(\beta_0, \beta_1, \beta_2 \mid \mathbf{h}^{(i)}_{1:T}, \mathbf{y}_{0:T})$.

(4) Generate $\mathbf{h}^{(i+1)}_{1:T} \sim p(h_{1:T} \mid (\beta_0, \beta_1, \beta_2)^{(i+1)}, (\mu, \phi, \sigma)^{(i+1)})$

(5) Set $i = i + 1$ and return to step (2) until convergence is achieved,

where, RMHMC is applied to steps (2) and (3), and HMC is applied to step (4). The detail of procedures to sample parameters and log-volatity are given in the Appendix.

4 Model Comparison Criteria

Given the wide range of candidate models, it has become increasingly important to be able to discriminate between competing models for a given application. A popular metric of summary statistics for Bayesian model comparison is the deviance information criterion (DIC) proposed by Spiegelhalter et al. (2002). This criterion is based on the posterior mean of the deviance. It can be approximated by $\overline{D} = \sum_{q=1}^{Q} D(\boldsymbol{\theta}_q)/Q$, where $D(\boldsymbol{\theta}) = -2\log f(\mathbf{y}_T \mid \boldsymbol{\theta}) = -2\log \mathcal{L}(\boldsymbol{\theta})$. The DIC can be estimated using the Monte Carlo output by $\widehat{\text{DIC}} = \overline{D} + \widehat{p_D} = 2\overline{D} - D(\bar{\boldsymbol{\theta}})$, where $\widehat{p_D}$ is the effective number of parameters, and can be evaluated as $\widehat{p_D} = \bar{D} - D(\bar{\boldsymbol{\theta}})$. Given the comparison of competing models, the model that best fits a dataset is the model with the smallest DIC value. It is important to integrate out all latent variables in the deviance calculation, since this yields a more appropriate penalty term $\widehat{p_D}$. For all these criteria, the evaluation of the likelihood function $\mathcal{L}(\boldsymbol{\theta})$ is a key aspect. However, the likelihood of the SVM can be evaluated using the results given in Appendix B (See Abanto-Valle et al., 2017, for additional details).

Finally, we use the Log-Predictive Score (LPS, Delatola and Griffin, 2011), which can be estimated as: $\widehat{LPS} = -\frac{1}{T}\sum_{t=1}^{T}\log p(y_t \mid \mathbf{y}_{t-1}, \bar{\boldsymbol{\theta}})$.

5 Empirical Application

We consider the daily closing prices of five Latin American stock market: MERVAL (Argentina), IBOVESPA (Brazil), IPSA (Chile), MEXBOL (Mexico) and IGBVL (Peru). We use the S&P 500 in order to compare the results with Latin American stock markets. One reason is that the U.S. stock market could be considered as a good benchmark. The data sets were obtained from the Yahoo finance web site available to download at `http://finance.yahoo.com`. The period of analysis is from January 6, 1998, until December 30, 2016. Stock returns are computed as $y_t = 100 \times (\log P_t - \log P_{t-1})$, where P_t is the (adjusted) closing price on day t. Table 1 shows the number of observation and summary descriptive statistics. The sample size differs between countries due to holidays and stock market non-trading days. According to Table 1, the IGBVL and S&P 500 are negatively skewed whereas the rest are positively skewed. The IGBVL return is the most negatively skewed with -0.3915 and the IBOVESPA return the most positively skewed with 0.5313. Regarding the kurtosis, all the daily returns of the five Latin American returns and the S&P 500 are leptokurtic, since all estimates of kurtosis exceed 3. Brazil, Peru and Chile are the markets with the highest degree of kurtosis with the USA near the value observed for Chile. Although there are high differences between the minimum and maximum values, the most outstanding values are those corresponding to Argentina and Brazil.

These two countries show to be the most volatile too, which can be attributed to extreme minimum and maximum values.

Table 1: Summary statistics for daily stock returns data.

INDEX	MERVAL	IBOVESPA	IPSA	MEXBOL	IGBVL	S & P 500
Size	4651	4698	4737	4759	4597	4777
Mean	0.0701	0.0376	0.0296	0.0464	0.0478	0.0177
S. D.	2.2125	2.0262	1.0695	1.4276	1.4111	1.2418
Minimum	-14.2896	-17.2082	-7.6381	-10.3410	-13.2908	-9.4695
Maximum	16.1165	28.8325	11.8034	12.1536	12.8156	10.9572
Skewness	2.2091	0.5313	0.1372	0.1458	-0.3915	-0.2086
Kurtosis	7.3418	16.8094	11.6866	8.7449	13.5715	10.6576
Returns						
$\hat{\rho}_1$	0.0550	0.0130	0.1840	0.0910	0.1890	-0.0700
$\hat{\rho}_2$	0.0020	-0.0180	0.0220	-0.0300	0.0080	-0.0450
$\hat{\rho}_3$	0.0240	-0.0390	-0.0190	-0.0301	0.0680	0.0100
$\hat{\rho}_4$	0.0070	-0.0320	0.0250	-0.0030	0.0640	-0.0080
$\hat{\rho}_5$	-0.0090	-0.0170	0.0270	-0.0150	0.0250	-0.0460
$Q(12)$	33.37	44.51	190.52	54.57	240.53	66.95
Squared Returns						
$\hat{\rho}_1$	0.2580	0.1990	0.2320	0.1430	0.4210	0.2040
$\hat{\rho}_2$	0.2160	0.1640	0.2130	0.1780	0.3890	0.3720
$\hat{\rho}_3$	0.1780	0.1860	0.1720	0.2540	0.3920	0.1920
$\hat{\rho}_4$	0.1660	0.1170	0.1550	0.1300	0.2840	0.2880
$\hat{\rho}_5$	0.2130	0.0990	0.2910	0.2420	0.2140	0.3220
$Q(12)$	1763.4	1069.3	2086.0	2147.1	3960.2	4643.7

We further observe that the IGBVL and IPSA returns show the highest level of first-order autocorrelation. These values decrease fast for the other orders of autocorrelation. In the case of returns, high first-order autocorrelation reflects the effects of non-synchronous or thin trading. The squared returns show high level of autocorrelation of order 1 which can be seen as an indication of volatility clustering. We further observe that high-order autocorrelations for squared returns are still high and decrease slowly[2].The $Q(12)$ test statistic, which is a joint test for the hypothesis that the first twelve autocorrelation coefficients are equal to zero, indicates that this hypothesis has to be rejected at the

[2] This behavior has suggested that the literature considers that there is a long memory in the volatility of returns, as well as the possibility that infrequent level shifts cause such behavior. For a discussion on this, see Diebold and Inoue (2001) and Perron and Qu (2010), among others. For applications to different financial markets in Latin America, see Rodríguez (2017) and the references mentioned therein.

Table 2: MCMC estimation of the SVM model.

Parameter	Mean	M.C. Error	95% interval	Inef	CD	Parameter	Mean	M.C. Error	95% interval	Inef	C.D
MERVAL (Argentina)						IBOVESPA (Brazil)					
μ	1.1647	0.0020	(0.9931,1.3291)	1.16	-1.79	μ	1.2898	0.0040	(0.9807,1.6022)	1.36	1.17
ϕ	0.9501	0.0007	(0.9353,0.9629)	19.32	-0.98	ϕ	0.9730	0.0007	(0.9565,0.9863)	14.83	0.26
σ	0.2688	0.0025	(0.2399,0.3044)	44.67	1.17	σ	0.1649	0.0025	(0.1325,0.1968)	44.23	-0.64
β_0	0.2052	0.0008	(0.1291,0.2815)	1.00	0.81	β_0	0.2575	0.0018	(0.1101,0.4091)	1.10	-0.07
β_1	0.0478	0.0003	(0.0157,0.0783)	1.00	-1.26	β_1	0.0309	0.0005	(-0.0161,0.0743)	1.10	-0.07
β_2	-0.0287	0.0025	(-0.0510,-0.0073)	1.00	-0.50	β_2	-0.0400	0.0004	(-0.0777,-0.0046)	1.00	-0.04
IPSA (Chile)						MEXBOL (Mexico)					
μ	-0.3596	0.0025	(-0.5706,-0.1555)	1.00	-0.33	μ	0.2316	0.0033	(-0.0634,0.5200)	1.00	-0.33
ϕ	0.9697	0.0004	(0.9599,0.9791)	15.91	0.06	ϕ	0.9803	0.0003	(0.9725,0.9870)	11.24	-0.56
σ	0.2111	0.0021	(0.1880,0.2385)	55.41	-0.11	σ	0.1859	0.0016	(0.1655,0.2102)	42.70	0.05
β_0	0.0710	0.0004	(0.0385,0.1057)	1.00	0.88	β_0	0.1039	0.0004	(0.1655,0.2102)	1.00	-0.16
β_1	0.1885	0.0003	(0.1578,0.2184)	1.00	-0.79	β_1	0.0742	0.0004	(0.0438,0.1051)	1.00	1.50
β_2	-0.0442	0.0003	(-0.0581,-0.0021)	1.00	-0.90	β_2	-0.0301	0.0003	(-0.0581,-0.0020)	1.00	-0.41
IGBVL (Peru)						S&P 500 (USA)					
μ	-0.0095	0.0025	(-0.2342,0.2044)	1.00	-1.08	μ	-0.1332	0.0032	(-0.4096,0.1375)	1.07	0.01
ϕ	0.9618	0.0006	(0.9490,0.9732)	19.82	-0.98	ϕ	0.9791	0.0005	(0.9705,0.9864)	28.76	0.91
σ	0.2725	0.0026	(0.2351,0.3102)	37.22	0.86	σ	0.1968	0.0035	(0.1686,0.2311)	100.00	-1.08
β_0	0.0596	0.0041	(0.0232,0.0960)	1.00	-0.01	β_0	0.1085	0.0004	(0.0754,0.1404)	1.00	-1.05
β_1	0.1875	0.0004	(0.1563,0.2199)	1.00	-1.21	β_1	-0.0504	0.0003	(-0.0853,-0.0232)	1.00	-0.89
β_2	-0.0114	0.0004	(-0.0414,0.0179)	1.11	0.86	β_2	-0.0595	0.0004	(-0.0924,-0.0272)	1.07	0.68

5% significance level for all return and squared return series.

We simulated the h_t's in one block using HMC method which was implemented by using 50 leapfrog steps and a step size of 0.015. $(\mu, \phi, \sigma)'$ and $(\beta_0, \beta_1, \beta_2)'$ are sampled in blocks using RMHMC method. In the case of $(\mu, \phi, \sigma)'$, we use a step size of 0.5, 20 leafprog steps and the number of fixed point iterations was 5. For $(\beta_0, \beta_1, \beta_2)'$ we use a step size of 0.1, 20 leapfrog steps and 5 fixed point iterations. We set the prior distributions of the common parameters as: $\beta_0 \sim \mathcal{N}(0, 10)$, $\frac{\beta_1+1}{2} \sim \mathcal{B}e(5, 1.5)$, $\beta_2 \sim \mathcal{N}(0, 10)$, $\mu \sim \mathcal{N}(0.0, 10)$, $\frac{\phi+1}{2} \sim \mathcal{B}e(20, 1.5)$, and $\sigma_\eta^2 \sim \mathcal{IG}(2.5, 0.025)$. These values imply that the prior mean and standard deviation of β_1 and ϕ are (0.5385,0.3077) and (0.8605,0.1074), respectively. For σ^2, the parameter setting implies a prior mean and prior standard deviation are (0.0167,0.0236).

We conducted the MCMC simulation for 30000 iterations. The first 10000 draws were discarded as a burn-in period. In order to reduce the autocorrelation between successive values of the simulated chain, only every $10th$ values of the chain are stored. With the resulting 2000 values, we calculated the posterior means, the 95% credible intervals, the Monte Carlo error of the posterior means, the inefficiency factors (Inef) and the convergence diagnostic (CD) statistics(Geweke, 1992). According to the CD values, the null hypothesis that the sequence of 2000 draws is stationary is accepted at 5% level for all the parameters and series considered here.

Table 2 summarizes these results for the five Latin American stock market and the S&P 500 returns. The value of ϕ is very similar among all markets, suggesting similar degrees of persistence (ranging from 0.9501 for Argentina to 0.9803 for Mexico). The MEXBOL, IBOVESPA and S&P 500 are more persistent. In fact, for the two former volatilities, the half-lives of the shocks have a duration of 34.9 and 25.3 days, respectively. In the cases of the IPSA, IGBVL and MERVAL, the durations are 22.5, 17.8 and 13.5 days, respectively. For the S&P 500, the half-life duration of a shock is around 32.8 days, which is very close to the result of the MEXBOL.

The posterior mean estimates of σ show that all returns have similar estimates in the range from 0.1649 to 0.2725. The higest value is 0.2725 for the IGBVL jointly with the estimate of ϕ indicates that IGBVL is the most volatile stock market index in the region. Regarding the posterior mean of μ, we found that the estimates are statistically significant for the MERVAL, IBOVESPA and IPSA indexes. For the MEXBOL, IGBVL and S & P 500 indexes, the parameter μ could be not significant because the credibility interval contains zero.

Table 3: Correlation matrix of posterior samples of the SVM model.

	μ	ϕ	σ	β_0	β_1	β_2		μ	ϕ	σ	β_0	β_1	β_2
MERVAL (Argentina)							IBOVESPA (Brazil)						
μ	1.0000						μ	1.0000					
ϕ	0.0438	1.0000					ϕ	-0.0153	1.0000				
σ	-0.0842	-0.6941	1.0000				σ	-0.0155	-0.5954	1.0000			
β_0	0.0229	-0.0517	0.0022	1.0000			β_0	-0.0434	-0.0452	0.0580	1.0000		
β_1	0.0247	-0.0331	0.0190	-0.0822	1.0000		β_1	0.0086	-0.0390	0.0015	-0.1493	1.0000	
β_2	-0.0007	0.0442	0.0121	-0.7504	0.0169	1.0000	β_2	0.0329	0.0340	-0.0235	-0.8114	0.0829	1.0000
IPSA (Chile)							MEXBOL (Mexico)						
μ	1.0000						μ	1.0000					
ϕ	-0.0079	1.0000					ϕ	0.0216	1.0000				
σ	-0.0390	-0.5806	1.0000				σ	-0.0290	-0.5456	1.0000			
β_0	-0.0086	-0.0556	0.0525	1.0000			β_0	0.0313	-0.0417	0.0588	1.0000		
β_1	0.0079	0.0381	-0.0525	-0.1391	1.0000		β_1	-0.0399	-0.0070	-0.0171	-0.0761	1.0000	
β_2	0.0001	0.0426	-0.0409	-0.7417	0.1021	1.0000	β_2	0.0151	0.0469	-0.0366	-0.6675	-0.0047	1.0000
IGBVL (Peru)							S & P 500 (USA)						
μ	1.0000						μ	1.0000					
ϕ	0.0105	1.0000					ϕ	0.0266	1.0000				
σ	-0.0495	-0.7258	1.0000				σ	-0.0619	-0.6820	1.0000			
β_0	0.0034	0.0593	-0.0603	1.0000			β_0	-0.0140	-0.0789	0.0826	1.0000		
β_1	-0.0052	0.0185	-0.0305	-0.0674	1.0000		β_1	0.0007	0.0364	-0.0599	-0.1315	1.0000	
β_2	0.0192	-0.0641	-0.0409	-0.6431	-0.0087	1.0000	β_2	0.0350	0.0273	0.0183	-0.6614	0.0764	1.0000

We observe that the posterior mean parameter β_0 is always positive and statistically significant for all series. The value of β_1 that measures the correlation of returns is as expected, small and very similar to the first-order autocorrelation coefficients reported in Table 1. The estimates of β_1 are statistically significant for all series with the exception of Brazil and, although in the cases of Chile and Peru these values are 0.188 and 0.1875, respectively, these values indicate a weak persistence with a rapid mean reversion. Regarding the parameter of interest (β_2), this is more negative in the cases of USA, Brazil and Chile. Intermediate values are observed in Argentina and Mexico while Peru presents the smallest value in absolute terms. Moreover, while all countries have a credibility interval that excludes the zero value, this does not happen in the case of Peru, so it is difficult to argue for an uncertainty effect in this market. It is important to note that the right side of the credibility interval is very close to zero in all markets except the U.S.. Therefore, the posterior mean of β_2 parameter, which measures both the ex ante relationship between returns and volatility and the volatility feedback effect, is negative for all series and statistically significant for all the series with the exception of Peru. Following Koopman and Uspensky (2002), the volatility feedback effect (negative) dominates the positive effect which links the returns with the expected volatility. Our estimates are more negative compared to those of Koopman and Uspensky (2002) where the hypothesis that $\beta_2 = 0$ can never be rejected at the conventional 5% siginificance level. Therefore, the volatility feedback effect is clearly dominant in our results (except for Peru) in comparison to those of Koopman and Uspensky (2002). These results confirm the hypothesis that investors require higher expected returns when unanticipated increases in future volatility are highly persistent. This is consistent with our findings of higher values of ϕ combined with larger negative values for the in-mean parameter. We have indirect evidence of a positive intertemporal relation between expected excess market returns and its volatility as this is one of the assumptions underlying the volatility feedback hypothesis.

Figures 1-6 show the MCMC output, the autocorrelation function (ACF) and the posterior densities of the parameters of the SVM model for all the indexes considered here. In all the cases, all the parameters showed well mixing properties with the exception of ϕ and σ. Regarding the inefficiency factors, showed in Table 2, these are higher in the ϕ and σ parameters. The U.S. shows the highest levels of inefficiency in the estimation of σ while in the case of Peru this is the lowest. A similar behavior is observed in the case of the ϕ parameter.

Now, we consider the correlation between all the parameters for indexes considered here, obtained from the MCMC output. From Table 3, we found small correlations for almost all the parameters with exception of ϕ and σ, and β_2 and β_0. In both cases, we found negative correlations. This fact indicates that if ϕ (β_2) increases σ (β_0) decreases or vice versa. For ϕ and σ, we found the most negative correlation for the IGBVL, followed by MERVAL and S&P 500. For β_2 and β_0, the most negative value is found in the IBOVESPA returns, followed by MERVAL and IPSA returns, respectively.

Figure 7 show the smoothed mean of $\mathrm{e}^{\frac{h_t}{2}}$ for all the indexes considered here. The smoothed mean is obtained as $\frac{1}{G}\sum_{i=1}^{G}\mathrm{e}^{\frac{h_t^{(i)}}{2}}$, where $h_t^{(i)}$ is the value of h_t for the $i-th$ MCMC iteration and G is the number of iterations. Figure 7 establishes a visual comparison of the evolutions of volatility and the absolute value of the returns. In both cases, the series show similar patterns at times of low, medium and high uncertainty, reflected in greater values for the estimated SVM model. It can be appreciated that the volatility of the Latin American stock market returns has been affected by the Russia and Brazilian and crises in 1998 and 1999, reflected in higher volatility. Likewise, the financial crisis in the United States (2008) has had serious repercussions on the behavior of the volatility of the Latin American stock markets. Moreover, it can also be appreciated that in 2010 there were no major shocks in these markets. However, in 2011 the crisis was accentuated in the countries of the European Community and North America, causing uncertainty in the markets of emerging economies such as Peru.

In order to compare the in-sample-fit of the SVM model, we estimate the GARCH-M(1,1) model defined by equations (4) and (5) in Appendix B. We conduct an MCMC simulation for 50000 iterations. The first 10000 draws were discarded as a burn-in period. In order to reduce the autocorrelation between successive values of the simulated chain, only every $10th$ values of the chain are stored.

The estimation results for the GARCH-M(1,1) model are given in Table 4, where we observe that estimates for the δ parameter are always positive and statistically significative and very similar to the corresponding parameter (β_1) in the SVM model (see Table 2) and the first-order autocorrelations in Table 1. The magnitude of the estimates of the in-mean parameter γ are similar to those obtained with the SVM model in absolute terms, so a large negative contemporaneous relationship in the SVM model is accompanied by a large positive ex-ante relationship in the GARCH-M model. However it is only observed for the cases of Brazil, Chile and Mexico because in the other cases, the null hypothesis of a zero ex ante relationship between excess returns and volatility can never be rejected at the 5%

Table 4: MCMC estimation of the GARCH-M(1,1) model.

Parameter	Mean	M.C. Error	95% interval	Inef	CD	Parameter	Mean	M.C. Error	95% interval	Inef	C.D
MERVAL (Argentina)						IBOVESPA (Brazil)					
ω	0.0322	0.0017	(-0.0520,0.1185)	1.10	0.07	ω	-0.0263	0.0020	(-0.1209,0.0424)	3.95	-0.53
δ	0.0730	0.0005	(0.0502,0.1064)	1.06	0.15	δ	0.0041	0.0003	(0.0000,0.0244)	1.51	1.07
γ	0.0171	0.0004	(-0.0063,0.0373)	1.00	-1.69	γ	0.0341	0.0007	(0.0102,0.0610)	2.81	0.00
α	0.1570	0.0009	(0.1115,0.2041)	1.21	-0.77	α	0.0692	0.0008	(0.0434,0.0932)	4.72	-0.58
β	0.1106	0.0004	(0.0932,0.1330)	1.22	0.84	β	0.0837	0.0005	(0.0691,0.0993)	3.33	0.37
θ	0.8589	0.0005	(0.8316,0.8797)	1.21	-0.12	θ	0.8973	0.0007	(0.8794,0.9170)	4.75	0.48
$\beta+\theta$	0.9695	0.0002	(0.9540,0.9814)	1.27	-1.31	$\beta+\theta$	0.9810	0.0003	(0.9714,0.9898)	5.93	0.48
IPSA (Chile)						MEXBOL (Mexico)					
ω	0.0158	0.0014	(-0.0042,0.0445)	29.17	0.53	ω	0.0225	0.0001	(-0.0183,0.0673)	1.00	0.05
δ	0.1909	0.0012	(0.1642,0.2124)	33.69	-0.50	δ	0.0818	0.0001	(0.0562,0.1105)	1.00	1.01
γ	0.0408	0.0013	(0.0056,0.0632)	22.70	-0.45	γ	0.0359	0.0001	(0.0097,0.0625)	1.00	-1.55
α	0.0245	0.0003	(0.0172,0.0346)	42.67	0.41	α	0.0165	0.0001	(0.0105,0.0258)	1.00	-0.06
β	0.1347	0.0010	(0.1176,0.1573)	29.15	1.62	β	0.0821	0.0004	(0.0675,0.0984)	1.00	0.33
θ	0.8436	0.0011	(0.8143,0.8617)	31.85	-1.06	θ	0.9103	0.0001	(0.8923,0.9623)	1.00	0.33
$\beta+\theta$	0.9783	0.0008	(0.9630,0.9897)	16.51	0.52	$\beta+\theta$	0.9923	0.0001	(0.9820,0.9973)	1.00	-0.65
IGBVL (Peru)						S&P 500 (USA)					
ω	0.0398	0.0006	(0.0026,0.0767)	1.00	-0.05	ω	0.0356	0.0003	(0.0005,0.0713)	1.00	0.68
δ	0.2076	0.0004	(0.1757,0.2387)	1.00	-0.46	δ	0.0059	0.0005	(0.0000,0.0375)	1.00	-0.42
γ	0.0185	0.0004	(-0.0084,0.0439)	1.00	0.13	γ	0.0299	0.0005	(-0.0023,0.0616)	1.00	-1.13
α	0.06186	0.0002	(0.0436,0.0850)	1.00	0.42	α	0.0233	0.0001	(0.0163,0.0302)	1.00	0.97
β	0.1853	0.0006	(0.1542,0.2222)	1.00	0.21	β	0.1018	0.0003	(0.0837,0.1203)	1.00	-0.78
θ	0.7868	0.0005	(0.7435,0.8230)	1.00	-0.10	θ	0.8810	0.0003	(0.8588,0.9006)	1.00	-0.19
$\beta+\theta$	0.9722	0.0003	(0.9542,0.9866)	1.00	0.18	$\beta+\theta$	0.9828	0.0001	(0.9730,0.9898)	1.00	-0.19

significance level. Compared to the SVM model, this result is consistent only for Peru. For the cases of U.S. and Argentina, results are in opposition with the in-mean parameter β_2 in the SVM models where it is statistically significant. The case of U.S. is surprising given that the SVM model suggests a statistically significant in-mean parameter whereas the GARCH-M model suggests not rejection of the zero value hypothesis. Overall, the results confirm the hypothesis that investors require higher expected returns when unanticipated increases in future volatility are highly persistent in the cases of Brazil, Chile and Mexico. According to the results of the GARCH-M model, these three markets present indirect evidence of a positive intertemporal relation between expected excess market returns and its volatility as this is one of the assumptions underlying the volatility feedback hypothesis. The SVM model suggests more and stronger evidence in favor of this hypothesis.

The volatility persistence parameters are comparable, but slightly more persistent to those found for the SVM model with near-unity values for $\beta + \theta$. It is more evident for Mexico.

We use the log-predictive score (LPS, Delatola and Griffin, 2011) and the deviance information criteria (DIC, Spiegelhalter et al., 2002) to compare all the competing models. In both cases, the best model has the smallest LPS (DIC). In order to evaluate the likelihood approximation in the SVM model, we apply the HMM machinery as described in Appendix C. To ensure a good approximation of the estimates, we use $b_m = -b_0 = 4.5$ and $m = 200$ (See Abanto-Valle et al., 2017, for more details). Table 5 shows the LPS and DIC for all the indexes considered here. Acoording to the LPS and DIC (see values in bold in Table 5) the SVM model outperforms the GARCH-M model for all the indexes.

6 Discussion

This article presented a Bayesian implementation in order to estimate the SVM model as proposed by Koopman and Uspensky (2002), via HMC and RMHMC methods. The SVM model enabled us to investigate the dynamic relationship between returns and their time-varying volatility. We illustrated our methods through an empirical application of five Latin American return series and the S&P 500 return. The β_2 estimate, which measures both the ex ante relationship between returns and volatility and the volatility feedback effect, was found to be negative and significant for all the indexes considered here with the exception of the IGBVL. The results are in line with those of French et al. (1987), who found a similar relationship between unexpected volatility dynamics and returns, and confirm the hypothesis that investors require higher expected returns when unanticipated increases in future volatility are highly persistent. This is consistent with our findings of higher values of ϕ combined with larger negative values for the in-mean parameter. We also estimated a GARCH-M model in order to compare estimates of the in-mean parameter. The results are consistent with those of the SVM model only for Brazil, Chile and Mexico. For the other countries the null hypothesis that the in-mean parameter is zero is not rejected which, at least for U.S. is surprising and rare.

Table 5: Model comparison criteria. Log-predictive score (LPS) and Deviance Information Criterion (DIC)

	LPS		DIC	
INDEX / MODEL	GARCH-M	SVM	GARCH-M	SVM
MERVAL	2.0981	**2.0720**	19528.50	**19285.50**
IBOVESPA	1.9763	**1.9720**	18580.21	**18540.66**
IPSA	1.2997	**1.2931**	12325.90	**12262.62**
MEXBOL	1.6038	**1.5875**	15257.72	**15121.16**
IGBVL	1.5192	**1.4942**	13979.51	**13749.80**
S P 500	1.4293	**1.4134**	13667.49	**13516.47**

Future research includes extending the model and algorithm to include time-varying parameters including the in-mean parameter. This would allow comparison with other algorithms such as the one proposed in Chan (2017). The second would be to incorporate heavy-tailedness as in Abanto-Valle et al. (2012) or skewness and heavy-tailedness simultaneously as in Leão et al. (2017).

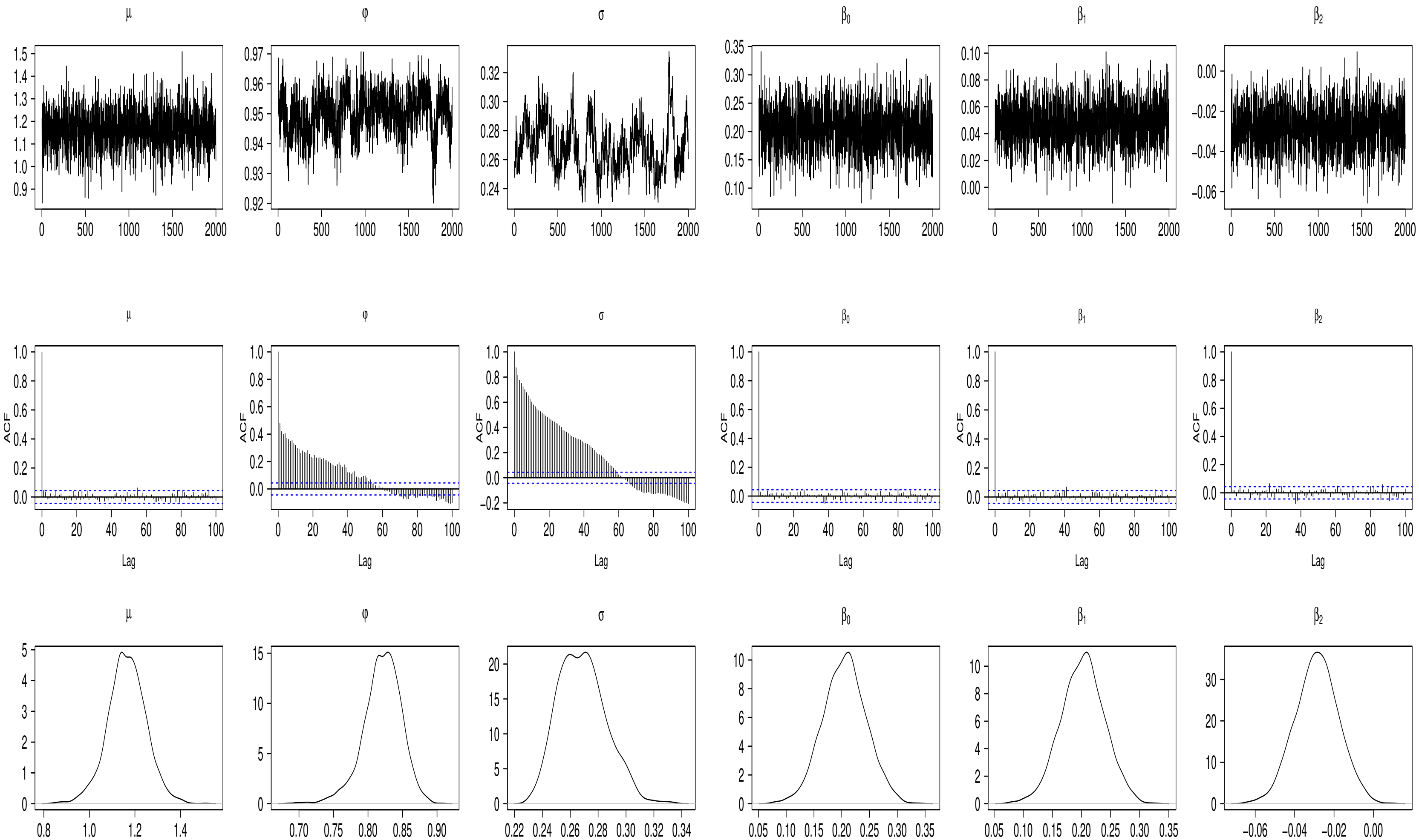

Figure 1: MCMC estimation results of the SVM model for MERVAL (Argentina). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

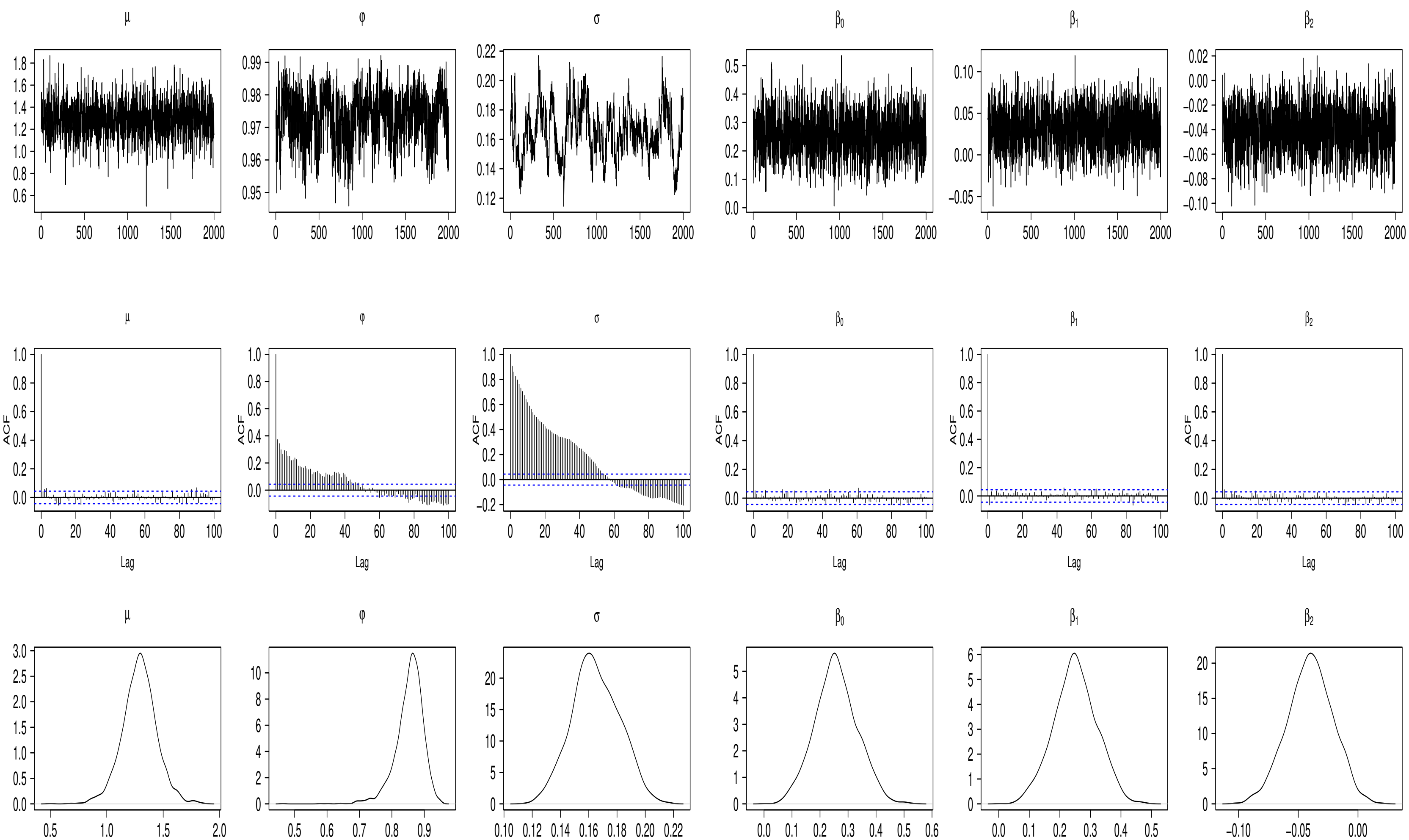

Figure 2: MCMC estimation results of the SVM model for BOVESPA (Brazil). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

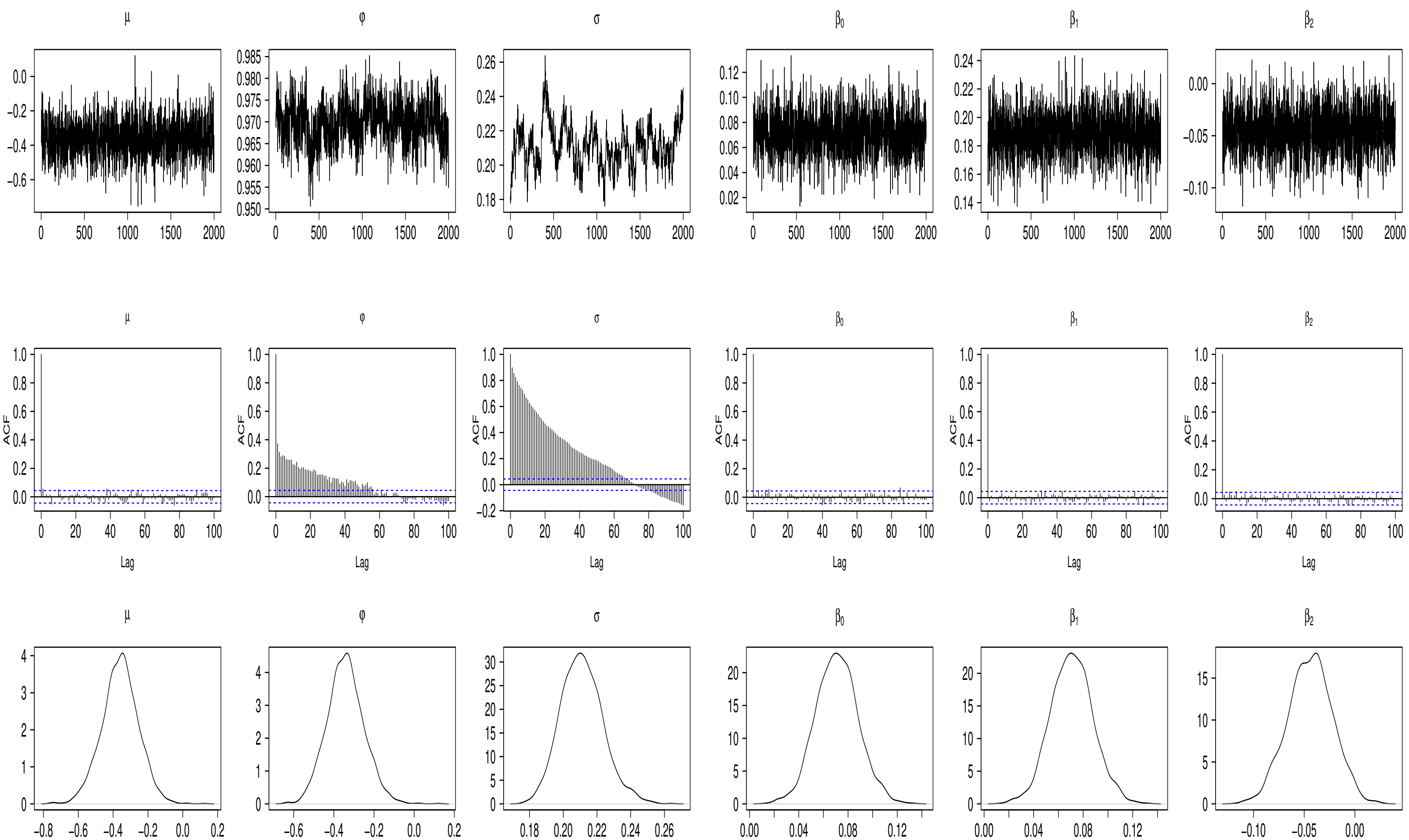

Figure 3: MCMC estimation results of the SVM model for IPSA (Chile). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

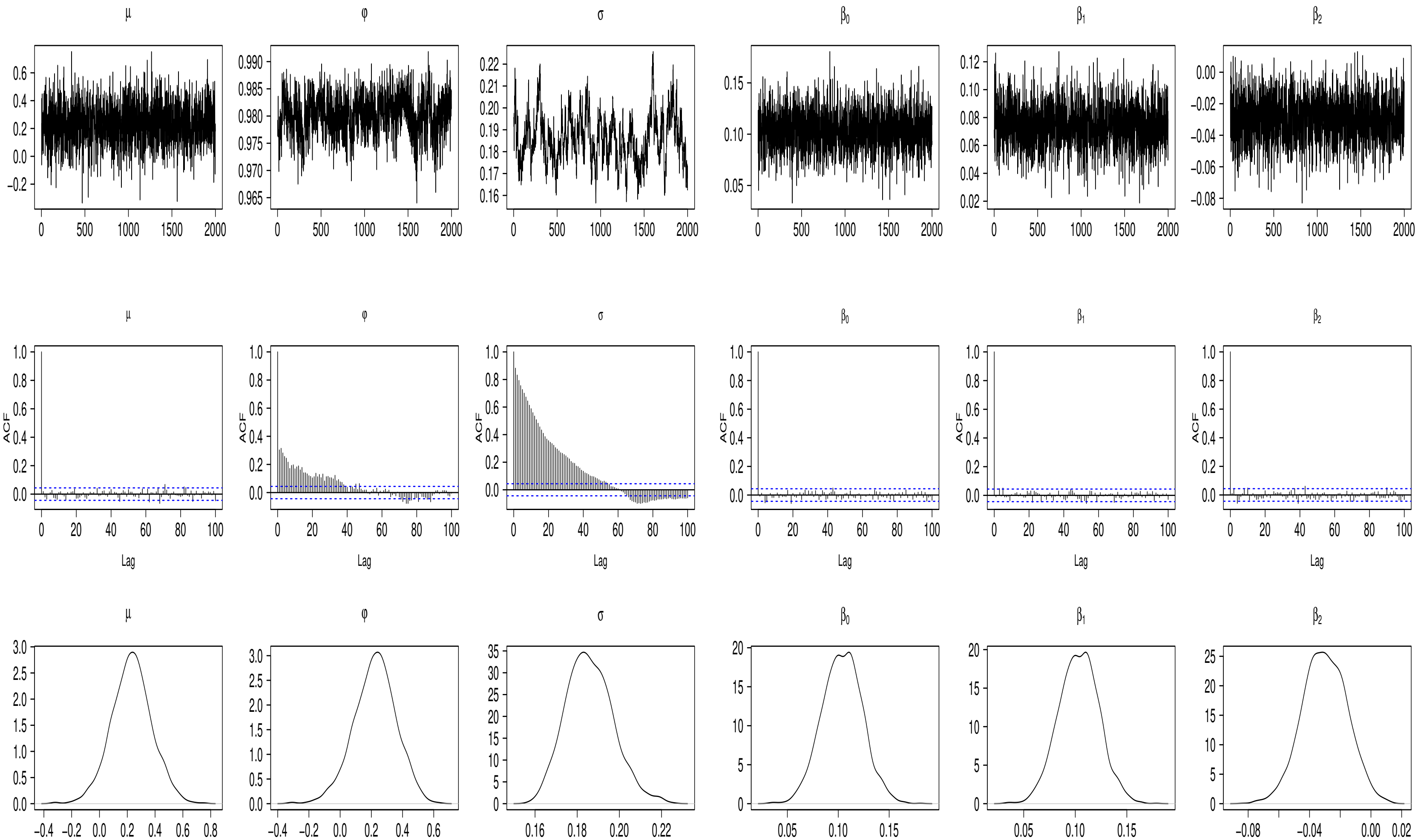

Figure 4: MCMC estimation results of the SVM model for MEXBOL (Mexico). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

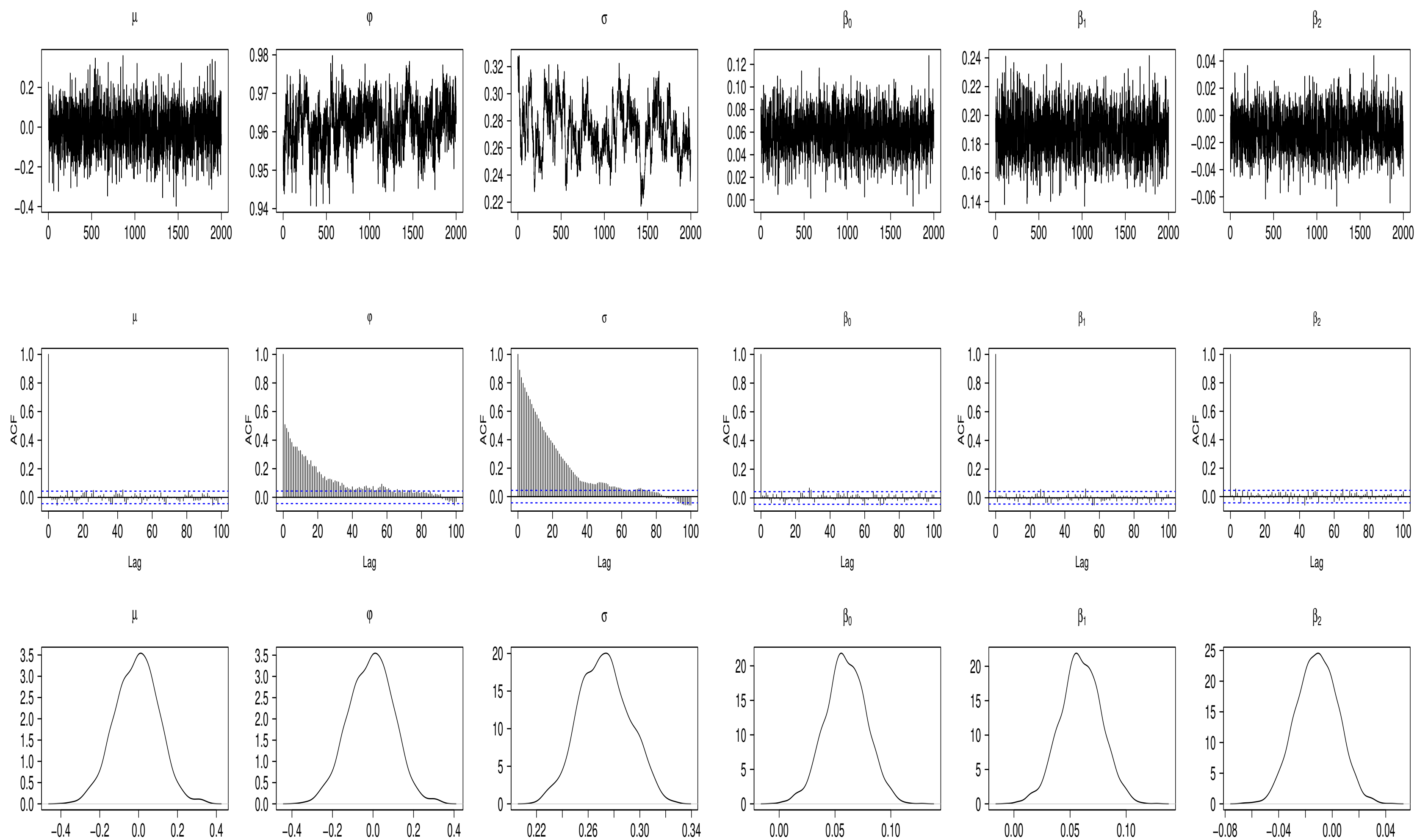

Figure 5: MCMC estimation results of the SVM model for IGBVL(Peru). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

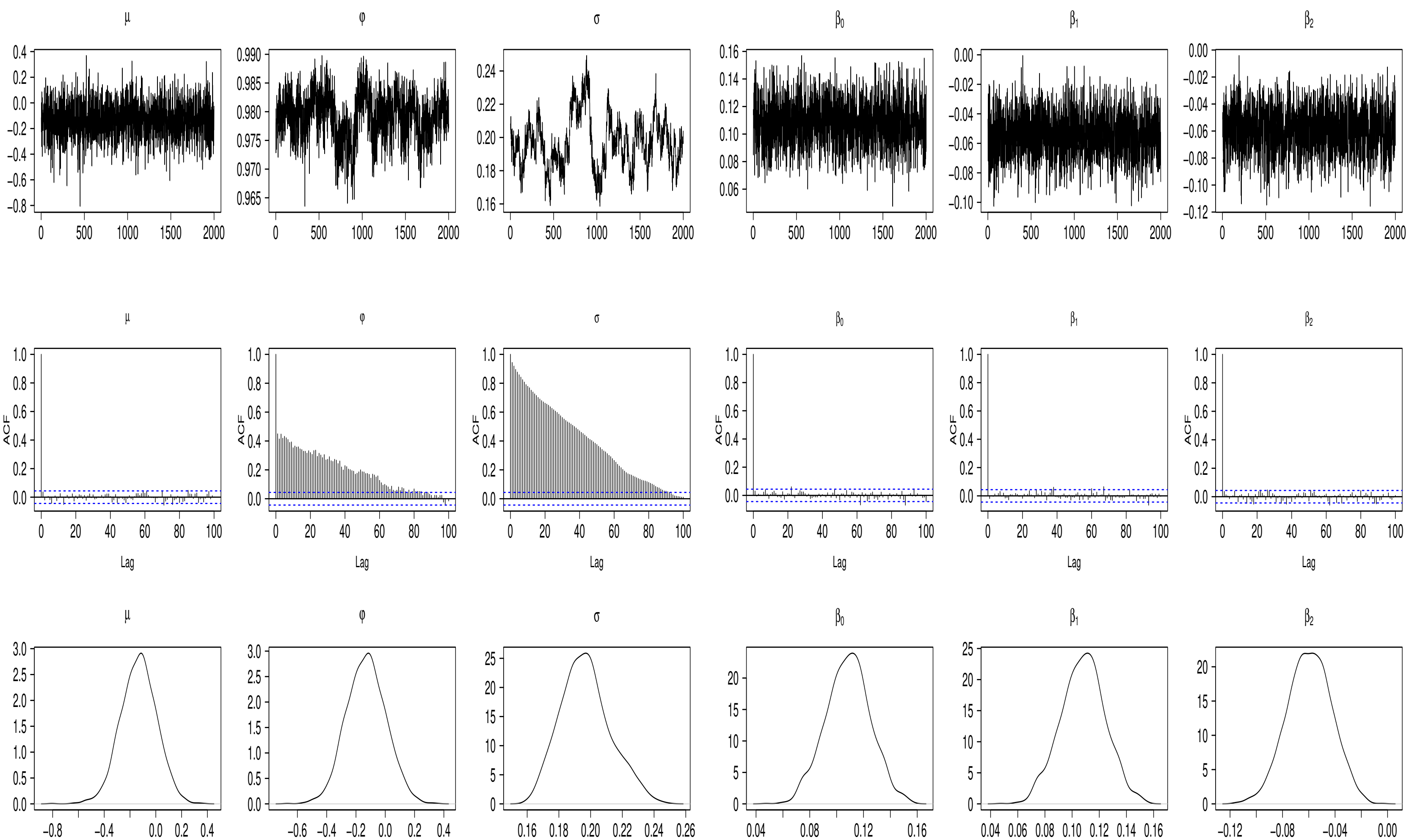

Figure 6: MCMC estimation results of the SVM model for S&P 500 (USA). Sample paths (top), autocorrelations (middle) and posterior densities (bottom)

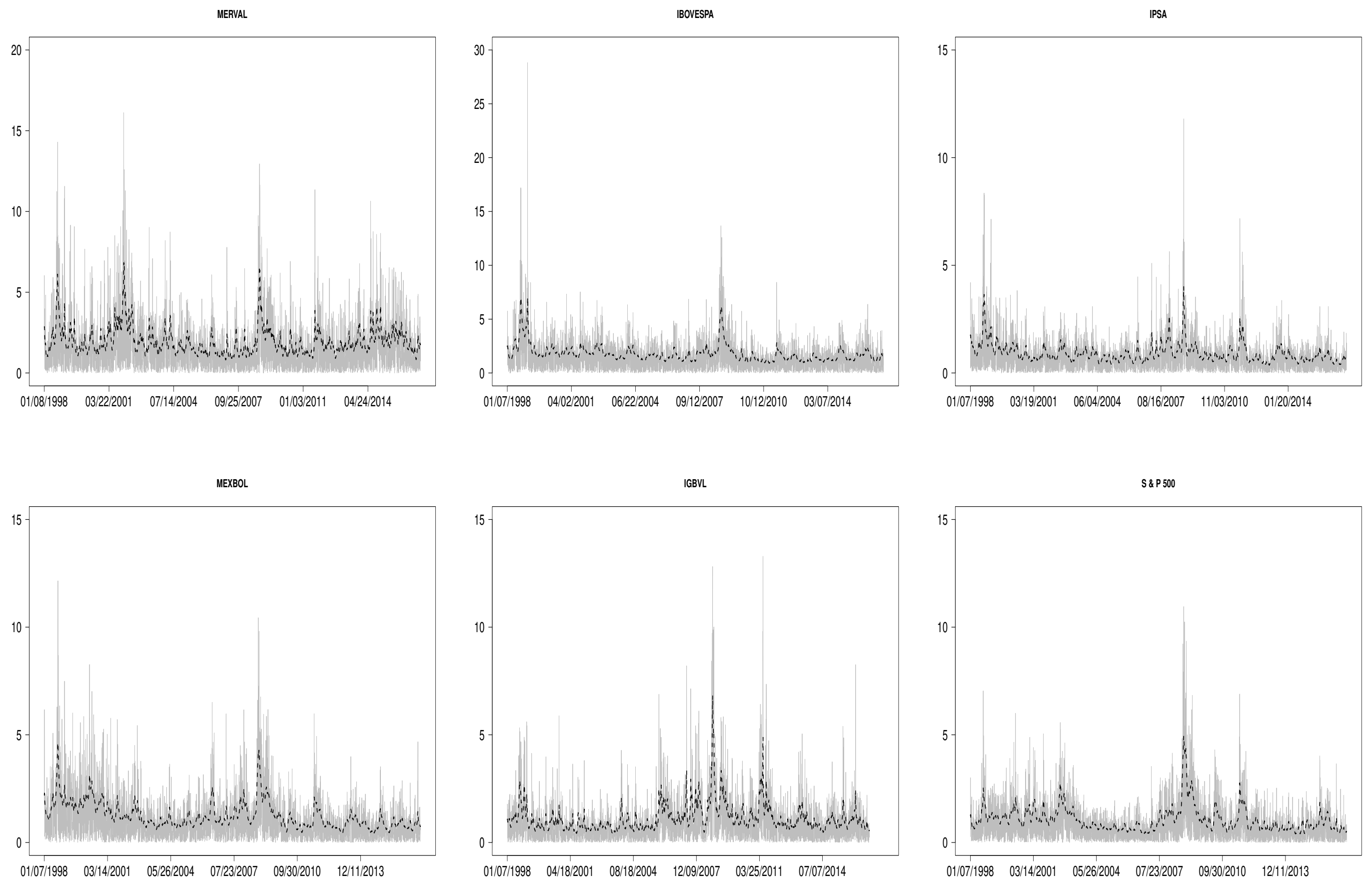

Figure 7: MERVAL, IBOVESPA, IPSA, MEXBOL, IGBVL and SP500 returns data sets: absolute returns (full gray line) and posterior smoothed mean of $e^{\frac{h_t}{2}}$ of the SVM model (dotted black line).

Appendix

A.1 Sampling scheme for $(\mu, \phi, \sigma)'$

We assume that $\mu \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$, $(\phi + 1)/2 \sim \mathcal{B}e(a_\phi, b_\phi)$, $\sigma^2 \sim \mathcal{IG}(a_\sigma, b_\sigma)$. To deal with constraints on the values of ϕ, σ, we make the transformation of these to the real line. So, we have $\phi = \tanh(\omega)$, $\sigma = \exp(\gamma)$. Let $\boldsymbol{\theta}_1 = (\mu, \omega, \gamma)'$ and from equation (3), we have

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}_1) &= \text{constant} + \frac{1}{2}\log(1-\phi^2) - \frac{1-\phi^2}{2\sigma^2}(h_1 - \mu)^2 - \frac{1}{2\sigma^2}\sum_{t=1}^{T-1}[h_{t+1} - \mu - \phi(h_t - \mu)]^2 \\
&- T\log(\sigma) + (a_\phi - 1)(1+\phi) + (b_\phi - 1)\log(1-\phi) - 2(a_\sigma + 1)\log(\sigma) - \frac{b_\sigma}{\sigma^2} \\
&- \frac{1}{2\sigma_\mu^2}(\mu - \mu_0)^2.
\end{aligned}
$$

As $\phi = \tanh(\omega)$ and $\sigma = \exp(\gamma)$, then $\frac{d\phi}{d\omega} = 1 - \phi^2$ and $\frac{d\sigma}{d\gamma} = \sigma$, so we have that the gradient is given by

$$
\triangledown_{\boldsymbol{\theta}_1}\mathcal{L}(\boldsymbol{\theta}_1) = \begin{bmatrix} \triangledown_\mu \mathcal{L}(\boldsymbol{\theta}_1) \\ \triangledown_\omega \mathcal{L}(\boldsymbol{\theta}_1) \\ \triangledown_\gamma \mathcal{L}(\boldsymbol{\theta}_1) \end{bmatrix},
$$

where

$$
\begin{aligned}
\triangledown_\mu \mathcal{L}(\boldsymbol{\theta}_1) &= \frac{1-\phi^2}{\sigma^2}(h_1 - \mu) + \frac{1-\phi}{\sigma^2}\sum_{t=1}^{T-1}[h_{t+1} - \mu - \phi(h_t - \mu)] - \frac{1}{\sigma_\mu^2}(\mu - \mu_0) \\
\triangledown_\omega \mathcal{L}(\boldsymbol{\theta}_1) &= -\phi + \frac{\phi(1-\phi^2)}{\sigma^2}(h_1 - \mu)^2 + \frac{1-\phi^2}{\sigma^2}\sum_{t=1}^{T-1}[h_{t+1} - \mu - \phi(h_t - \mu)][h_t - \mu] \\
&+ (a_\phi - 1)(1-\phi) - (b_\phi - 1)(1+\phi) \\
\triangledown_\gamma \mathcal{L}(\boldsymbol{\theta}_1) &= -T + \frac{1-\phi^2}{\sigma^2}(h_1 - \mu)^2 + \frac{1}{\sigma^2}\sum_{t=1}^{T-1}[h_{t+1} - \mu - \phi(h_t - \mu)]^2 \\
&- 2(a_\sigma + 1) + 2\frac{b_\sigma}{\sigma^2}.
\end{aligned}
$$

Then, we have the tensor matrix:

$$
\mathbf{M}(\boldsymbol{\theta}_1) = \begin{bmatrix} \frac{(1-\phi)^2(T-1)}{\sigma^2} + \frac{1-\phi^2}{\sigma^2} + \frac{1}{\sigma_\mu^2} & 0 & 0 \\ 0 & 2\phi^2 + (T-1)(1-\phi^2) + (a_\phi + b_\phi - 2)(1-\phi^2) & 2\phi \\ 0 & 2\phi & 2T + \frac{4b_\sigma}{\sigma^2} \end{bmatrix}
$$

and

$$
\begin{aligned}
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_1)}{\partial \mu} &= \mathbf{0}_{3\times 3} \\
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_1)}{\partial \omega} &= \begin{bmatrix} -\frac{2(1-\phi^2)}{\sigma^2}[(1-\phi)(T-1)+\phi] & 0 & 0 \\ 0 & 2\phi(1-\phi^2)[4-(T-1)-a_\phi-b_\phi] & 2(1-\phi^2) \\ 0 & 2(1-\phi^2) & 0 \end{bmatrix} \\
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_1)}{\partial \gamma} &= \begin{bmatrix} -\frac{2}{\sigma^2}[(1-\phi)^2(T-1)+1-\phi^2] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{-8b_\sigma}{\sigma^2} \end{bmatrix}
\end{aligned}
$$

A.2 Sampling scheme for $(\beta_0, \beta_1, \beta_2)'$

We assume the priors distribution as follows: $\beta_0 \sim \mathcal{N}(\bar{\beta}_0, \sigma^2_{\beta_0})$, $(\beta_1+1)/2 \sim \mathcal{B}e(a_{\beta_1}, b_{\beta_1})$ and $\mathcal{N}(\bar{\beta}_2, \sigma^2_{\beta_2})$. We make the transformation $\beta_1 = \tanh(\delta)$ and let $\boldsymbol{\theta}_2 = (\beta_0, \delta, \beta_2)'$ and from equation (3), we have:

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{\theta}_2) &= \text{constant} - \frac{1}{2}\sum_{t=1}^{T} \lambda_t \mathrm{e}^{-h_t}[y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 \mathrm{e}^{h_t}]^2 - \frac{1}{2\sigma_{\beta_0^2}}(\beta_0 - \bar{\beta}_0)^2 \\
&+ (a_{\beta_1} - 1)\log(1+\beta_1) + (b_{\beta_1} - 1)\log(1-\beta_1) - \frac{1}{2\sigma_{\beta_2^2}}(\beta_2 - \bar{\beta}_2)^2.
\end{aligned}
$$

As, $\beta_1 = \tanh(\delta)$ $\frac{d\beta_1}{d\delta} = 1 - \beta_1^2$. Then, we have the gradient is given by

$$
\bigtriangledown_{\boldsymbol{\theta}_2}\mathcal{L}(\boldsymbol{\theta}_2) = \begin{bmatrix} \bigtriangledown_{\beta_0}\mathcal{L}(\boldsymbol{\theta}_2) \\ \bigtriangledown_{\delta}\mathcal{L}(\boldsymbol{\theta}_2) \\ \bigtriangledown_{\beta_2}\mathcal{L}(\boldsymbol{\theta}_1) \end{bmatrix},
$$

where

$$
\begin{aligned}
\bigtriangledown_{\beta_0}\mathcal{L}(\boldsymbol{\theta}_2) &= \sum_{t=1}^{T} \lambda_t \mathrm{e}^{-h_t}[y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 \mathrm{e}^{h_t}] - \frac{1}{\sigma_{\beta_0^2}}(\beta_0 - \bar{\beta}_0) \\
\bigtriangledown_{\delta}\mathcal{L}(\boldsymbol{\theta}_2) &= (1-\beta_1^2)\sum_{t=1}^{T} \lambda_t \mathrm{e}^{-h_t}[y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 \mathrm{e}^{h_t}]y_{t-1} \\
&+ (a_{\beta_1} - 1)(1-\beta_1) - (b_{\beta_1} - 1)(1+\beta_1) \\
\bigtriangledown_{\beta_2}\mathcal{L}(\boldsymbol{\theta}_2) &= \sum_{t=1}^{T} \lambda_t [y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 \mathrm{e}^{h_t}] - \frac{1}{\sigma_{\beta_2^2}}(\beta_0 - \bar{\beta}_2).
\end{aligned}
$$

Then, we have the tensor matrix:

$$\mathbf{M}(\boldsymbol{\theta}_2) = \begin{bmatrix} \sum_{t=1}^{T} \mathrm{e}^{-h_t} + \frac{1}{\sigma_{\beta_0^2}} & (1-\beta_1^2)\sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1} & T \\ (1-\beta_1^2)\sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1} & (1-\beta_1^2)^2 \sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1}^2 + (a_{\beta_1} + b_{\beta_1} - 2)(1-\beta_1^2) & (1-\beta_1^2)\sum_{t=1}^{T} y_{t-1} \\ \sum_{t=1}^{T} & (1-\beta_1^2)\sum_{t=1}^{T} \lambda_t y_{t-1} & \sum_{t=1}^{T} \mathrm{e}^{h_t} + \frac{1}{\sigma_{\beta_2^2}} \end{bmatrix}$$

$$\begin{aligned}
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_2)}{\partial \mu} &= \mathbf{0}_{3\times 3} \\
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_2)}{\partial \omega} &= \begin{bmatrix} 0 & -2\beta_1(1-\beta_1^2)\sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1} & 0 \\ -2\beta_1(1-\beta_1^2)\sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1} & -4\beta_1(1-\beta_1^2)^2 \sum_{t=1}^{T} \mathrm{e}^{-h_t} y_{t-1}^2 - 2\beta_1(a_{\beta_1} + b_{\beta_1} - 2)(1-\beta_1^2) & -2\beta_1(1-\beta_1^2)\sum_{t=1}^{T} y_{t-1} \\ 0 & -2\beta_1(1-\beta_1^2)\sum_{t=1}^{T} y_{t-1} & 0 \end{bmatrix} \\
\frac{\partial \mathbf{M}(\boldsymbol{\theta}_1)}{\partial \beta_2} &= \mathbf{0}_{3\times 3}
\end{aligned}$$

A3. Sampling $\mathbf{h}_{1:T}$

Let $\mathcal{L}(\mathbf{h}_{1:T})$ be defined by

$$\begin{aligned}
\mathcal{L}(\mathbf{h}_{1:T}) &= \text{constant} + \sum_{t=1}^{T} \frac{h_t}{2} - \frac{1}{2\sigma^2}\sum_{t=1}^{T-1}[h_{t+1} - \mu - \phi(h_t - \mu)]^2 - \frac{1-\phi^2}{2\sigma^2}(h_1 - \mu)^2 \\
&- \frac{1}{2}\sum_{t=1}^{T} \mathrm{e}^{-h_t}(y_t - \beta_0 - \beta_1 y_{t-1} - \beta_2 \mathrm{e}^{h_t})^2.
\end{aligned}$$

Then, we have the gradient $\bigtriangledown_{\mathbf{h}_{1:T}}\mathcal{L}(\mathbf{h}_{1:T})$ is given by

$$\bigtriangledown_{\mathbf{h}_{1:T}}\mathcal{L}(\mathbf{h}_{1:T}) = \begin{bmatrix} \bigtriangledown_{h_1}\mathcal{L}(\mathbf{h}_{1:T}) \\ \vdots \\ \bigtriangledown_{h_t}\mathcal{L}(\mathbf{h}_{1:T}) \\ \vdots \\ \bigtriangledown_{h_T}\mathcal{L}(\mathbf{h}_{1:T}) \end{bmatrix},$$

where

$$
\begin{aligned}
\bigtriangledown_{h_1}\mathcal{L}(\mathbf{h}_{1:T}) &= -\frac{1}{2}+\frac{1}{2}\mathrm{e}^{-h_1}(y_1-\beta_0-\beta_1 y_0-\beta_2\mathrm{e}^{h_1})^2+\beta_2(y_1-\beta_0-\beta_1 y_0-\beta_2\mathrm{e}^{h_1})\\
&+ \frac{\phi}{\sigma^2}[h_2-\mu-\phi(h_1-\mu)]-\frac{1-\phi^2}{\sigma^2}(h_1-\mu)\\
\bigtriangledown_{h_t}\mathcal{L}(\mathbf{h}_{1:T}) &= -\frac{1}{2}+\frac{1}{2}\mathrm{e}^{-h_t}(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2\mathrm{e}^{h_t})^2+\beta_2(y_t-\beta_0-\beta_1 y_{t-1}-\beta_2\mathrm{e}^{\ h_t})\\
&+ \frac{\phi}{\sigma^2}[h_{t+1}-\mu-\phi(h_t-\mu)]-\frac{1}{\sigma^2}[h_t-\mu-\phi(h_{t-1}-\mu)] \qquad \text{for } 1<t<T\\
\bigtriangledown_{h_T}\mathcal{L}(\mathbf{h}_{1:T}) &= -\frac{1}{2}+\frac{1}{2}\mathrm{e}^{-h_T}(y_T-\beta_0-\beta_1 y_{T-1}-\beta_2\mathrm{e}^{h_T})^2+\beta_2(y_T-\beta_0-\beta_1 y_T-\beta_2\mathrm{e}^{h_T})\\
&- \frac{1}{\sigma^2}[h_T-\mu-\phi(h_{T-1}-\mu)]
\end{aligned}
$$

B. Sampling scheme of the parameters in the GARCH-M(1,1) model

The GARCH(1,1) in mean model is defined by

$$
\begin{aligned}
y_t &= \omega+\delta y_{t-1}+\gamma h_t+h_t^{1/2}\epsilon_t \qquad \epsilon_t\sim\mathcal{N}(0,1) &(4)\\
h_t &= \alpha+\beta(h_t\epsilon_{t-1}^2)+\theta h_{t-1} &(5)
\end{aligned}
$$

where $|\delta|<1$, $\alpha\geq 0$, $\beta,\theta>0$ and $\beta+\theta<1$. We use the following reparameterization $\psi=\log(\delta/(1-\delta))$, $\kappa=\log(\alpha)$, $\lambda=\log(\beta/(1-\beta))$ and $\rho=\log(\theta/(1-\beta-\theta))$. Let $\boldsymbol{\theta}=(\omega,\psi,\gamma,\kappa,\lambda,\rho)'$. We assume $\boldsymbol{\theta}\sim\mathcal{N}_6(\boldsymbol{\theta}_0,\boldsymbol{\Sigma}_0)$, where $\mathcal{N}_q(.,.)$ denotes the q-variate normal distribution. Then the posterior distribution of $\boldsymbol{\theta}$ is given by

$$
\begin{aligned}
\pi(\boldsymbol{\theta}\mid y_{-1},y_0,y_1,\ldots,y_t) &\propto \prod_{t=1}^{T} h_t^{-1/2}\exp\{-\frac{1}{2h_t}(y_t-\omega-\delta y_{t-1}-\gamma h_t)^2\}\\
&\times \exp\{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}_0)'\boldsymbol{\Sigma}_0^{-1}(\boldsymbol{\theta}-\boldsymbol{\theta}_0)\}.
\end{aligned}
$$

As $\pi(\boldsymbol{\theta}\mid y_{-1},y_0,y_1,\ldots,y_t)$ does not have closed form, we sample form using a Metropolis-Hastings algorithm with proposal $N_6(\boldsymbol{\theta}_1,\boldsymbol{\Sigma}_1)$, where $\boldsymbol{\theta}_1$ is the maximum a posteriori and $\boldsymbol{\Sigma}_1$ the inverse of the Hessian matrix evaluated at $\boldsymbol{\theta}_1$

C. Likelihood evaluation by iterated numerical integration and fast evaluation of the approximate likelihood using HMM techniques

To formulate the likelihood, we require the conditional *pdfs* of the random variables y_t, given h_t and y_{t-1} $(t = 1, \dots, T)$, and of the random variables h_t, given h_{t-1} $(t = 2, \dots, T)$. We denote these by $p(y_t \mid y_{t-1}, h_t)$ and $p(h_t \mid h_{t-1})$, respectively. For a member of the class of SMN distributions, the likelihood of the model defined by equations (2a) and (2b) can then be derived as

$$\begin{aligned}
\mathcal{L} &= \int \dots \int p(y_1, \dots, y_T, h_1, \dots, h_T \mid y_0) dh_T \dots dh_1 \\
&= \int \dots \int p(y_1, \dots, y_T \mid y_0, h_1, \dots, h_T) p(h_1, \dots, h_T) dh_T \dots dh_1 \\
&= \int \dots \int p(h_1) p(y_1 \mid y_0, h_1) \prod_{t=2}^{T} p(y_t \mid y_{t-1}, h_t) p(h_t \mid h_{t-1}) dh_T \dots dh_1,
\end{aligned}$$

exploiting the dependence structure of the stochastic volatility models in the last step. Hence, the likelihood is a high-order multiple integral that cannot be evaluated analytically. Through numerical integration, using a simple rectangular rule based on m equidistant intervals, $B_i = (b_{i-1}, b_i)$, $i = 1, \dots, m$, with midpoints b_i^* and length b, the likelihood can be approximated as follows:

$$\begin{aligned}
\mathcal{L} \approx b^T \sum_{i_1=1}^{m} \dots \sum_{i_T=1}^{m} & p(h_1 = b_{i_1}^*) p(y_1 \mid y_0, h_1 = b_{i_1}^*) \\
& \times \prod_{t=2}^{T} p(h_t = b_{i_t}^* \mid h_{t-1} = b_{i_{t-1}}^*) p(y_t \mid y_{t-1}, h_t = b_{i_t}^*) = \mathcal{L}_{\text{approx}} \,. \qquad (6)
\end{aligned}$$

This approximation can be made arbitrarily accurate by increasing m, provided that the interval (b_0, b_m) covers the essential range of the log-volatility process. We note that this simple midpoint quadrature is by no means the only way in which the integral can be approximated (cf. Langrock et al., 2012).

In the form given in (6), the approximate likelihood can be evaluated numerically, but the evaluation will usually be computationally intractable since it involves m^T summands. However, instead of the brute force summation in (6), an efficient recursive scheme can be used to evaluate the approximate likelihood. To see this, we note that the numerical integration essentially corresponds to a discretization of the state space, i.e., the support of the log-volatility process h_t. Therefore, the approximate likelihood given in (6) can be evaluated using the tools well-established for HMMs, which are the models that have exactly the same dependence structure as the stochastic volatility models, but with a finite and hence discrete state space (cf. Langrock, 2011; Langrock et al., 2012). In the given scenario, the discrete states correspond to the intervals B_i, $i = 1, \ldots, m$, in which the state space has been partitioned. A key property of HMM, which we exploit here, is that the likelihood can be evaluated efficiently using the so-called forward algorithm, a recursive scheme which iteratively traverses forward along the time series, updating the likelihood and the state probabilities in each step (Zucchini et al., 2016). For an HMM, applying the forward algorithm results in a convenient closed-form matrix product expression for the likelihood, and this is exactly what is obtained also for the SVM model:

$$\mathcal{L}_{\text{approx}} = \boldsymbol{\delta}\mathbf{P}(y_1)\boldsymbol{\Gamma}\mathbf{P}(y_2)\boldsymbol{\Gamma}\mathbf{P}(y_3)\cdots\boldsymbol{\Gamma}\mathbf{P}(y_{T-1})\boldsymbol{\Gamma}\mathbf{P}(y_T)\mathbf{1}'\,. \tag{7}$$

Here, the $m \times m$-matrix $\boldsymbol{\Gamma} = \left(\gamma_{ij}\right)$ is the analogue to the transition probability matrix in case of an HMM, defined by $\gamma_{ij} = p(h_t = b_j^* \mid h_{t-1} = b_i^*) \cdot b$, which is an approximation of the probability of the log-volatility process changing from some value in the interval B_i to some value in the interval B_j; this conditional probability is determined by (2b). The vector $\boldsymbol{\delta}$ is the analogue to the Markov chain initial distribution in case of an HMM, here defined such that δ_i is the density of the $\mathcal{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$-distribution — the stationary distribution of the log-volatility process — multiplied by b. Furthermore, $\mathbf{P}(y_t)$ is an $m \times m$ diagonal matrix with the ith diagonal entry $p(y_t \mid y_{t-1}, h_t = b_i^*)$, hence the analogue to the matrix comprising the state-dependent probabilities in case of an HMM; this conditional probability is determined by (2a). Finally, $\mathbf{1}'$ is a column vector of ones. Using the matrix product expression given in (7), the computational effort required to evaluate the approximate likelihood is linear in the number of observations, T, and quadratic in the number of intervals used in the discretization, m.

In practice, this means that the likelihood can typically be calculated in a fraction of a second, even for T in the thousands and say $m = 100$, a value which renders the approximation virtually exact. Furthermore, the approximation can be made arbitrarily accurate by increasing m (and potentially widening the interval $[b_0, b_m]$).

It should perhaps be noted here that, although we are using the HMM forward algorithm to evaluate the (approximate) likelihood, the specifications of $\boldsymbol{\delta}$, $\boldsymbol{\Gamma}$ and $\mathbf{P}(x_t)$ given above do not define exactly an HMM, since in general the row sums of $\boldsymbol{\Gamma}$ will only approximately equal one, and the components of the vector $\boldsymbol{\delta}$ will only approximately sum to one. If desired, this can be remedied by scaling each row of $\boldsymbol{\Gamma}$ and the vector $\boldsymbol{\delta}$ to total 1.

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