

N° 541

PUBLIC DEBT DYNAMICS
AND SUSTAINABILITY:
A FRAMEWORK
FOR ANALYSIS

Waldo Mendoza, Marko Razzo
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Documento de Trabajo 541

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Editado:

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Primera edición – Diciembre, 2024

ISSN 2079-8474 (En línea)

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**Departamento
Académico de Economía**

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<http://doi.org/10.18800/2079-8474.0541>

Public Debt Dynamics and Sustainability: A Framework for Analysis

Waldo Mendoza, Marko Razzo and Rafael Vilca¹

ABSTRACT

This paper presents a macro-fiscal model for examining the public sector primary surplus and the dynamics and sustainability of public debt in closed and open economies. The model simulates how changes in the primary surplus affect public debt, highlighting key differences between these economic contexts. Notably, open economies can finance fiscal deficits with foreign currency-denominated debt, introducing additional sources of instability in public debt dynamics. The analysis demonstrates how a permanent reduction in the primary surplus undermines public debt sustainability, with outcomes shaped by economic conditions and the features of open and closed economies. Furthermore, it confirms that delays in implementing fiscal adjustments following a destabilizing shock result in increasingly severe corrective measures over time.

JEL Classification: E62, H62, H63

Keywords: Debt, Dynamic, Sustainability, Fiscal Adjustment

RESUMEN

En este trabajo, se presenta un modelo macro fiscal que permite abordar los conceptos de superávit primario, dinámica y sostenibilidad de la deuda pública en dos contextos: economía cerrada y una economía abierta. El modelo se ha construido para simular los efectos de una variación del superávit primario sobre la dinámica de la deuda pública. La principal diferencia entre ambas versiones del modelo radica en que, en el caso de la

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economía abierta, el déficit fiscal también puede financiarse con deuda pública en moneda extranjera, lo cual introduce nuevos mecanismos de desestabilización de la trayectoria estable de la deuda pública. Los resultados muestran que, bajo una serie de condiciones, una reducción permanente del superávit primario tiene efectos significativos en la sostenibilidad de la deuda pública. Asimismo, se muestra que, en ambos casos, los ajustes fiscales son más grandes, conforme más tiempo transcurre desde el shock desestabilizador original.

Clasificación JEL: E62, H62, H63

Palabras clave: Deuda, dinámica, sostenibilidad, ajuste fiscal

INTRODUCTION

Fiscal policy, implemented by the government—in Peru’s case, the Ministry of Economy and Finance (MEF)—involves managing public spending, tax rates, and public debt to maintain output near potential without jeopardizing public debt sustainability.

In macroeconomic literature, particularly in the study of business cycles, public spending and tax rates are frequently discussed as tools to guide aggregate demand toward potential output. These analyses typically assume the sustainability of public debt, as it is a prerequisite for the discretionary use of public spending and tax rates.

A country’s public debt is considered sustainable if:

“[...] the government is able to meet all its current and future payment obligations without exceptional financial assistance or going into default.”

(Hakura, 2020, p. 60)

That is, public debt is sustainable when:

"[...] the government has a high likelihood of solvency, meaning it can meet its current and future financial obligations without resorting to unfeasible or undesirable policies. Additionally, the government must also have sufficient liquidity." (Hakura, 2020, p. 60).

For the purposes of this text, designed for undergraduate students, we adopt a more straightforward definition. Public debt will be considered sustainable if the government can maintain it at a constant level, as a percentage of GDP, that is deemed appropriate. Along with sustainability, this analysis explores how public debt evolves over time—a concept referred to as public debt dynamics.

1. MACRO-FISCAL MODEL OF A CLOSED ECONOMY

1.1 Public Debt Dynamics²

The fiscal deficit (DF_t), though an imperfect measure,³ is a key indicator of fiscal policy. In any given period t , it can be defined in two complementary ways. First, it represents the difference between public sector spending and revenue. Second, since the deficit must be financed, it is equal to the net borrowing flow, or the change in the public debt stock. This flow accounts for disbursements received net of repayments made during the same period. Borrowing can occur through bond issuance, bilateral agreements, or loans from multilateral organizations such as the International Monetary Fund (IMF), the World Bank (WB), or the Inter-American Development Bank (IDB).

Under the first definition, the fiscal deficit reflects the difference between total public spending and revenue. Spending includes primary (non-financial) spending in real terms (G_t) and financial spending, which consists of interest payments on public debt—the real interest rate multiplied by the previous period's debt stock (rB_{t-1}^g). Revenue comes from tax collection, equal to the tax rate multiplied by real GDP (tY_t). Abstracting from international considerations, the real fiscal deficit can be expressed as:

$$DF_t = G_t + rB_{t-1}^g - tY_t \quad (1)$$

Debt is measured as of the previous period for two reasons. First, this approach reflects the reality that interest payments result from past borrowing. Second, it facilitates the development of a dynamic model in discrete time to examine debt behavior over time. A model is considered dynamic when variables span multiple time periods.

Using the primary surplus ($SP_t = tY_t - G_t$), which excludes interest payments, this equation can be rewritten as:

$$DF_t = rB_{t-1}^g - SP_t \quad (2)$$

² For a general overview, see Blanchard (2023).

³ This result reflects not only fiscal policy but also the state of the economic cycle—whether expansionary or contractionary—as well as the interest rate cycle and international commodity price trends in an open economy.

The second definition of the fiscal deficit focuses on financing. Under this approach, the deficit is equal to the net borrowing flow, or the change in the public debt stock between periods t and $t - 1$:

$$B_t^g - B_{t-1}^g = DF_t \quad (3)$$

Combining these definitions yields:

$$B_t^g - B_{t-1}^g = DF_t = rB_{t-1}^g - SP_t \quad (4)$$

To facilitate international comparisons, it is useful to express these variables as percentages of GDP. Dividing both sides of equation (4) by real GDP (Y_t) and applying standard algebraic transformations leads to:

$$\frac{B_t^g}{Y_t} - \frac{B_{t-1}^g}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} = \frac{DF_t}{Y_t} = r \frac{B_{t-1}^g}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} - \frac{SP_t}{Y_t} \quad (5)$$

Expressing growth as:

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = g \quad (6)$$

or equivalently:

$$\frac{Y_t}{Y_{t-1}} = 1 + g \quad (7)$$

provides the basis for reformulating equation (5) as:

$$b_t^g - \frac{b_{t-1}^g}{1+g} = df_t = r \frac{b_{t-1}^g}{1+g} - sp_t \quad (8)$$

This formulation underpins the equation that describes the dynamics of public debt:

$$b_t^g = \frac{1+r}{1+g} b_{t-1}^g - sp_t \quad (9)$$

Equation (9) shows that public debt, as a percentage of GDP, is determined by the previous period's debt (adjusted for interest and growth) minus the primary surplus. This framework captures the dynamics of debt accumulation over time, assuming the absence of borrowing constraints—a point discussed further below.

Plotted as the relationship between current (b_t^g) and previous (b_{t-1}^g) debt levels, both expressed as percentages of GDP, equation (9) forms the bb line. The slope of this line varies under different conditions, which will be explored in subsequent sections.

1.2 Public Debt Sustainability

In discrete-time dynamic equations, a steady-state (long-run) equilibrium is reached when the dynamic endogenous variable stabilizes—that is, when it stops changing. The concept of public debt sustainability is closely tied to the idea of steady-state equilibrium.

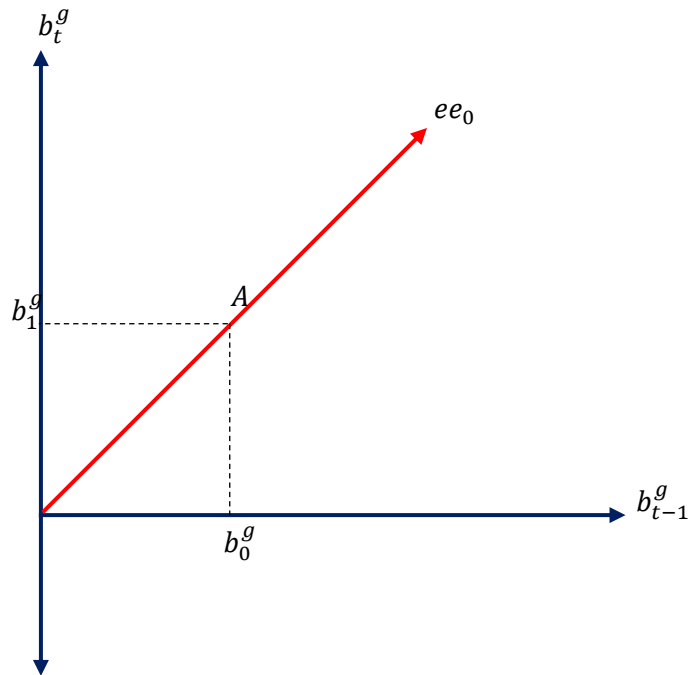
In equation (9), the dynamic endogenous variable is the public debt stock as a percentage of GDP. Achieving steady-state equilibrium means that the debt-to-GDP ratio no longer changes, implying that public debt in one period is equal to that of the previous period. This represents the most fundamental concept of public debt sustainability: the government's ability to stabilize public debt as a percentage of GDP. The condition for public debt sustainability, or steady-state equilibrium, is expressed in equation (10):

$$b_t^g = b_{t-1}^g \tag{10}$$

Graphically, this condition is represented by the ee line, which has a positive slope of one, in the (b_{t-1}^g, b_t^g) plane.

Figure 1

Public Debt Sustainability



What is required for an economy to achieve steady-state equilibrium—that is, to keep the public debt stock constant as a percentage of GDP? Combining equation (9), which describes debt dynamics, with equation (10), which defines the sustainability condition, yields the equation for the primary surplus needed to stabilize public debt:

$$sp_{ee} = \frac{r-g}{1+g} b_{t-1}^g \quad (11)$$

Equation (11) has significant implications for macroeconomic policy. Highly indebted governments—those with very high public debt-to-GDP ratios and/or those where the real interest rate significantly exceeds the GDP growth rate ($r - g > 0$)—must allocate substantial resources to debt interest payments. Consequently, they are required to maintain large primary surpluses, which can only be achieved through significant reductions in primary spending and/or major increases in revenue, both as percentages of GDP.

It is important to note that the assumption of a constant interest rate is highly concessive. In practice, as public debt increases, interest rates tend to rise as well.

To summarize, the framework includes the debt dynamics equation (9), the steady-state equilibrium condition (10), and the primary surplus required to stabilize public debt at equilibrium, expressed in equation (11):

$$b_t^g = \frac{1+r}{1+g} b_{t-1}^g - sp_t \quad (9)$$

$$b_t^g = b_{t-1}^g \quad (10)$$

$$sp_{ee} = \frac{r-g}{1+g} b_{t-1}^g \quad (11)$$

1.3 Public Debt Dynamics and Sustainability

If a government disregards or downplays the importance of public debt sustainability in fiscal policy—perhaps due to confidence in its ability to finance fiscal deficits without difficulty—it operates solely under equation (9). In contrast, a government concerned with sustainability also considers equation (10).

Fiscal balance, referred to here as intertemporal fiscal balance, results from combining equations (9) and (10).⁴ Solving these equations leads to three possible scenarios. To graph these cases, it is essential to first determine the initial steady-state equilibrium point and the slope of the *bb* line in each case. Using the primary surplus at steady-state equilibrium (equation 11) and the slope of the *bb* line (equation 9), we identify three cases:⁵

Case 1: $r = g$

$$sp_{ee} = \frac{r-g}{1+g} b_{ee}^g = 0 \quad (12.1)$$

Slope of *bb* = slope of *ee*.

Case 2: $r > g$

$$sp_{ee} = \frac{r-g}{1+g} b_{ee}^g > 0 \quad (12.2)$$

⁴ Given the values of public debt and the primary surplus, the fiscal deficit can be determined.

⁵ In the steady-state equilibrium, $b_t^g = b_{t-1}^g = b_{ee}^g$. From this, the expressions for the steady-state surplus in the different cases can be derived.

Slope of $bb >$ slope of ee .

Case 3: $r < g$

$$sp_{ee} = \frac{r-g}{1+g} b_{ee}^g < 0 \quad (12.3)$$

Slope of $bb <$ slope of ee .

Figures 2, 3, and 4 illustrate these cases.

Figure 2

Public Debt Dynamics and Sustainability – Case 1: $r = g$

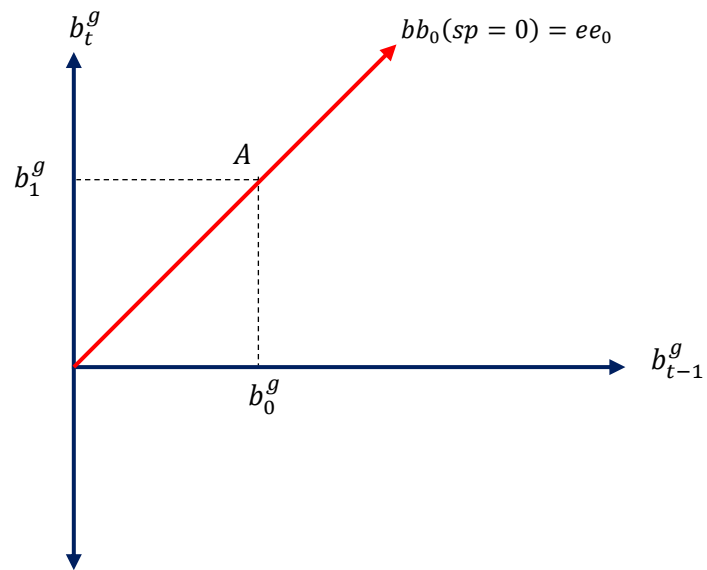


Figure 3

Public Debt Dynamics and Sustainability – Case 2: $r > g$

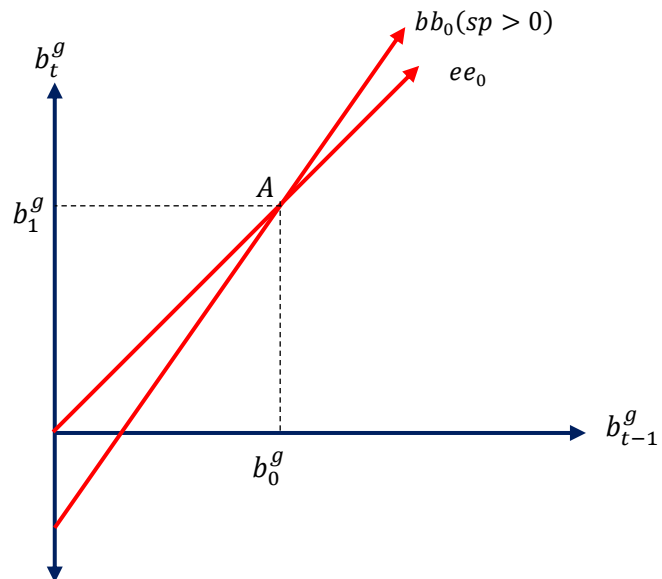
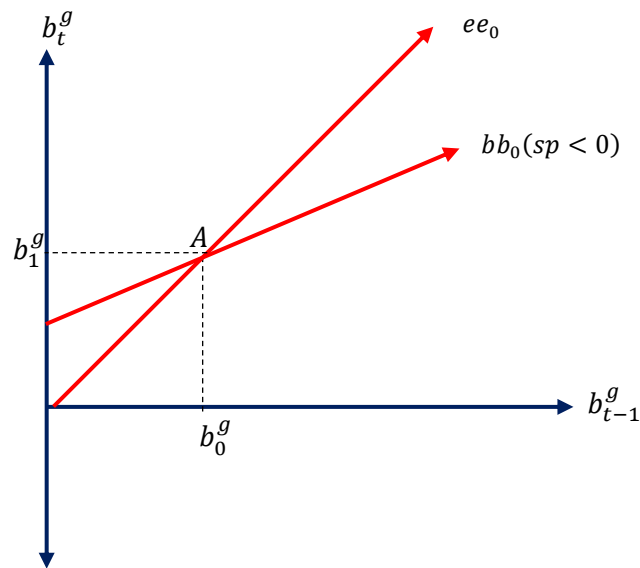


Figure 4

Public Debt Dynamics and Sustainability – Case 3: $r < g$



In these figures, the public debt dynamics line (equation 9, bb), the steady-state equilibrium line (equation 10, ee), and their slopes are compared. The bb line represents how public debt evolves over time, while the ee line indicates the condition where public debt in one period equals that of the previous period.

In Figure 2, the bb and ee lines overlap, as their slopes are identical; in Figure 3, the slope of the bb line exceeds that of the ee line; and in Figure 4, the slope of the bb line is less than that of the ee line.

At point A , intertemporal fiscal balance is achieved. At this point, public debt remains constant because interest payments on debt are fully supported by the existing primary surplus (sp_0). If this level of primary surplus is maintained over time, and no shocks occur to the interest rate or GDP growth rate, public debt sustainability is ensured. In such a case, the public debt-to-GDP ratio remains constant across periods.

1.4. Primary Surplus, Public Debt Dynamics, and Sustainability

This section examines the effects of a permanent increase in public spending as a share of GDP, which reduces the primary surplus below the level required to stabilize public

debt as a percentage of GDP ($sp_1 < sp_0$). The response of public debt dynamics depends on the specific case: $r = g$, $r > g$, or $r < g$.

Assuming an initial steady-state equilibrium, public debt is constant at its steady-state level, and the primary surplus is sufficient to stabilize public debt as a percentage of GDP. Public debt in period 1 equals that in period 0 ($b_1^g = b_0^g$) and the primary surplus is sp_0 .

This analysis considers three phases: the short-term impact, the transition toward a new equilibrium, and the long-term outcome (if a new steady-state equilibrium exists).

Case 1: $r = g$

When the real GDP growth rate equals the real interest rate, equation (9) implies:

$$\frac{1+r}{1+g} = 1$$

Short-Term Impact

Differentiating equation (9) to determine the short-term effects gives:

$$db_1^g = -dsp_1 > 0$$

This indicates that a reduction in the primary surplus during the first period increases public debt by the same amount.

Transition to Steady State

In subsequent periods, public debt increases by the same magnitude as in the first period. This occurs because the condition $\frac{1+r}{1+g} = 1$ implies that the debt increment remains constant across periods. In period 2, the increase in public debt from period 1 causes a further increase in debt by the same amount.⁶ This pattern repeats in period 3 and subsequent periods. Each increase in public debt equals the initial reduction in the primary surplus, and the debt-to-GDP ratio continues to rise indefinitely.

⁶ This is because, in the public debt dynamics equation, the public debt from the previous period is multiplied by a coefficient equal to 1.

Differentiating equation (9) yields:

$$db_2^g = -dsp_1 > 0$$

$$db_3^g = -dsp_1 > 0$$

Generalizing for any period n :

$$db_n^g = -dsp_1 > 0$$

Steady-State Equilibrium

To analyze the long-term behavior of public debt, equation (9) is expressed as:

$$b_n^g = b_0^g + \sum_{t=1}^n db_t^g \quad (13)$$

Since $\frac{1+r}{1+g} = 1$, the debt increment is constant ($db_n^g = -dsp_1, \forall n \geq 1$). Substituting into the equation gives:

$$b_n^g = b_0^g - \sum_{t=1}^n dsp_1 \quad (13.1)$$

Taking the limit as $n \rightarrow \infty$:

$$\lim_{n \rightarrow \infty} b_n^g = \infty_+ > 0$$

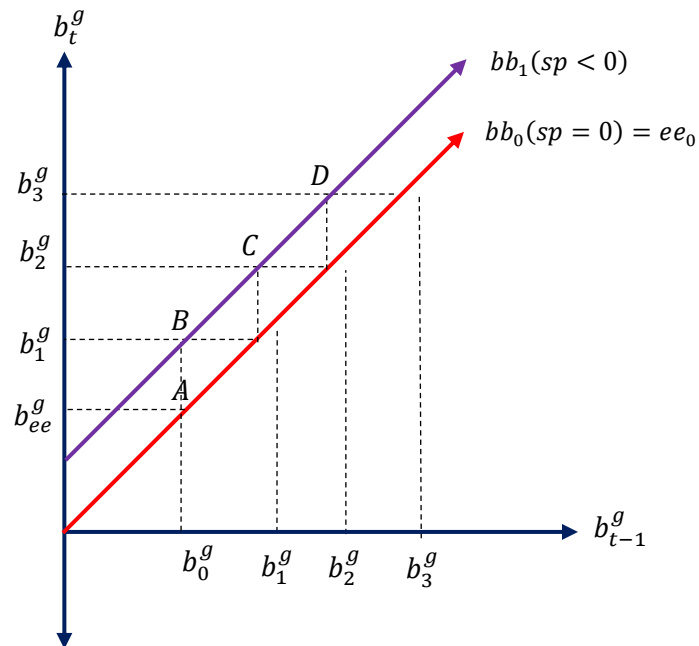
This confirms that a permanent reduction in the primary surplus⁷ leads to a divergent trajectory for public debt.

Figure 5 illustrates this case. Initially, the bb_0 and ee_0 lines overlap, representing the steady-state equilibrium at point A , where public debt is stable at b_{ee}^g . In period 1, when the primary surplus is reduced, the bb_0 line shifts upward to bb_1 , and public debt increases to point B . In subsequent periods, the economy moves to points C, D, \dots , with public debt increasing indefinitely. This dynamic system is linearly unstable, and public debt becomes unsustainable.

⁷ That is, the primary surplus as a percentage of GDP decreases from its initial level, sp_0 , to a lower level, sp_1 , and remains at that new level indefinitely.

Figure 5

Reduction in the Primary Surplus: Case $r = g$



Case 2: $r > g$

When the real interest rate exceeds the GDP growth rate, equation (9) implies:

$$\frac{1+r}{1+g} > 1$$

Short-Term Impact

Differentiating equation (9) to determine the short-term effects gives:

$$db_1^g = -dsp_1 > 0$$

This result indicates that public debt increases by the same magnitude as the reduction in the primary surplus.

Transition to Steady State

In period 2, public debt increases again, but by a multiple of the increment observed in period 1. This pattern arises because, under the condition $r > g$, the numerator in the

debt dynamics equation exceeds the denominator.⁸ In period 3, the debt rises again, magnified by the same factor applied to the increase from period 2. This compounding process continues in subsequent periods, with public debt growing at an accelerating rate.

Differentiating equation (9) for subsequent periods yields:

$$db_2^g = -\left(\frac{1+r}{1+g}\right) dsp_1 > 0$$

$$db_3^g = -\left(\frac{1+r}{1+g}\right)^2 dsp_1 > 0$$

Generalizing for any period n :

$$db_n^g = -\left(\frac{1+r}{1+g}\right)^{n-1} dsp_1 > 0$$

Steady-State Equilibrium

In this case, where $\frac{1+r}{1+g} > 1$, the change in public debt is given by:

$$db_n^g = -\left(\frac{1+r}{1+g}\right)^{n-1} dsp_1 > 0, \quad \forall n \geq 1$$

From this expression, the total public debt in period n is given by:

$$b_n^g = b_0^g - \sum_{t=1}^n \left(\frac{1+r}{1+g}\right)^{t-1} dsp_1 \tag{13.2}$$

Taking the limit as $n \rightarrow \infty$:

$$\lim_{n \rightarrow \infty} b_n^g = \infty_+ > 0$$

This confirms that a permanent reduction in the primary surplus leads to a divergent trajectory for public debt.

Figure 6 illustrates this case. At the initial steady-state equilibrium, the public debt dynamics line (bb_0) intersects with the steady-state equilibrium line (ee_0) at point A . The

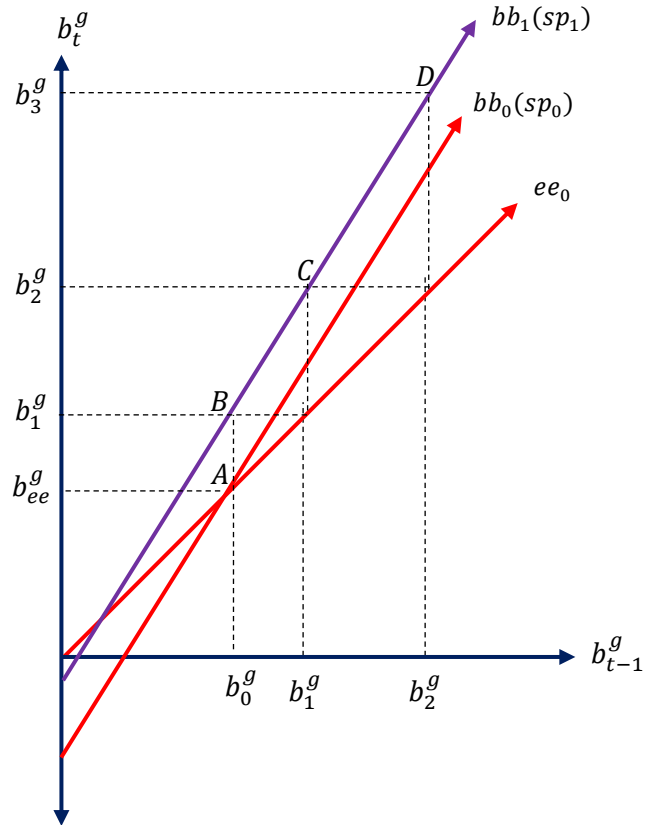
⁸ In equation (9), public debt from the previous period is multiplied by a factor greater than one.

slope of bb is steeper than that of ee , reflecting the greater weight of interest payments relative to GDP growth. Public debt at this initial equilibrium is b_{ee}^g .

In period 1, the reduction in the primary surplus shifts the public debt dynamics line upward, from, from bb_0 to bb_1 . Debt increases to point B , above the previous level. In subsequent periods, as the debt and fiscal deficit grow, the economy moves to points C , D , ..., with debt increasing indefinitely. Without changes to the primary surplus, the economy fails to reach a new steady-state equilibrium.

Figure 6

Reduction in the Primary Surplus: Case $r > g$



Case 3: $r < g$

When the GDP growth rate exceeds the real interest rate, equation (9) implies:

$$\left(\frac{1+r}{1+g}\right) < 1$$

Short-Term Impact

Differentiating equation (9) to determine the short-term effects gives:

$$db_1^g = -dsp_1 > 0$$

This result indicates that public debt increases by the same magnitude as the reduction in the primary surplus during the first period.

Transition to Steady State

In period 2, the increase in public debt from period 1 causes a further increase, but by a fraction of the initial rise.⁹ In period 3, the debt increases again, but only by a fraction of the rise in period 2. In subsequent periods, this periodic increase in public debt continues, with the increments becoming progressively smaller until the debt-to-GDP ratio stabilizes. At the new steady-state equilibrium, public debt is higher than in the initial equilibrium.

Differentiating equation (9) for subsequent periods yields:

$$db_2^g = -\left(\frac{1+r}{1+g}\right) dsp_1 > 0$$

$$db_3^g = -\left(\frac{1+r}{1+g}\right)^2 dsp_1 > 0$$

Generalizing for any period n :

$$db_n^g = -\left(\frac{1+r}{1+g}\right)^{n-1} dsp_1 > 0$$

Steady-State Equilibrium

For this case, where $\left(\frac{1+r}{1+g}\right) < 1$, the change in public debt is given by:

$$db_n^g = -\left(\frac{1+r}{1+g}\right)^{n-1} dsp_1 > 0, \quad \forall n \geq 1$$

From this expression, the total public debt in period n is given by:

$$b_n^g = b_0^g - \sum_{t=1}^n \left(\frac{1+r}{1+g}\right)^{t-1} dsp_1 \quad (13.3)$$

Taking the limit as $n \rightarrow \infty$:

$$\begin{aligned} \lim_{n \rightarrow \infty} b_n^g &= b_0^g - dsp_1 \lim_{n \rightarrow \infty} \sum_{t=1}^n \left(\frac{1+r}{1+g}\right)^{t-1} \\ &= b_0^g - dsp_1 \left(\frac{1+g}{g-r}\right) > 0 \end{aligned}$$

⁹ When $g > r$, $\frac{1+r}{1+g}$ is a fraction.

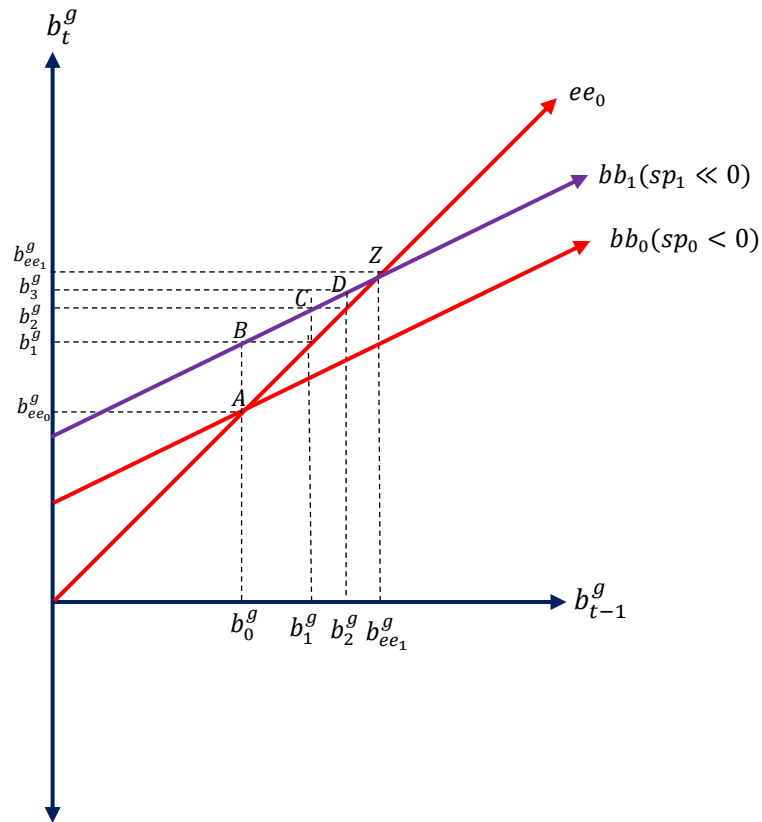
This confirms that, although a permanent reduction in the primary surplus leads to an increase in public debt, it eventually stabilizes at a new steady-state level.

Figure 7 illustrates this case. At the initial steady-state equilibrium, the public debt dynamics line (bb_0) intersects the steady-state equilibrium line (ee_0) at point A , with the slope of bb being less steep than that of ee . Public debt at this equilibrium is b_{ee}^g .

In period 1, the reduction in the primary surplus shifts the public debt dynamics line upward, from bb_0 to bb_1 , causing public debt to rise above its previous level, reaching point B . In subsequent periods, the rate of debt growth slows, and the economy transitions through points C, D, \dots , until public debt stabilizes at point Z , the new steady-state equilibrium, characterized by a higher debt level.

Figure 7

Reduction in the Primary Surplus: Case $r < g$



In Case 3, where the real GDP growth rate exceeds the real interest rate, a permanent reduction in the primary surplus leads to a convergent trajectory for public debt. Public debt stabilizes at a new steady-state equilibrium, which is higher than the initial equilibrium. This outcome highlights the stabilizing influence of GDP growth exceeding the real interest rate, even under conditions of fiscal expansion.

If real-world economies behaved as described in Cases 1 and 2, we would observe widespread explosive debt trajectories among governments. However, this is not the case. A continuous growth of public debt over time, as outlined in these cases, presupposes economies with unlimited borrowing capacity—a condition that is unrealistic. In reality, all countries face borrowing constraints, typically expressed as a maximum public debt-to-GDP ratio. Developed economies, benefiting from stronger reputations, generally have higher debt limits compared to emerging or developing nations.

A key observation is that countries neglecting sustainability by maintaining a primary surplus below the level needed to stabilize debt often enter a phase of annual debt accumulation. This trend is eventually interrupted when access to credit is curtailed, both in financial markets—preventing the issuance of public debt—and through multilateral or bilateral lenders, such as the IMF, WB, or IDB.¹⁰

When these borrowing limits are reached, and fiscal deficits can no longer be financed, governments face two choices:

1. Immediate Fiscal Adjustment:

Governments without international support—either because assistance is unavailable or, more commonly, because they choose not to seek it for ideological reasons—must undertake abrupt fiscal adjustments. This requires immediate expenditure cuts and tax increases to raise the primary surplus. Public debt is frozen at its current level, the fiscal deficit is eliminated, and the primary surplus stabilizes debt.

2. Gradual Adjustment with IMF Support:

Governments anticipating the consequences of reaching their borrowing limit may turn to the IMF preemptively. The IMF facilitates a more gradual fiscal adjustment, allowing deficits to narrow progressively rather than being eliminated at once. Expenditures are reduced, and revenues are increased incrementally, slowing the growth of public debt until it stabilizes at its upper limit. While this approach still entails social and political costs, these are often less severe than those of an abrupt adjustment. No fiscal adjustment is cost-free.

Hendrik (2022) highlights that IMF programs often extend beyond direct financial assistance, providing a “catalytic effect.” Agreements with the IMF frequently act as a signal to other multilateral institutions, such as the WB or IDB, of a country’s commitment to meeting its debt obligations.

¹⁰ We exclude the option of central bank financing of the government through primary issuance. In such cases, the debt explosion could be accompanied by hyperinflation.

1.5 The Stabilizing Shock

When a fiscal deficit can no longer be financed, governments face no alternative but to adjust public spending and revenue. This section examines a scenario in which the government implements an abrupt stabilization of public debt—a shock treatment. The analysis focuses on Case 2, where the real interest rate exceeds the real GDP growth rate ($\frac{1+r}{1+g} > 1$), a particularly relevant scenario for economies with low growth rates and high interest rates.

The starting point is an initial steady-state equilibrium where the primary surplus is sufficient to maintain public debt stability. In period 1, a permanent reduction in the primary surplus occurs, falling below the level required to stabilize debt. This is assumed to result from a permanent increase in public spending as a percentage of GDP. The immediate consequence is an increase in public debt during that period. This rise, driven by the dynamics described in equation (9), leads to further increases in public debt in period 2, at a faster rate than in period 1. In period 3, debt rises again, growing even more rapidly, until it reaches the maximum limit imposed by financial markets and international institutions.¹¹

At this point, public debt cannot increase further, and the fiscal deficit must be eliminated. Achieving a zero deficit requires a sharp increase in the primary surplus through spending cuts and revenue increases, raising it to the level necessary to freeze debt at its upper limit. The new primary surplus must cover total interest payments on the debt in period 3. In the absence of external shocks, this would represent a new steady-state equilibrium. If the economy can sustain the required primary surplus each year and avoid adverse changes to interest rates or GDP growth, the debt level in period 3 can be considered sustainable.

However, the primary challenge with this “market-driven” adjustment is the size of the primary surplus needed to stabilize public debt at its new, elevated level (rb_3^g).¹² The

¹¹ The government may also choose not to seek IMF support for ideological reasons or other considerations.

¹² Stabilizing public debt at its original level, b_0^g , would require a significantly greater fiscal effort. During the adjustment period, the primary surplus would need to be substantially higher than rb_3^g , an unrealistic objective. With such a surplus, the model’s dynamics would allow public debt to gradually decrease over time, and upon

higher debt volume can result in a significantly larger primary surplus requirement, with severe social and political implications.

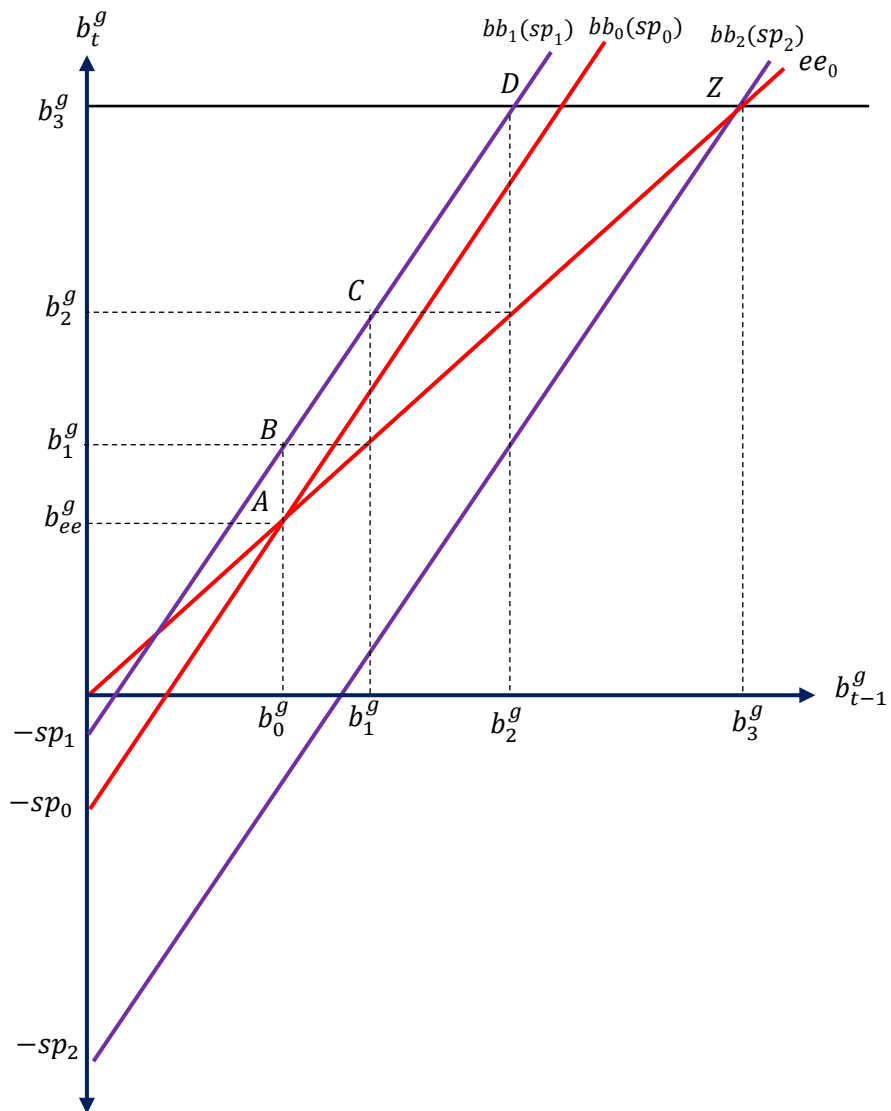
Figure 8 illustrates this situation. In period 1, a permanent reduction in the primary surplus, below the level required to stabilize public debt, shifts the debt dynamics line upward from bb_0 to bb_1 , increasing debt from b_0^g to b_1^g . The increase in debt during period 1 leads to further growth in period 2, reaching b_2^g , and again in period 3, reaching b_3^g , which we assume represents the debt limit beyond which debt cannot increase.

To stabilize debt at b_3^g , the primary surplus must increase from sp_0 to sp_3 , causing the debt dynamics line to shift downward from bb_1 to bb_2 . The intercept of this new line reflects the new primary surplus, which is significantly higher than the initial level. The new steady-state equilibrium is reached at point Z , where the debt dynamics line (bb_2) intersects the sustainable debt line (ee_0).

reaching its initial level, the government could maintain the exact primary surplus needed to stabilize debt at that level. This analysis focuses on the more realistic and feasible scenario of stabilizing public debt at a new, higher level.

Figure 8

The Stabilizing Shock in a Closed Economy



The analysis assumes that the country has lost access to credit from financial markets and international institutions. Additionally, the possibility of the central bank acting as the government's lender of last resort is excluded. Under these conditions, the country would need to implement a drastic fiscal adjustment, potentially exacerbating challenges beyond the economic sphere. The scale of spending cuts and tax increases required could lead to significant social unrest.

2. MACRO-FISCAL MODEL FOR AN OPEN ECONOMY¹³

2.1. Public Debt Dynamics

In an open economy, the macro-fiscal model introduces two key changes. On the revenue side, the government may receive income from taxes on export earnings, particularly from commodities. On the expenditure side, access to external borrowing introduces foreign-denominated debt and associated interest payments. Therefore, equation (4) in Section 1, which combines the two definitions of the fiscal deficit—as the change in public debt and as the difference between government expenditures and revenues—becomes:

$$B_t^g - B_{t-1}^g + e_t(B_t^{g*} - B_{t-1}^{g*}) = DF_t = rB_{t-1}^g + e_t r^* B_{t-1}^{g*} - SP_t \quad (15)$$

Where:

e_t is the real exchange rate in period t

B_t^{g*} is public debt in foreign currency in period t

B_{t-1}^{g*} is public debt in foreign currency in period $t - 1$

r^* is the international interest rate on foreign-denominated debt

SP_t is the primary surplus

The primary surplus (SP_t) now also includes revenues from commodity exports, which depend on international prices (P^*) and export volumes (X_0).¹⁴

$$SP_t = tY_t + te_t P^* X_0 - G_t \quad (16)$$

Applying the algebraic transformations outlined in Section 1 and expressing the variables in equation (15) as percentages of GDP results in:

$$\frac{B_t^g}{Y_t} - \frac{B_{t-1}^g}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} + \frac{e_t B_t^{g*}}{Y_t} - \frac{e_t B_{t-1}^{g*}}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} \frac{e_{t-1}}{e_{t-1}} = \frac{DF_t}{Y_t} = r \frac{B_{t-1}^g}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} + \frac{e_t r^* B_{t-1}^{g*}}{Y_t} \frac{Y_{t-1}}{Y_{t-1}} \frac{e_{t-1}}{e_{t-1}} - \frac{SP_t}{Y_t} \quad (17)$$

¹³ Jiménez (2008).

¹⁴ Income tax on exporters.

We denote x_t as the share of X_t in GDP ($x_t = \frac{X_t}{Y_t}$). Additionally, the growth rate of the real exchange rate is defined as:

$$\frac{e_t - e_{t-1}}{e_{t-1}} = g_e \quad (18)$$

or equivalently:

$$\frac{e_t}{e_{t-1}} = 1 + g_e \quad (19)$$

Building on equation (17), and converting the variables into percentages of GDP using equations (7) and (19), we arrive at equation (20), which summarizes the two ways of expressing the fiscal deficit, now as percentages of GDP:

$$b_t^g - \frac{1}{1+g} b_{t-1}^g + b_t^{g*} - \frac{(1+g_e)}{(1+g)} b_{t-1}^{g*} = df_t = r \frac{1}{1+g} b_{t-1}^g + r^* \frac{(1+g_e)}{(1+g)} b_{t-1}^{g*} - sp_t \quad (20)$$

From equation (20), we derive the equation describing the behavior of public debt over time—the public debt dynamics equation, which is central to our analysis:

$$b_t^g + b_t^{g*} = \frac{1+r}{1+g} b_{t-1}^g + \frac{(1+r^*)(1+g_e)}{1+g} b_{t-1}^{g*} - sp_t \quad (21)$$

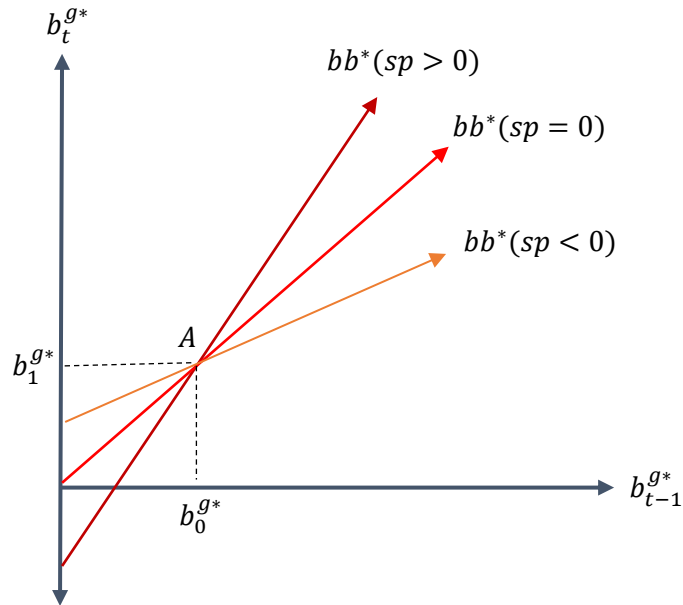
To graphically represent the relationship between current and previous-period domestic public debt, the equation can be rearranged as follows:

$$b_t^{g*} = \frac{(1+r^*)(1+g_e)}{1+g} b_{t-1}^{g*} - b_t^g + \frac{1+r}{1+g} b_{t-1}^g - sp_t \quad (22)$$

Equation (22), which is plotted in terms of current foreign-denominated public debt (b_t^{g*}) and previous-period foreign-denominated public debt (b_{t-1}^{g*}), defines the bb^* line. This line has a positive slope, but for different slope values, three cases arise, which will be discussed later. Figure 9 depicts this relationship.

Figure 9

Public Debt Dynamics in an Open Economy



The open economy model is represented by equation (21), which characterizes the dynamics of public debt. This model incorporates two endogenous variables: domestic public debt and external public debt, both expressed as percentages of GDP.

2.2 Public Debt Sustainability

As in the closed economy model, steady-state or long-term equilibrium is achieved when the dynamic endogenous variables stabilize and no longer change. In equation (21), these variables—domestic public debt and external public debt, both expressed as percentages of GDP—reach equilibrium when their levels remain constant over time, meaning the debt in one period equals the debt in the previous period. This condition reflects the concept of public debt sustainability used in this analysis: the government’s ability to stabilize public debt as a percentage of GDP. This relationship is expressed in equation (23):

$$b_t^g + b_t^{g*} = b_{t-1}^g + b_{t-1}^{g*} \tag{23}$$

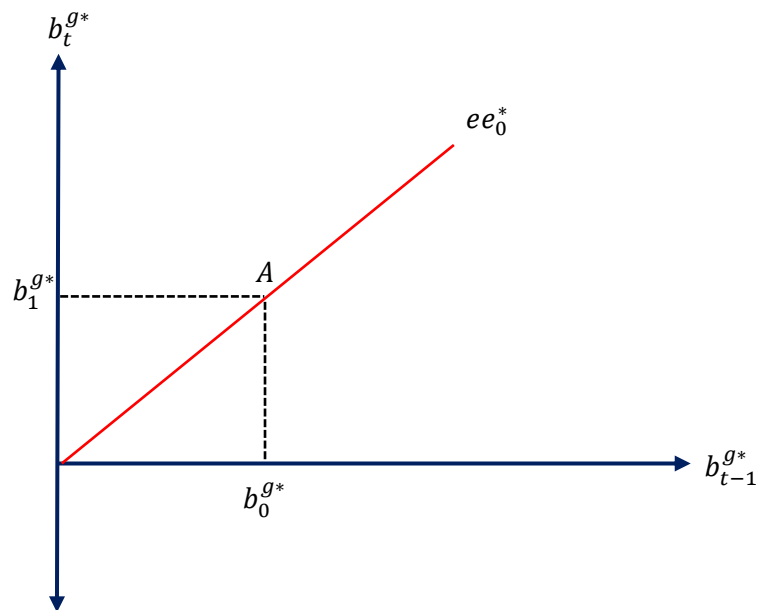
For consistency with the closed economy model, equation (23) can be rearranged as:

$$b_t^g = (b_{t-1}^{g*} - b_t^{g*}) + b_{t-1}^g \quad (24)$$

In this form, the ee line, shown in Figure 10, represents the condition for public debt sustainability in the (b_t^{g*}, b_{t-1}^{g*}) plane. The line has a positive slope of one and passes through the origin, reflecting the assumption of an initial steady-state equilibrium where $b_{t-1}^{g*} - b_t^{g*} = 0$.

Figure 10

Public Debt Sustainability in an Open Economy



The primary surplus required to stabilize both domestic and external public debt as percentages of GDP can be determined by combining the debt dynamics equation (21) with the sustainability condition in equation (24). This yields the steady-state fiscal surplus:

$$sp_{ee} = \frac{r-g}{1+g} b_{t-1}^g + \frac{r^*(1+g_e)+(g_e-g)}{1+g} b_{t-1}^{g*} \quad (25)$$

Similar to equation (11) in the closed economy model, equation (25) has significant implications for macroeconomic policy. Governments with high debt-to-GDP ratios, real interest rates far exceeding GDP growth ($r - g > 0$), or exchange rate depreciation

surpassing GDP growth,¹⁵ face substantial interest obligations. Such governments must maintain large primary surpluses, achievable only through significant reductions in primary expenditures or substantial increases in revenue, both as percentages of GDP.

To summarize, the open economy model¹⁶ is characterized by:

- The debt dynamics equation (21)
- The condition for steady-state equilibrium or debt sustainability (equation 24)
- The steady-state primary surplus required to stabilize public debt (equation 25)

$$b_t^g + b_t^{g*} = \frac{1+r}{1+g} b_{t-1}^g + \frac{(1+r^*)(1+g_e)}{1+g} b_{t-1}^{g*} - sp_t \quad (21)$$

$$b_t^g = (b_{t-1}^{g*} - b_t^{g*}) + b_{t-1}^g \quad (24)$$

$$sp_{ee} = \frac{r-g}{1+g} b_{t-1}^g + \frac{r^*(1+g_e)-(g-g_e)}{1+g} b_{t-1}^{g*} \quad (25)$$

When external borrowing is absent ($b_{t-1}^{g*}, b_t^{g*} = 0$), the equations governing public debt dynamics in an open economy become identical to those of a closed economy.

2.3 Debt Dynamics and Sustainability

Building on this framework, three cases can be identified, similar to those discussed in the closed economy model.¹⁷ However, in the open economy context, in addition to the relationship between the domestic interest rate (r) and the GDP growth rate (g), it is necessary to account for the relationship between the international interest rate, (r^*), the real depreciation rate of the domestic currency (g_e), and the GDP growth rate (g).

Case 1:

(i) $r = g, 0$

¹⁵ The transmission mechanism operates as follows: when a country's currency depreciates faster than its GDP grows, its domestic-currency revenues (which depend on GDP growth, Y) are insufficient to offset exchange rate risk. This necessitates a higher primary surplus. This is the 'balance sheet effect' applied to public debt.

¹⁶ This model assumes imperfect capital mobility, following Mendoza (2019) and Mendoza and Vilca (2024). Consequently, domestic and foreign interest rates are determined independently, and the uncovered interest rate parity equation is not applied.

¹⁷ In an open economy, additional cases may arise.

$$(ii) r^* + g_e = g^{18}$$

$$sp_{ee} = 0$$

$$bb^* \text{ slope} = ee^* \text{ slope}$$

Case 2:

$$(i) r > g, 0$$

$$(ii) r^* + g_e > g$$

$$sp_{ee} > 0$$

$$bb^* \text{ slope} > ee^* \text{ slope}$$

Case 3:

$$(i) r < g, 0$$

$$(ii) r^* + g_e < g$$

$$sp_{ee} < 0$$

$$bb^* \text{ slope} < ee^* \text{ slope}$$

Figures 11, 12, and 13 illustrate Cases 1, 2, and 3, respectively. The choice of axes depends on the type of debt used to finance the fiscal deficit. If external public debt is used, the axes will be $b_{t-1}^{g^*}$ and $b_t^{g^*}$, allowing an analysis of the trajectory of external public debt in response to deviations from the initial primary surplus equilibrium. In this scenario, the relationship between r^* , g_e and g is central.

Conversely, if the fiscal deficit is financed through domestic public debt, the axes will be the same as in the closed economy model (b_{t-1}^g, b_t^g). In this case, the relevant relationship is between r and g .

¹⁸ To ensure that the steady-state primary surplus equals zero, we consider the relationship $(1 + g_e)(1 + r^*) = (1 + g)$. Since the values of g_e , r^* , and g are typically small, a linear approximation can be applied. Taking natural logarithms and using Taylor's approximation around $x = 0$, where $\ln(1 + x) \approx x$, the original equation, $(1 + r^*)(1 + g_e) = (1 + g)$, simplifies to the following linearized expression: $r^* + g_e = g$. Hereafter, we will adopt this linearized version.

Figure 11

Public Debt Dynamics and Sustainability in an Open Economy

$$r = g \text{ or } r^* + g_e = g$$

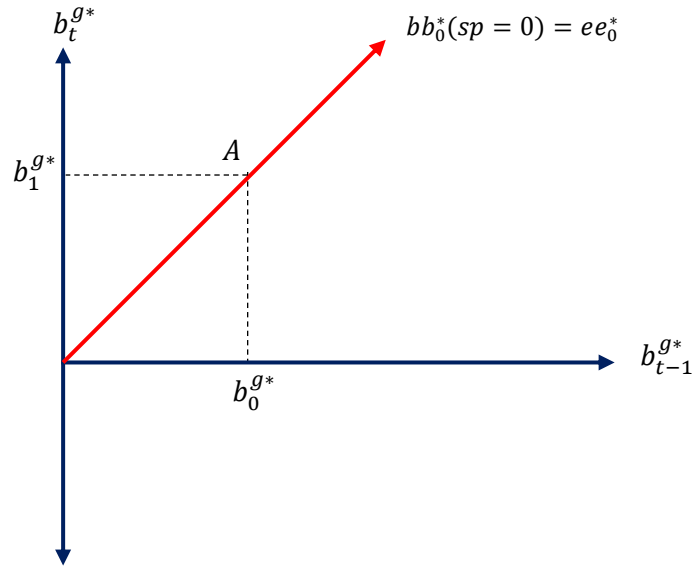


Figure 12

Public Debt Dynamics and Sustainability in an Open Economy

$$r > g \text{ or } r^* + g_e > g$$

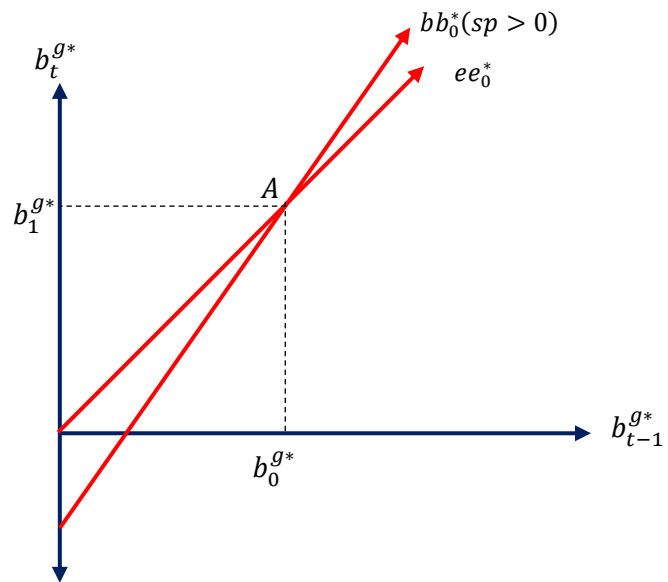
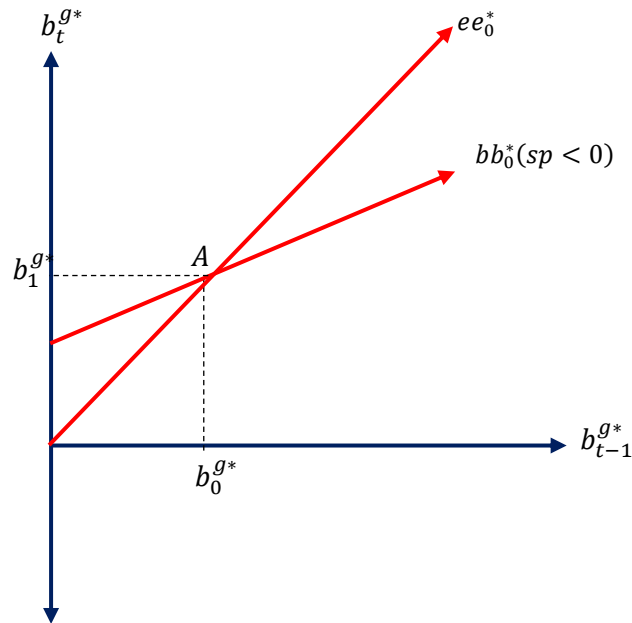


Figure 13

Public Debt Dynamics and Sustainability in an Open Economy

$$r < g \text{ or } r^* + g_e < g$$



In these figures, the public debt dynamics line (bb^*), defined by equation (22), and the steady-state equilibrium line (ee^*), defined by equation (24), are combined. The bb^* line, with a positive slope, illustrates the evolution of public debt over time, while the ee^* line, with a slope of one, represents the condition for public debt sustainability, where public debt remains unchanged between periods.

The main difference between these figures and those in the closed economy model lies in the intercepts of the public debt dynamics line. These intercepts now include components of external public debt, both from the dynamics equation and the steady-state analysis. However, the slopes of the lines remain unchanged, preserving the fundamental similarity to the closed economy model.

Finally, at point A , intertemporal fiscal balance is achieved. At this point, public debt remains constant because interest payments on both domestic and external public debt are fully covered by the existing primary surplus (sp_0). If this level of primary surplus is sustained over time, public debt sustainability is ensured, as public debt remains constant as a percentage of GDP across periods.

2.4. Primary Surplus, Debt Dynamics, and Sustainability in an Open Economy

In an open economy at intertemporal fiscal equilibrium, a shock that reduces the primary surplus below the level required to stabilize public debt as a percentage of GDP ($sp_1 < sp_0$) can have significant effects. This analysis considers a permanent reduction in the primary surplus resulting from a decline in export prices ($dP^* < 0$), under the assumption that the fiscal deficit is financed entirely through external public debt as a percentage of GDP (b_t^{g*}).¹⁹ The outcomes depend on the relationships among r , g , r^* , and g_e . Since external public debt is the model's endogenous variable, the relationship between r and g is excluded from this analysis.

Case 1: $r^* + g_e = g$

When the sum of the international interest rate and the real depreciation rate equals the GDP growth rate, equation (21) simplifies to $\frac{(1+r^*)(1+g_e)}{(1+g)} = 1$. In this scenario, a reduction in the primary surplus caused by lower export prices increases external public debt in period 1 by the same magnitude as the reduction in the surplus.

This analysis examines the effects across three periods: short-term (period 1), transition to steady state (subsequent periods), and steady-state equilibrium. The reduction in the primary surplus is defined as $dsp_1 < 0$, $dsp_n = 0$, $\forall n > 1$, under the condition $r^* + g_e \cong g$.

Short-Term Impact

From equation (21):

$$b_t^g + b_t^{g*} = \frac{(1+r^*)(1+g_e)}{1+g} b_{t-1}^{g*} - sp_t + \frac{1+r}{1+g} b_{t-1}^g$$

Differentiating:

$$db_1^{g*} = -dsp_1 > 0$$

¹⁹ Alternatively, the deficit could be financed through the stock of domestic public debt as a percentage of GDP (b_t^g) or even through a combination of both. However, financing with external debt requires an adequate supply of dollars in the economy.

Thus, the reduction in the primary surplus immediately increases external public debt in period 1.

Transition to Steady State

Differentiating equation (21) to analyze changes in subsequent periods yields the following results:

$$db_2^{g*} = -dsp_1 > 0$$

$$db_3^{g*} = -dsp_1 > 0$$

Generalizing for any period n :

$$db_n^{g*} = -dsp_1 > 0$$

Steady-State Equilibrium

To determine the long-term outcome, we simulate the case where $n \rightarrow \infty$. Using a similar approach to equation (13) in the closed economy model, we derive the following expression:

$$b_n^{g*} = b_0^{g*} + \sum_{t=1}^n db_t^{g*}$$

Given that the change in external public debt is constant across all periods, this simplifies to:

$$b_n^{g*} = b_0^{g*} - \sum_{t=1}^n dsp_1$$

Taking the limit as $n \rightarrow \infty$:

$$\lim_{n \rightarrow \infty} b_n^{g*} = \infty_+ > 0$$

Thus, as in the closed economy model, a permanent reduction in sp_t generates an explosive trajectory for external public debt, increasing indefinitely without reaching a steady-state equilibrium. It is also worth noting that if the lower primary surplus were financed through domestic public debt or a combination of domestic and external public debt, the same divergent trajectory would be observed, precluding a new steady-state equilibrium.

Case 2: $r^* + g_e > g$

When the sum of the international interest rate and the real depreciation rate exceeds the GDP growth rate, equation (21) implies:

$$\frac{(1+r^*)(1+g_e)}{(1+g)} > 1$$

As in Case 1, a reduction in the primary surplus in period 1—resulting from a decline in export prices—causes an increase in external public debt during that period, equal to the reduction in the surplus.

Short-Term Impact

Differentiating equation (21) to assess the short-term to assess the short-term impact yields the following result:

$$db_1^{g^*} = -dsp_1 > 0$$

This indicates an immediate increase in external public debt as a result of the lower primary surplus.

Transition to Steady State

In period 2, external public debt increases further, rising by a multiple of the previous period's increment. This pattern continues in subsequent periods, with debt growth accelerating because the numerator in the debt dynamics equation exceeds the denominator under the condition $r^* + g_e > g$. Differentiating equation (21) for subsequent periods yields:

$$db_2^{g^*} = -\left(\frac{(1+r^*)(1+g_e)}{(1+g)}\right) dsp_1 > 0$$

$$db_3^{g^*} = -\left(\frac{(1+r^*)(1+g_e)}{(1+g)}\right)^2 dsp_1 > 0$$

Generalizing for any period n :

$$db_n^{g^*} = -\left(\frac{(1+r^*)(1+g_e)}{(1+g)}\right)^{n-1} dsp_1 > 0$$

Because $\frac{(1+r^*)(1+g_e)}{(1+g)} > 1$, the increments in external public debt grow progressively larger, preventing convergence to a steady-state equilibrium.

Steady-State Equilibrium

In this case, where $\frac{(1+r^*)(1+g_e)}{(1+g)} > 1$, the change in external public debt is given by:

$$db_n^g = -\left(\frac{(1+r^*)(1+g_e)}{(1+g)}\right)^{n-1} dsp_1 > 0, \quad \forall n \geq 1$$

From this, we derive the following equation:

$$b_n^{g*} = b_0^{g*} - \sum_{t=1}^n \left(\frac{(1+r^*)(1+g_e)}{(1+g)}\right)^{t-1} dsp_1$$

Since $\frac{(1+r^*)(1+g_e)}{(1+g)} > 1$ when $r^* + g_e > g$, taking the limit as $n \rightarrow \infty$ yields:

$$\lim_{n \rightarrow \infty} b_n^{g*} = \infty_+ > 0$$

This result confirms that a permanent reduction in the primary surplus (sp_t) results in a divergent trajectory for external public debt (b_t^{g*}).

Case 3: $r^* + g_e < g$

In an open economy, the desirable scenario for fiscal policy arises when the sum of the international interest rate and the depreciation rate is lower than the real GDP growth rate. From equation (21), this condition is expressed as:

$$\frac{(1+r^*)(1+g_e)}{(1+g)} < 1$$

In this scenario, a reduction in the primary surplus, caused by a decline in export prices, leads to an increase in external public debt during the same period, equal to the magnitude of the surplus reduction.

Short-Term Impact

Differentiating equation (21) to assess the short-term impact yields:

$$db_1^{g*} = -dsp_1 > 0$$

This result indicates that external public debt rises in direct proportion to the reduction in the primary surplus during the first period.

Transition to Steady State

In period 2, external public debt increases again but by only a fraction of the increment observed in period 1. This pattern arises because, under the condition $r^* + g_e < g$, the numerator in the debt dynamics equation is smaller than the denominator.

Differentiating equation (21) for subsequent periods yields:

$$db_2^{g*} = - \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right) dsp_1 > 0$$

$$db_3^{g*} = - \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right)^2 dsp_1 > 0$$

Generalizing for any period n :

$$db_n^{g*} = - \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right)^{n-1} dsp_1 > 0$$

Steady-State Equilibrium

For this case, where $\frac{(1+r^*)(1+g_e)}{(1+g)} < 1$, the change in external public debt is given by:

$$db_n^g = - \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right)^{n-1} dsp_1 > 0, \quad \forall n \geq 1$$

From this, the total external public debt in period n can be expressed as:

$$b_n^{g*} = b_0^{g*} - \sum_{t=1}^n \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right)^{t-1} dsp_1$$

Since $\frac{(1+r^*)(1+g_e)}{(1+g)} < 1$ when $r^* + g_e < g$, taking the limit as $n \rightarrow \infty$ yields:

$$\begin{aligned} \lim_{n \rightarrow \infty} b_n^{g*} &= b_0^{g*} - dsp_1 \lim_{n \rightarrow \infty} \sum_{t=1}^n \left(\frac{(1+r^*)(1+g_e)}{(1+g)} \right)^{t-1} \\ &= b_0^g - dsp_1 \left(\frac{1+g}{(1+g) - (1+r^*)(1+g_e)} \right) > 0 \end{aligned}$$

This result confirms that, although a permanent reduction in the primary surplus (sp_t) leads to an increase in external public debt, it eventually stabilizes at a new steady-state level.

2.5 The Stabilizing Shock

This section examines the possibility of halting an explosive trajectory of external public debt, similar to the analysis conducted for a closed economy. In this context, adjustments must occur through either expenditure reductions or revenue increases.

The analysis begins with an initial steady-state equilibrium in which the primary surplus is sufficient to maintain external public debt at a stable level. The focus is on Case 2, where the international interest rate, combined with the real depreciation rate of the domestic currency, exceeds the real GDP growth rate, $\frac{(1+r^*)(1+g_e)}{1+g} > 1$. In this scenario, as previously demonstrated, the trajectory of public debt does not naturally converge to a new equilibrium but instead grows at an increasingly rapid pace.

During period 1, a permanent reduction in the primary surplus occurs, driven by a decline in export prices, which reduces government revenues as a percentage of GDP. This leads to an immediate increase in external public debt during the same period. According to the dynamics outlined in equation (21), the debt rises further in period 2 at a greater rate than in period 1. In period 3, external public debt grows again, surpassing the prior increase and reaching the maximum level allowed by markets and international organizations.

Consequently, debt can no longer grow, making it impossible to continue financing fiscal deficits as in previous periods. At this point, the fiscal deficit must be eliminated entirely. To achieve this, the primary surplus must increase to a level sufficient to cover interest payments on the external public debt at its new upper limit in period 3. Absent any adverse external shocks, this marks a new steady-state equilibrium.

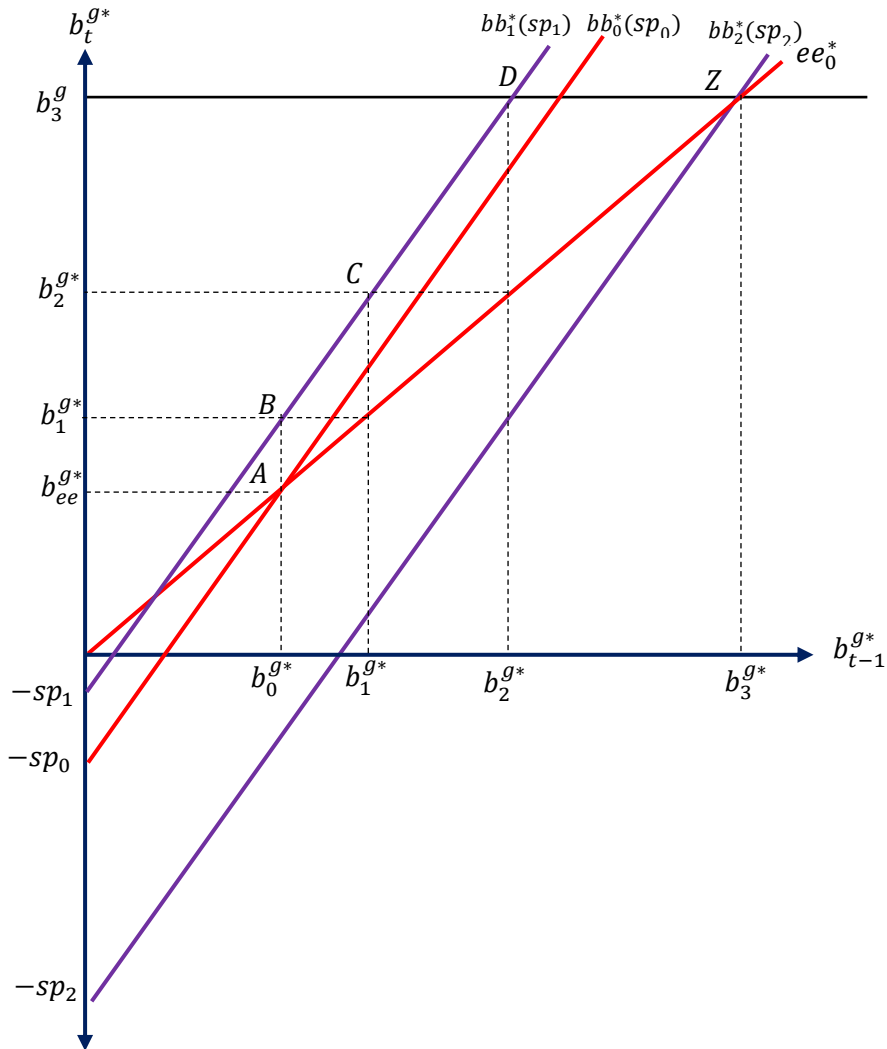
If the economy can sustain the described primary surplus annually and no unfavorable shocks occur to the international interest rate, the domestic currency's depreciation rate, or the real GDP growth rate, the external debt level in period 3 becomes sustainable.

However, the primary surplus required to stabilize external public debt at this higher level, rb_3^{g*} , is significant due to the elevated debt volume. This implies substantial social and political costs. This dynamic is depicted in Figure 14. In period 1, a permanent reduction in the primary surplus below the level required to stabilize public debt shifts the debt dynamics line bb_0^* to bb_1^* , raising external debt from b_0^{g*} to b_1^{g*} . The increased debt in period 1 drives further growth in period 2 to b_2^{g*} , which, in turn, leads to a further increase in period 3 to b_3^{g*} , the debt ceiling. As noted earlier, debt cannot grow beyond this level.

To stabilize debt at b_3^{g*} , the primary surplus must increase from sp_0 to sp_3 , shifting the debt dynamics line from bb_1^* to bb_2^* . The new steady-state equilibrium is represented by point Z , where the debt dynamics line, bb_2^* , intersects the public debt sustainability line, ee_0^* .

Figure 14

The Stabilizing Shock in an Open Economy



3. CONCLUSIONS AND IMPLICATIONS

This paper presents a model in two versions—closed economy and open economy—to analyze the dynamics and sustainability of public debt. The model was used to examine how an economy responds when public debt reaches a critical threshold, leading to the closure of all credit markets. In such a scenario, where the deficit must be eliminated, the only option is to implement a fiscal adjustment that stabilizes public debt as a percentage of GDP.

In the closed economy version, public debt sustainability depends primarily on the relationship between the real interest rate (r) and the GDP growth rate (g). In the open economy version, the fiscal deficit can be financed through domestic public debt or external public debt. If the deficit is financed domestically, the situation mirrors the closed economy case, where sustainability depends on the relationship between r and g . In the open economy model, a key distinction emerges when the deficit is financed through external public debt, in which case debt dynamics depend on the relationship between the international interest rate (r^*), the depreciation rate of the domestic currency (g_e), and the GDP growth rate (g).

Simulations were conducted in both versions to examine the impact of a permanent reduction in the primary surplus (sp_t). The results indicate that when $r > g$ in the closed economy and $r^* + g_e > g$ in the open economy, public debt follows an explosive trajectory, failing to reach a new steady-state equilibrium. However, this trajectory is constrained by limits imposed by markets and multilateral organizations. At this point, achieving a new equilibrium requires a sufficiently large primary surplus to freeze public debt at its upper limit. Attaining this equilibrium entails substantial fiscal adjustments, either through significant reductions in public spending or increases in taxation.

In summary, public debt sustainability in both closed and open economies depends on maintaining adequate primary surpluses and carefully managing the relationship between the GDP growth rate, interest rates, and the depreciation of the domestic currency.

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