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DOCUMENTO DE TRABAJO

Nº 554

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Departamento
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Febrero, 2026

DOCUMENTO DE TRABAJO 554
http://doi.org/10.18800/2079-8474.0554

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Editado:

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Encargado de la Serie: Gabriel Rodríguez
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Primera edición – Febrero, 2026

ISSN 2079-8474 (En línea)

Forecasting Value at Risk and Expected Shortfall in Equity Markets of High-Income and Latin American Countries*

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Version: February 16, 2026

Abstract

Using daily equity market data for Latin American (Latam) and high-income (HI) countries over 2008-2023, this paper estimates GARCH and GJR models to forecast Value at Risk (VaR) and Expected Shortfall (ES). The performance of a broad set of heavy-tailed and asymmetric distributions is evaluated, including the Normal ($\mathcal{N}$), Skewed Normal ($sk\mathcal{N}$), Student's t ($\mathcal{S}$), skewed $\mathcal{S}$ ($sk\mathcal{S}$), generalized hyperbolic $sk\mathcal{S}$ ($\mathcal{GH}sk\mathcal{S}$), normal inverse Gaussian ($\mathcal{NIG}$), skewed $\mathcal{NIG}$ ($sk\mathcal{NIG}$), normal reciprocal inverse Gaussian ($\mathcal{NRIG}$), and skewed $\mathcal{NRIG}$ ($sk\mathcal{NRIG}$). The key findings can be summarized as follows: (i) for VaR forecasting, asymmetric distributions are preferred at both confidence levels, and at the 99% level heavy tails are also required; (ii) for ES forecasting, at both confidence levels the selected models rely on asymmetric heavy-tailed distributions, with $\mathcal{GH}sk\mathcal{S}$ emerging as the dominant specification; (iii) for VaR forecasting, modeling leverage effects is necessary for most HI countries, whereas this is required for only about half of the Latam countries; and (iv) for ES forecasting, volatility specification plays a more limited role than in VaR forecasting.

JEL classification: C52, C53, G17

Keywords: Value at Risk, Expected Shortfall, GARCH Models, Heavy-Tailed Distributions, Latin American Countries, High-Income Countries, Equity Markets, Forex Markets.

*This paper is based on Fiorela Liza's Thesis at the Faculty of Social Sciences of the Pontificia Universidad Católica del Perú (PUCP). The authors thank Paul Castillo B. (Central Reserve Bank of Peru and PUCP) for helpful comments. Any remaining errors are the authors' responsibility.

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Pronóstico del Valor en Riesgo y de la Pérdida Esperada en los Mercados Bursátiles de Países de Altos Ingresos y de América Latina*

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Versión: Febrero 4, 2026

Resumen

Utilizando datos diarios de los mercados bursátiles de países de América Latina (Latam) y de economías de altos ingresos (HI) para el período 2008-2023, este trabajo estima modelos GARCH y GJR con el fin de pronosticar el Valor en Riesgo (VaR) y la Pérdida Esperada (Expected Shortfall, ES). Se evalúa el desempeño de un amplio conjunto de distribuciones con colas pesadas y asimetría, entre las que se incluyen la Normal ($\mathcal{N}$), Sesgada Normal ($sk\mathcal{N}$), Student's t ($\mathcal{S}$), Sesgada$\mathcal{S}$ ($sk\mathcal{S}$), Hiperbólica Generalizada $sk\mathcal{S}$ ($\mathcal{GH}sk\mathcal{S}$), Normal Inversa Gausiana ($\mathcal{NIG}$),Sesgada $\mathcal{NIG}$ ($sk\mathcal{NIG}$), Normal Recíproca Inversa Gausiana ($\mathcal{NRIG}$), y Sesgada $\mathcal{NRIG}$ ($sk\mathcal{NRIG}$). Los principales resultados pueden resumirse de la siguiente manera: (i) para la predicción del VaR, las distribuciones asimétricas son preferidas en ambos niveles de confianza, y en el nivel del 99% también se requiere la presencia de colas pesadas; (ii) para la predicción de la ES, en ambos niveles de confianza los modelos seleccionados se basan en distribuciones asimétricas con colas pesadas, destacándose $GHsk\mathcal{S}$ como la especificación dominante; (iii) en la predicción del VaR, la modelación de efectos de apalancamiento resulta necesaria para la mayoría de los países HI, mientras que en el caso de los países de Latam ello es requerido solo en aproximadamente la mitad de los casos; y (iv) para la predicción de la ES, la especificación de la volatilidad desempeña un papel más limitado que en la predicción del VaR.

Clasificación JEL: C52, C53, G17.

Palabras Claves: Valor al Riesgo, Pérdida Esperada, Modelos GARCH, Distribuciones de Colas Pesadas, Países LATAM, Países de Altos Ingresos, Mercados Bursátiles, Mercados Forex.

*Este documento está basado en la Tesis de Licenciatura de Fiorela Liza de la Facultad de Ciencias Sociales de la Ponticia Universidad Católica del Perú (PUCP). Apreciamos los comentarios de Paul Castillo B. (Banco Central de Reserva del Perú y PUCP). Los puntos de vista expresados en este documento son de los autores y no necesariamente representan la posición oficial de la CAF. Los errores subsistentes son responsabilidad de los autores.

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1 Introduction

Market risk is one of the main sources of uncertainty for banking and financial institutions, as portfolio values can incur substantial losses due to fluctuations in asset prices such as equities, foreign exchange, and bonds. Accordingly, the Basel regulatory framework requires banks to estimate potential losses and hold sufficient capital buffers to absorb them (Basel Committee on Banking Supervision, 2019).

Over the past three decades, the measurement of market risk has gained prominence for several reasons. First, trading volumes in equity and foreign exchange markets increased markedly over 1990–2008, particularly in emerging economies in Latin America and Asia, which offer higher returns to investors (Bank for International Settlements, 2007; Organization for Economic Co-operation and Development, 2019). These higher expected returns, however, are associated with greater asset price volatility and heightened economic and political uncertainty. Second, recent financial crises—including the 2008 global financial crisis and the COVID-19 crisis in 2020—have adversely affected the liquidity and stability of financial institutions, underscoring the need for models that quantify market risk with greater accuracy.

The empirical literature treats Value at Risk (VaR) and Expected Shortfall (ES) as standard tools for risk management. One approach to computing VaR and ES is the parametric framework, which assumes a probability distribution for returns and estimates volatility. The GARCH family of models falls within this framework and is widely used to model daily return volatility while capturing key stylized facts of financial markets. A substantial body of work argues for assuming heavy-tailed and asymmetric return distributions, as the Normal distribution tends to underestimate market risk (Giot and Laurent, 2003, 2004; Aas and Haff, 2006; Diamandis et al., 2006; Kuester et al., 2006; Su and Knowles, 2006; Degiannakis et al., 2012; Orhan and Köksal, 2012; Guo, 2019). In addition, empirical evidence indicates that volatility models should capture leverage effects in returns. Prominent specifications include the AGARCH model (Engle, 1990), EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan, and Runkle, 1993), and TGARCH (Zakoian, 1994).

This paper evaluates equity market risk using VaR and ES. The sample is divided into two groups: high-income (HI) markets—Canada, the United States, Denmark, Norway, Australia, Switzerland, the United Kingdom, Japan, and Europe—and Latin American (Latam) markets—Argentina, Brazil, Chile, Colombia, Mexico, and Peru. The analysis estimates the Bollerslev (1986) GARCH model and the GJR model of Glosten et al. (1993), which incorporates leverage effects. Both models are combined with a range of return distributions, including Normal ($\mathcal{N}$), Student's t ($\mathcal{S}$), skewed $\mathcal{N}$ ($sk\mathcal{N}$), skewed Student's t ($sk\mathcal{S}$), skewed generalized hyperbolic Student's t ($\mathcal{GH}sk\mathcal{S}$), Normal Inverse Gaussian ($\mathcal{NIG}$), skewed $\mathcal{NIG}$ ($sk\mathcal{NIG}$), Normal Reciprocal Inverse Gaussian ($\mathcal{NRIG}$), and skewed $\mathcal{NRIG}$ ($sk\mathcal{NRIG}$). Model performance is assessed using backtesting procedures and the Model Confidence Set of Hansen et al. (2011) to identify specifications with superior predictive accuracy.

To the best of current knowledge, this is the first study to compare a broad set of HI and Latam countries while employing an extensive menu of heavy-tailed and asymmetric return distributions. The main findings are as follows. First, for VaR forecasts, asymmetric distributions are preferred at both confidence levels (95% and 99%), and at the 99% level heavy tails are also required. Second, for ES forecasts, the selected models at both confidence levels feature asymmetric heavy-tailed distributions, with the $\mathcal{GH}sk\mathcal{S}$ distribution prevailing. Third, regarding volatility specification, leverage effects are necessary for VaR prediction in most HI countries, but only for about half of the

Latam sample. Moreover, for HI markets, the evidence suggests that volatility specification matters more for VaR forecasts than the choice of return distribution, provided the latter is asymmetric and heavy-tailed. For ES forecasts, volatility specification plays a smaller role than in the VaR case.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the methodology. Section 4 presents and discusses the results. Section 5 concludes.

2 Literature Review

Value at Risk (VaR) and Expected Shortfall (ES) are the principal measures of market risk used in the empirical literature and in portfolio management. Both can be estimated using nonparametric, parametric, and semiparametric approaches, which differ in their assumptions about the underlying return distribution. The nonparametric approach relies on the empirical distribution of returns, the parametric approach assumes a specific probability distribution, and the semiparametric approach combines elements of both (Abad et al., 2014). Within the parametric framework, the GARCH model of Bollerslev (1986) and its extensions are widely used to model return volatility while capturing key stylized facts of financial markets. This review focuses on that class of models.

Two main strands of research apply GARCH-type models to the measurement of market risk: (i) the choice of the return distribution, and (ii) the specification of the volatility process. The first strand advocates the use of heavy-tailed and asymmetric return distributions. Van den Goorbergh and Vlaar (1999), Huang and Lin (2004), Dimitrakopoulos et al. (2010), Xekalaki and Degiannakis (2010), Orhan and Köksal (2012), and Miletic and Miletic (2015) propose heavy-tailed distributions such as the Student's t ($\mathcal{S}$) for VaR estimation, showing that they reduce the number of violations relative to the Normal distribution. Several studies, however, argue that this improvement reflects a more conservative specification that tends to overestimate risk (Angelidis and Benos, 2008; Lee and Su, 2012). For ES, Kjellson (2013) and Alexander and Sheedy (2008) caution that models based on the $\mathcal{S}$ distribution may be spuriously favored by backtesting procedures that detect only underestimation—such as the one-sided test of McNeil and Frey (2000)—and fail to penalize overestimation.

In response to these limitations, Kuester et al. (2006) and Su and Knowles (2006) propose the generalized error distribution ($\mathcal{GED}$), showing that GARCH models with GED errors outperform Normal-based models in VaR estimation. Similarly, Angelidis et al. (2004), using returns from the Japanese Nikkei index, find that the tendency to overestimate VaR is weaker under the $\mathcal{GED}$ than under the $\mathcal{S}$ distribution. Other proposed alternatives include the Normal mixture distributions of Xu and Wirjanto (2010) and the double Gamma distribution of Chiu and Chuang (2020), both of which deliver performance comparable to that of the $\mathcal{S}$ distribution.

A related literature evaluates the gains from allowing for asymmetry in heavy-tailed return distributions, reflecting the differing behavior of the left and right tails in empirical data. The skewed Student's t distribution ($sk\mathcal{S}$) of Fernández and Steel (1998) has been widely used for VaR estimation (Giot and Laurent, 2003, 2004; Diamandis et al., 2006; Tu et al., 2008; Bubák, 2008; Kang and Yoon, 2009; Maghyereh and Awartani, 2012; Degiannakis et al., 2012). These studies report improvements in violation rates and backtesting outcomes relative to the $sk\mathcal{S}$ and/or $\mathcal{N}$ distributions. Giot and Laurent (2003, 2004) further note that overestimation of VaR is less frequent under the $sk\mathcal{S}$ specification. Watanabe (2012) shows that the distinction between the $\mathcal{S}$ and $sk\mathcal{S}$ distributions is even more pronounced for ES, as this measure integrates losses over the entire left tail. Additional asymmetric extensions of the $\mathcal{S}$ distribution are discussed in Bali and Theodossiou (2006), Bali et al. (2008), and Altun et al. (2018).

Despite the popularity of $sk\mathcal{S}$-type distributions, Aas and Haff (2006) argue that these specifications may fail to capture return dynamics when empirical skewness is pronounced. They propose a GARCH model with a skewed generalized hyperbolic Student's t distribution ($\mathcal{GH}sk\mathcal{S}$) and show, using the Norwegian krone–euro exchange rate, that $\mathcal{GH}sk\mathcal{S}$ outperforms competing distributions for ES estimation and is also marginally superior for VaR. In a related vein, Forsberg and Bollerslev (2002) advocate the Normal Inverse Gaussian ($\mathcal{NIG}$) distribution, while Guo (2017) introduces the Normal Reciprocal Inverse Gaussian ($\mathcal{NRIG}$) distribution. Comparing the two, Guo (2019) finds that both are suitable for risk management applications, but the $\mathcal{NRIG}$ distribution offers greater flexibility for fitting assets with very heavy tails.

Although comparative studies across multiple heavy-tailed and asymmetric distributions are relatively scarce, several general conclusions emerge. First, heavy-tailed distributions consistently outperform the $\mathcal{N}$ distribution, but no single specification dominates across applications (Bali and Theodossiou, 2008). Second, Slim et al. (2017) show that heavy-tailed distributions are particularly important during crisis periods, whereas in post-crisis episodes most models tend to pass backtesting procedures. Third, when comparing symmetric and asymmetric specifications, De Oliveira and Maia (2017) and García-Jorcano and Novales (2021) find that asymmetric distributions yield more accurate VaR forecasts, although Sobreira and Louro (2020) report no systematic differences.

The second strand of the literature focuses on more flexible volatility specifications that capture observed features of return volatility, notably the leverage effect.[1] First documented by Black (1976), the leverage effect refers to the negative correlation between returns and volatility, whereby negative shocks generate larger increases in volatility than positive shocks of the same magnitude. This phenomenon has motivated numerous extensions of the Bollerslev (1986) GARCH model, including AGARCH (Engle, 1990), EGARCH (Nelson, 1991), GJR (Glosten et al., 1993), APARCH (Ding et al., 1993), and TGARCH (Zakoian, 1994); see Bollerslev (2010) for a detailed review. While these models were initially developed to describe and forecast volatility, subsequent work has examined their relevance for market risk prediction across asset classes.

For equity markets, Nieto and Ruiz (2008) compare the standard GARCH model with asymmetric extensions—GJR, EGARCH, TGARCH, and APARCH—in forecasting the VaR of the S&P 500 index, finding that EGARCH and GJR perform best. A broader set of studies similarly emphasizes the importance of asymmetric volatility models for VaR prediction in both developed equity markets (Abad and Benito, 2013; Krause and Paolella, 2014; Wong et al., 2016) and emerging markets (Angelidis and Degiannakis, 2008; Demireli, 2010; Şener et al., 2012; Bucevska, 2013; Cerović et al., 2017).

At the same time, several studies argue that the assumed return distribution plays a more decisive role in predicting VaR and ES than the volatility specification itself. For example, García-Jorcano (2018) compares a symmetric GARCH model with three asymmetric volatility specifications using equity and foreign exchange returns and finds that violation rates change little across volatility models but vary substantially across distributions. Similar conclusions are reported by Angelidis and Degiannakis (2007b) and Braione and Scholtes (2016), suggesting that distributional

[1]Empirical studies also examine the role of long memory in VaR forecasting, but the evidence is mixed. Some authors support its relevance (Mittnik and Paolella, 2000; Degiannakis, 2004; Härdle and Mungo, 2008; Mabrouk and Saadi, 2012; Mabrouk, 2016; Aloui and Ben Hamida, 2015; Walther, 2017), while others find no clear support (Ané, 2006; So and Yu, 2006; Angelidis and Degiannakis, 2007a; Nikolić et al., 2011; Katzke and Garbers, 2016; García-Jorcano and Novales, 2020).

assumptions dominate volatility specification in market risk forecasting.

Two gaps emerge from this literature. First, most studies focus on developed markets or Asian emerging economies, while Latam markets receive limited attention (Dimitrakopoulos et al., 2010; Diamandis et al., 2011; Orhan and Köksal, 2012; Şener et al., 2012; De Oliveira and Maia, 2017). Second, relatively few papers adopt a unified GARCH framework with a broad set of alternative distributions to compare performance in VaR and ES forecasting. This study contributes to the literature by evaluating equity market risk in HI and Latam countries using GARCH and GJR models combined with a wide range of heavy-tailed and asymmetric return distributions.

3 Methodology

This section presents the volatility models and return distributions. It also describes the two measures used to quantify market risk and the corresponding backtesting procedures.

3.1 Models

Let $r_t = 100 \times [log(p_t) - log(p_{t-1})]$ denote the logarithmic return of a financial asset at time t, where p_t is the value of a stock market index. For simplicity, returns are assumed to have zero mean and no autocorrelation. Under these assumptions, r_t is written as

$$r_t \sim \mathcal{D}(0, h_t, \mathbf{\Phi}), \tag{1}$$

where $\mathcal{D}(0, h_t, \mathbf{\Phi})$ denotes a continuous distribution with zero mean, conditional variance h_t, and additional parameters governing skewness and kurtosis collected in the vector $\mathbf{\Phi}$.

In general, the conditional variance h_t depends on past returns r_{t-1} , its own lag h_{t-1}, and a parameter vector $\boldsymbol{\theta}$:

$$h_t = h(r_{t-1} \,, h_{t-1}, \boldsymbol{\theta}), \tag{2}$$

where $h(.)$ is a measurable function that ensures positivity of the variance. The specific functional form depends on the chosen volatility model.

3.1.1 GARCH Model

The GARCH(1,1) model proposed by Bollerslev (1986) is specified as

$$r_t = \sqrt{h_t}\eta_t \tag{3a}$$

$$\eta_t \sim i.i.d.\ \mathcal{D}(0, 1, \mathbf{\Phi}) \tag{3b}$$

$$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1} \tag{3c}$$

Here, η_t denotes the model innovations. The parameter restrictions $\alpha_0 > 0$, $\alpha_1 \geqslant 0$, $\beta_1 \geqslant 0$, and $\alpha_1 + \beta_1 < 1$ ensure a positive variance process. In this case, the parameter vector is $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \beta_1)'$.

3.1.2 GJR Model

The specification of Glosten et al. (1993) allows for asymmetries in volatility:

$$h_t = \alpha_0 + (\alpha_1 + \gamma I_{[r_{t-1}<0]})r_{t-1}^2 + \beta_1 h_{t-1}, \tag{4}$$

where $I_{[.]}$ is an indicator function that equals one when the condition holds and zero otherwise. The parameter γ captures asymmetry in volatility. The constraints $\alpha_0 > 0$, $\alpha_1 \geqslant 0$, $\gamma \geqslant 0$, $\beta_1 \geqslant 0$, and $\alpha_1 + \gamma E[\eta_t^2 I_{\{\eta_t<0\}}] + \beta_1 < 1$ ensure a positive and stationary variance process. For this model, the parameter vector is $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \gamma, \beta_1)'$.

3.2 Return Distributions

For the volatility models described above, several alternatives are considered for the distribution of the innovations $\mathcal{D}(0, 1, \boldsymbol{\Phi})$.

3.2.1 Normal ($\mathcal{N}$) Distribution

When the innovations η_t follow a standard $\mathcal{N}$ distribution, the density function is

$$f(\eta) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}\eta^2). \tag{5}$$

An alternative specification is proposed by Fernández and Steel (1998), who introduce a skewness parameter to obtain the skewed Normal distribution ($sk\mathcal{N}$):

$$f(\eta;\xi) = \begin{cases} \frac{2\xi}{1+\xi^2} f\left(\xi\left(s\eta + m\right)\right), & \eta < -m/s \\ \\ \frac{2\xi}{1+\xi^2} f\left(\xi^{-1}\left(s\eta + m\right)\right) & \eta \geq -m/s \end{cases} \tag{6}$$

where $f(.)$ denotes the density of a standardized symmetric $\mathcal{N}$ distribution and ξ is the skewness parameter. Values $\xi > 1$ indicate positive skewness, while $\xi < 1$ imply negative skewness. The coefficients $m = \mu + \frac{\sqrt{2}\sigma(\xi^2-1)}{\sqrt{\pi}\xi}$ and $s = \sqrt{\frac{\sigma^2[(\pi-2)\xi^6+2\xi^2(\xi^2+1)+\pi+2]}{\pi(1+\xi^2)}}$ are, respectively, the mean and standard deviation of the non-standardized skewed $\mathcal{N}$ distribution. In this case, $\boldsymbol{\Phi} = \xi$.

3.2.2 Student's t ($\mathcal{S}$) Distribution

To allow for heavy tails, the innovations may follow an $\mathcal{S}$ distribution, with density

$$f(\eta;\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\left(\nu-2\right)}} \left(1 + \frac{\eta^2}{(\nu-2)}\right)^{-\left(\frac{\nu+1}{2}\right)}, \tag{7}$$

where $\Gamma(.)$ denotes the Gamma function and $\nu > 2$ is required for the variance to be finite. The parameter ν governs tail thickness, with smaller values implying heavier tails. For this distribution, $\boldsymbol{\Phi} = \nu$.

The skewed $\mathcal{S}$distribution ($sk\mathcal{S}$) is obtained following Fernández and Steel (1998):

$$f(\eta;\nu,\xi) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}} sf\left(\xi\left(s\eta + m\right); \nu\right), & \eta < -m/s \\ \\ \frac{2}{\xi+\frac{1}{\xi}} sf\left(\xi^{-1}\left(s\eta + m\right); \nu\right) & \eta \geq -m/s \end{cases} \tag{8}$$

where $f(\eta;\nu)$ denotes the density of the standardized symmetric $\mathcal{S}$ distribution and ξ is the skewness parameter defined above. The constants $m = \frac{\Gamma(\frac{\nu-1}{2})\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\frac{\nu}{2})}(\xi - \frac{1}{\xi})$ and $s = \sqrt{(\xi^2 + \frac{1}{\xi^2} - 1) - m^2}$ are, respectively, the mean and standard deviation of the non-standardized skewed $\mathcal{S}$ distribution. In this case, $\mathbf{\Phi} = (\xi, \nu)'$.

3.2.3 Generalized Hyperbolic ($\mathcal{GH}$) Distribution

Prause (1999) introduces the $\mathcal{GH}$ distribution, whose density is given by:

$$f(\eta;\lambda,\delta,\alpha,\mu,\beta) = \frac{(\sqrt{\alpha^2-\beta^2}/\delta)^{\lambda} K_{\lambda-1/2}(\alpha\sqrt{\delta^2+(\eta-\mu)^2})}{\sqrt{2\pi}(\sqrt{\delta^2+(\eta-\mu)^2}/\alpha)^{1/2-\lambda} K_{\lambda}(\delta\sqrt{\alpha^2-\beta^2})} \times \exp(\beta(\eta-\mu)), \tag{9}$$

where K_j denotes the modified Bessel function of the third kind of order j. The parameters δ, α, μ, and β control scale, tail thickness, location, and skewness, respectively. The admissible parameter space requires $\delta \geq 0$ and $|\beta| < \alpha$ if $\lambda > 0$; $\delta > 0$ and $|\beta| < \alpha$ if $\lambda = 0$; and $\delta > 0$ and $|\beta| \leq \alpha$ if $\lambda < 0$. Different choices of λ yield the subclasses described below.

3.2.3.1 Skewed Generalized Hyperbolic Student's t ($\mathcal{GH}sk\mathcal{S}$) Distribution

The $\mathcal{GH}sk\mathcal{S}$ distribution is a subclass of the $\mathcal{GH}$ family obtained by setting $\lambda = -\nu/2$ and $\alpha \to |\beta|$. Its density is

$$f(\eta;\delta,\nu,\beta,\mu) = \frac{2^{\frac{1-\nu}{2}} \delta^{\nu} |\beta|^{\frac{\nu+1}{2}} K_{\frac{\nu+1}{2}}\left(\sqrt{\beta^2(\delta^2+(x-\mu)^2)}\right)}{\Gamma(\frac{\nu}{2})\sqrt{\pi}(\sqrt{\delta^2+(x-\mu)^2})^{\frac{\nu+1}{2}}} \times \exp(\beta(x-\mu)), \qquad \beta \neq 0, \tag{10}$$

The mean and variance are $\mu + \frac{\delta^2\beta}{\nu-2}$ and $\frac{2\beta\delta^4}{(\nu-2)^2(\nu-4)} + \frac{\delta^2}{\nu-2}$, respectively. The standardized version is obtained by setting $\mu = -\frac{\delta^2\beta}{\nu-2}$, while δ is chosen as the positive real solution that normalizes the variance to unity. The distribution exhibits positive (negative) skewness when $\beta > 0$ ($\beta < 0$). When $\beta = 0$, the $\mathcal{GH}sk\mathcal{S}$ distribution collapses to the $\mathcal{S}$ distribution in (7). The parameter vector is $\mathbf{\Phi} = (\beta, \nu)'$.

3.2.3.2 Normal Inverse Gaussian ($\mathcal{NIG}$) Distribution

The $\mathcal{NIG}$ distribution is another $\mathcal{GH}$ subclass obtained by setting $\lambda = -1/2$. The standardized version is obtained by choosing $\mu = -\frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}$ and $\delta = \frac{\left(\sqrt{\alpha^2-\beta^2}\right)^3}{\alpha^2}$. When $\beta = 0$, the symmetric $\mathcal{NIG}$ distribution has density

$$f(\eta;\alpha) = \frac{\alpha^2 K_1\left(\alpha\sqrt{\alpha^2+\eta^2}\right)}{\pi\sqrt{\alpha^2+\eta^2}} \times \exp(\alpha^2), \tag{11}$$

with parameter vector $\mathbf{\Phi} = \alpha$.

When $\beta \neq 0$, the skewed $\mathcal{NIG}$ distribution ($sk\mathcal{NIG}$) has density

$$f(\eta;\alpha,\beta)=\frac{(\alpha^2-\beta^2)^{3/2}K_1\left(\alpha\sqrt{\frac{(\alpha^2-\beta^2)^3}{\alpha^4}+\left(\eta+\frac{\beta(\alpha^2-\beta^2)}{\alpha^2}\right)^2}\right)}{\alpha\pi\sqrt{\frac{(\alpha^2-\beta^2)^3}{\alpha^4}+\left(\eta+\frac{\beta(\alpha^2-\beta^2)}{\alpha^2}\right)^2}}\times\exp\left(\beta\eta+(\alpha^2-\beta^2)\right) \quad (12)$$

where α governs tail behavior and β determines the degree of skewness. Positive (negative) skewness arises when $\beta>0$ ($\beta<0$). For this distribution, $\mathbf{\Phi}=(\beta,\alpha)'$.

3.2.3.3 Normal Reciprocal Inverse Gaussian ($\mathcal{NRIG}$) Distribution

The $\mathcal{NRIG}$ distribution is another subclass of the $\mathcal{GH}$ family, obtained by setting $\lambda=1/2$. Its mean and variance are given by $\mu+\frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}+\frac{\beta}{\alpha^2-\beta^2}$ and $\frac{\alpha^2\delta\sqrt{\alpha^2-\beta^2}+\alpha^2+\beta^2}{(\alpha^2-\beta^2)^2}$, respectively. The $\mathcal{NRIG}$ distribution is therefore standardized by setting $\mu=-(\frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}}+\frac{\beta}{\alpha^2-\beta^2})$, $\delta=\frac{1}{\alpha^2\sqrt{\alpha^2-\beta^2}}[(\alpha^2-\beta^2)^2-\alpha^2-\beta^2]$. When $\beta=0$, the resulting symmetric $\mathcal{NRIG}$ distribution is:

$$f(\eta;\alpha)=\frac{\alpha K_0(\sqrt{(\alpha^2-1)^2+\alpha^2\eta_t^2})}{\pi}\times\exp(\alpha^2-1), \quad (13)$$

where $\mathbf{\Phi}=\alpha$.
Otherwise, the distribution becomes asymmetric ($sk\mathcal{NRIG}$) and takes the following form:

$$f(\eta;\alpha,\beta) = \frac{\sqrt{\alpha^2-\beta^2}K_0\left(\alpha\sqrt{\frac{\left((\alpha^2-\beta^2)^2-\alpha^2-\beta^2\right)^2}{\alpha^4(\alpha^2-\beta^2)}+\left(\eta+\frac{\beta\left(\alpha^2-\beta^2\right)^2-\beta^3}{\alpha^2(\alpha^2-\beta^2)}\right)^2}\right)}{\pi}, \quad (14)$$
$$\times\exp\left(\beta\eta+\frac{\left(\alpha^2-\beta^2\right)^2-\alpha^2}{(\alpha^2-\beta^2)}\right)$$

where the parameters α and β have the same interpretation as in the $sk\mathcal{NIG}$ distribution and $\mathbf{\Phi}=(\beta,\alpha)'$.

3.3 Risk Measures

The risk measures considered are VaR and ES.

3.3.1 VaR

VaR measures the maximum loss that a portfolio may incur over a given time horizon at a specified confidence level. Formally, for a confidence level $\alpha\in(0,1)$, VaR is defined as the smallest value l such that the probability that the loss L exceeds l is less than or equal to $1-\alpha$.

$$VaR_t^\alpha=\inf\{l\in\mathbb{R}:P(L>l)\le 1-\alpha\} \quad (15)$$

In probabilistic terms, VaR corresponds to the α-quantile of the return (or loss) distribution.

3.3.2 ES

ES measures the average loss over a given horizon conditional on losses exceeding the VaR threshold. Accordingly, at confidence level $\alpha \in (0, 1)$, ES is given by:

$$ES_t^{\alpha} = E\left\{L | L \geq VaR_t^{\alpha}\right\}, \tag{16}$$

In this study, both measures are evaluated at confidence levels (α) of 0.95 and 0.99.

3.4 VaR Backtesting Tests

Table 1 summarizes the test statistics and null hypotheses of the VaR backtesting procedures. These tests can be grouped into three categories according to the null hypothesis under evaluation. The first category assesses unconditional coverage, which tests whether the observed proportion of VaR violations differs significantly from the expected rate $(1 - \alpha)$. This group includes the binomial test of Jorion (2003), the proportion-of-failures (POF) test of Kupiec (1995), and the Basel Committee's traffic light approach (Basel Committee on Banking Supervision, 1996). The binomial and POF tests allow rejection of models that either underestimate or overestimate risk, whereas the Basel traffic light test only rejects models that underestimate risk. A limitation of the POF test is that its power depends heavily on sample size, implying that large samples are required to detect inadequate models.

The second category comprises tests of independence, which examine whether VaR violations occur independently over time and are uncorrelated with past violations. These tests reject models in which violations cluster, a pattern that may be interpreted as delayed adjustment to changes in market conditions. The conditional coverage independence (CCI) test of Christoffersen (1998) falls into this category.

The third category comprises conditional coverage (CC) tests, which jointly assess the two preceding hypotheses—unconditional coverage and independence. The main tests in this category are the CC test proposed by Christoffersen (1998) and the dynamic quantile (DQ) test developed by Engle and Manganelli (2004). Although both tests consider the incidence of exceedances and their dependence, the DQ test allows for more general forms of dependence (clustering) in exceedances, whereas the CC test is limited to dependence between consecutive periods.

3.5 ES Backtesting Tests

Table 2 reports the test statistics and null hypotheses used for ES backtesting. These procedures can be grouped into two sets. The first set consists of one-sided tests, which evaluate the null that ES forecasts are at least as conservative as the true ES (i.e., market risk is not underestimated). This set includes the one-sided exceedance residual (*ER 1s*) test of McNeil and Frey (2000); the one-sided Simple (*Sim 1s*) and General (*Gen 1s*) conditional calibration tests of Nolde and Ziegel (2017); and the one-sided Intercept (*Int 1s*) test of Bayer and Dimitriadis (2022). Although these tests share the same null hypothesis, the *Int 1s* dominates the others in terms of size and power.

The second set consists of two-sided tests, whose null is that ES is correctly specified. Accordingly, these tests reject when ES forecasts either overestimate or underestimate risk. This set includes the two-sided exceedance residual (*ER 2s*) test of McNeil and Frey (2000); the two-sided Simple (*Sim 2s*) and General (*Gen 2s*) conditional calibration tests of Nolde and Ziegel (2017); and the Auxiliary (*Aux*), Strict (*Str*), and Intercept (*Int 2s*) tests of Bayer and Dimitriadis (2022). As in the one-sided case, the *Aux* and *Str* tests exhibit superior size and power properties relative to the *Sim 2s* and *Gen 2s* tests. Bayer and Dimitriadis (2022) note, however, that the *Int 2s* test has relatively lower power, as it is less effective at detecting certain forms of misspecification.

4 Empirical Evidence

4.1 Data and Preliminary Statistics

Daily equity index data are obtained from *Bloomberg Financial Data* for HI markets—Canada, the United States, Denmark, Norway, Australia, Switzerland, the United Kingdom, Japan, and Europe—and Latam markets—Argentina, Brazil, Chile, Colombia, Mexico, and Peru. All series end on February 28, 2023, while starting dates vary due to data availability. For Canada (September 10), the United States (September 24), Denmark (October 16), Norway (December 17), Switzerland (November 18), the United Kingdom (December 16), Japan (February 7), Argentina (January 16), Brazil (May 26), Chile (July 18), Mexico (October 10), and Peru (October 13), the sample begins in 2008. The remaining markets start on January 6, 2009 (Australia), June 1, 2009 (Europe), and November 15, 2007 (Colombia).

For each country, the sample is split into an in-sample period of 1,500 observations and an out-of-sample period of 2,000 observations. Following Ardia et al. (2018), Table 3 reports descriptive statistics for out-of-sample returns for HI and Latam equity markets. Panels (a) and (b) correspond to HI and Latam markets, respectively. Latam markets exhibit higher standard deviations and higher average returns than HI markets, reflecting the risk–return trade-off associated with higher expected profitability. Skewness is negative across all markets, underscoring the relevance of modeling a heavier left tail when computing risk measures. Among HI markets, Canada displays the largest skewness in absolute value (−1.65), while Argentina exhibits the largest skewness among Latam markets (−3.45). Kurtosis exceeds three in all cases, indicating leptokurtic return distributions. Latam markets generally display higher kurtosis, with Argentina standing out with a value of 62.58. Consistent with these features, the last six columns of Table 3 show that VaR and ES levels are higher in Latam markets.

4.2 Model Estimation

Models are estimated by maximum likelihood. A total of 270 specifications are considered, reflecting four symmetric distributions, five asymmetric distributions, fifteen equity markets, and two volatility models (GARCH and GJR). One-day-ahead VaR and ES forecasts are generated using a rolling-window procedure. The in-sample window contains 1,500 observations, and the out-of-sample evaluation spans 2,000 observations. In the first iteration, models are estimated using the initial in-sample window to obtain risk forecasts. At each subsequent step, one observation is added and the oldest observation is removed, maintaining a fixed window length. Model parameters are re-estimated every ten observations. This procedure continues until all out-of-sample observations are exhausted.

The estimated parameters yield several insights.[2] First, the estimates of $\hat{\nu}$ and $\hat{\alpha}$ are small across all markets, indicating pronounced tail heaviness. These parameters are systematically smaller for

[2] Since there are 2,000 out-of-sample observations and the models are re-estimated every 10 observations, 200 parameter sets are obtained. Accordingly, the median of the estimated parameters is used in the analysis. In order to save space, the Tables reporting complete set of parameters estimates are provided in an Appendix available upon request.

Latam countries, suggesting heavier tails—and hence higher risk—than in HI markets. Regarding skewness, $\hat{\beta} > 0$ implies positive skewness and $\hat{\beta} < 0$ negative skewness; equivalently, $\hat{\xi} > 1$ indicates positive skewness and $\hat{\xi} < 1$ negative skewness. The results show $\hat{\xi} < 1$ and $\hat{\beta} < 0$ in all cases, confirming pervasive negative skewness and reinforcing the use of asymmetric return distributions. HI markets exhibit more negative values of $\hat{\beta}$ and lower values of $\hat{\xi}$ (farther from the threshold of one), pointing to stronger tail asymmetry relative to Latam markets. Finally, estimates of the leverage parameter satisfy $\hat{\gamma} > 0$ in both groups, providing evidence of leverage effects. These effects are weaker in Latam markets (ranging from 0.026 to 0.103) than in HI markets (ranging from 0.106 to 0.306).

Using the estimated coefficients, density functions are plotted to illustrate how each distribution captures tail behavior and to highlight differences across distributions. Owing to space constraints, only four equity markets are shown.

Figure 1 displays empirical densities alongside symmetric fitted distributions for two HI markets (Canada and the United States) and two Latam markets (Mexico and Peru).[3] Around the center, the empirical density is closer to the Normal $\mathcal{N}(0,1)$, as both exhibit lower peaks relative to heavy-tailed alternatives. In the tails, however, the $\mathcal{S}$, $\mathcal{NIG}$, and $\mathcal{NRIG}$ distributions more closely track the empirical density, reflecting their heavier tails relative to $\mathcal{N}(0,1)$. Because these distributions are symmetric, they fail to capture the pronounced left tail of the empirical distribution, suggesting the need for asymmetric specifications.

Figure 2 reports empirical densities and asymmetric fitted distributions for the same set of markets. While the central part of the estimated distributions ($sk\mathcal{N}$,$sk\mathcal{NIG}$, $sk\mathcal{NRIG}$, $\mathcal{GH}sk\mathcal{S}$, and $sk\mathcal{S}$) shows little change relative to Figure 1, the left tail becomes heavier when $\beta < 0$ ($\xi < 1$), closely matching the empirical return distribution. Asymmetric heavy-tailed distributions provide a better fit than $sk\mathcal{N}$, mirroring the comparison between symmetric distributions in Figure 1. Differences across the $sk\mathcal{NIG}$, $sk\mathcal{NRIG}$, $\mathcal{GH}sk\mathcal{S}$, and $sk\mathcal{S}$ tails, however, are not visually pronounced.

Comparing Figures 1 and 2 yields three main conclusions. First, the $\mathcal{N}$ and $sk\mathcal{N}$ distributions exhibit clear limitations in capturing extreme returns, confirming the need for heavy-tailed specifications. Second, asymmetric heavy-tailed distributions fit the left tail of the empirical density better than their symmetric counterparts, indicating that asymmetry is also essential. Third, given the lack of marked differences among the tails of the $sk\mathcal{NIG}$, $sk\mathcal{NRIG}$, $\mathcal{GH}sk\mathcal{S}$, and $sk\mathcal{S}$, distributions, their backtesting performance is expected to be broadly similar.

4.3 VaR Backtesting

Tables 4 (HI) and 5 (Latam) report the VaR backtesting results. For each country, the first row corresponds to the benchmark GARCH-$\mathcal{N}$ specification, the second to the selected model, and the remaining rows to models with comparable performance. Within each test, blue entries indicate the model with the highest p-value, while red entries denote rejected specifications.

Model selection follows a three-stage evaluation procedure. The first stage considers unconditional coverage tests (Binomial, POF, and traffic light). Models are required to achieve p-values above 0.05 in the Binomial and POF tests and a "green" (or, at a minimum, "yellow") classification under the traffic light approach. The second stage assesses independence using the CCI test, which models must pass with a p-value above, or close to, 0.05. The third stage evaluates conditional coverage using the CC and DQ tests. Among the models that pass this stage, the preferred specifi-

[3] In the interest of space economy, the remaining Figures are provided in an Appendix available upon request.

cation is the one with the highest p-values in both tests or, at least, in the DQ test, given its ability to detect more general forms of violation clustering. If the selected model displays a violation rate that deviates markedly from the expected level $1 - \alpha$, priority is given to the specification with a violation frequency closer to that benchmark. The discussion below is organized around these three stages for confidence levels of 95% and 99%.

For HI countries, several patterns emerge. The unconditional coverage tests (Binomial, traffic light, and POF) indicate that, at the 95% level, most models deliver adequate coverage: Binomial and POF p-values exceed 0.05, and the traffic light test classifies them as green. For most countries, even specifications based on the $\mathcal{N}$ and $sk\mathcal{N}$ distributions pass all three tests. At the 99% level, by contrast, only heavy-tailed distributions pass the three tests simultaneously ($p > 0.05$ for Binomial and POF, and a green traffic light outcome). Comparing symmetric and asymmetric specifications, asymmetric distributions systematically yield higher p-values at both confidence levels, indicating superior coverage when return asymmetry is allowed. This finding is consistent with De Oliveira and Maia (2017) and García-Jorcano and Novales (2021), who show that asymmetric distributions provide more accurate VaR forecasts than symmetric ones.

The CCI test shows that, among models with adequate unconditional coverage, independence of violations is generally not rejected. As a result, the CCI test provides limited additional discrimination within this subset of models. This pattern holds for Canada, the United States, Denmark, Norway, Switzerland, Japan, the United Kingdom, and Europe at both confidence levels. For Australia, at the 95% level, none of the models with correct coverage satisfy the independence condition, although several exhibit p-values close to 0.05. At the 99% level, only the GJR-$sk\mathcal{NRIG}$ specification jointly passes the Binomial, traffic light, POF, and CCI tests.

The conditional coverage tests (CC and DQ) further discriminate among models that already satisfy coverage and independence. The preferred specification is the one with the highest p-values in both tests or, alternatively, the highest p-value in the DQ test. Applying these criteria, the selected models at the 95% level are GJR-$sk\mathcal{N}$ (Canada, the United States, and Denmark), GJR-$sk\mathcal{NRIG}$ (Norway and the United Kingdom), GARCH-$sk\mathcal{NRIG}$ (Australia), GJR-$\mathcal{GH}$-skS (Switzerland), and GJR-skS (Japan and Europe). At the 99% level, the selected models are GJR-$sk\mathcal{NRIG}$ (Canada, Australia, and Switzerland), GJR-$\mathcal{GH}$-skS (the United States and Japan), GJR-skS (Denmark and the United Kingdom), GARCH-$\mathcal{GH}$-skS (Norway), and GARCH-$sk\mathcal{NIG}$ (Europe).

Regarding the selected specifications, two aspects related to volatility modeling are noteworthy. First, the GJR model is chosen in 6 of the 9 markets at both the 95% and 99% confidence levels, indicating that incorporating leverage effects improves VaR predictive performance, consistent with Abad and Benito (2013). Second, for several countries there exist close alternatives with p-values and violation rates very similar to those of the selected model. This occurs, for example, in the United States, Australia, and Switzerland at the 95% level, and at the 99% level in 7 of the 9 markets (Canada, the United States, Norway, Australia, Switzerland, the United Kingdom, and Europe). In these cases, the competing models share the same volatility specification but differ in the return distribution. This pattern indicates that, conditional on using asymmetric heavy-tailed distributions, the choice of volatility model matters more for VaR forecasts than the choice of the return distribution. This finding contrasts with García-Jorcano (2018), Angelidis and Degiannakis (2007b), and Braione and Scholtes (2016), who argue that volatility specification plays a secondary role relative to distributional assumptions.

Figure 3 plots VaR forecasts from the selected models alongside realized returns for HI countries.

Volatility clustering is evident throughout the sample, with the COVID-19 crisis in 2020 standing out as the most pronounced stress episode. At the 95% confidence level, two main features emerge. First, across all markets, the selected model and the GARCH-$\mathcal{N}$ benchmark deliver similar forecasts during periods of low volatility, but in high-volatility episodes the selected model tracks return dynamics more closely. This result is consistent with Slim et al. (2017), who emphasize that heavy-tailed distributions become particularly relevant during crisis periods, as they better capture extreme realizations that arise under heightened volatility. Second, in several markets—most notably Switzerland, the United Kingdom, Japan, and Europe—a noticeable divergence between GARCH-$\mathcal{N}$ forecasts (red line) and those of the selected model (yellow line) appears in 2015 and 2020. These patterns are consistent with the fact that the selected models pass coverage and independence tests, calibrating both the frequency and the timing of violations and adjusting without delay to changing market conditions.

At the 99% confidence level, larger discrepancies emerge between the selected models and the benchmark, both in low- and high-volatility periods. At this level, more extreme quantiles of the distribution are forecast, and models with asymmetric heavy-tailed distributions better capture tail behavior. In addition, for the United Kingdom, Japan, and Europe, the GARCH-$\mathcal{N}$ specification exhibits delayed responses to changes in volatility, in contrast to the selected models, which more accurately reproduce the sequence of violations.

For the Latam group, the unconditional coverage tests yield the following results. At the 95% confidence level, model performance is satisfactory for Argentina, Brazil, Chile, Mexico, and Peru, as most specifications pass all three tests simultaneously. For Colombia, by contrast, only the GARCH-$sk\mathcal{N}$ specification satisfies this condition. Raising the confidence level to 99% substantially reduces the set of models with adequate coverage. For Argentina, Brazil, Mexico, and Peru, only specifications with heavy-tailed distributions pass the three tests jointly. For Chile and Colombia, since no model satisfies this criterion, priority is given to specifications that (i) display p-values close to 0.05 in the Binomial and POF tests and (ii) receive a "yellow" classification under the traffic light approach. Under this rule, the two best-performing models for both countries are GARCH-$sk\mathcal{NIG}$ and GJR-$sk\mathcal{NRIG}$, both featuring heavy-tailed distributions.

In the second stage, the CCI test is applied to assess independence of violations among models that passed the first stage. At the 95% level, independence is not rejected for Argentina, Brazil, Chile, and Mexico. For Colombia, since only the GARCH-$sk\mathcal{N}$ specification passes the first stage, it proceeds directly to the third stage. For Peru, the CCI test rejects all models; therefore, only GJR-$\mathcal{N}$ and GJR-$sk\mathcal{N}$ are allowed to advance, as they exhibit the highest p-values among the rejected specifications. At the 99% level, models reaching this stage are generally not rejected by the CCI test. For Peru, however, none of the models pass, and p-values are very similar across specifications; accordingly, all models that passed the first stage are allowed to proceed to the third.

In the third stage, conditional coverage tests (CC and DQ) are used to discriminate among the remaining models. Among those that pass both tests, the preferred specification is the one with the highest p-values in both tests or, at a minimum, in the DQ test. If the selected model exhibits a violation rate that deviates substantially from the expected level $1-\alpha$, priority is given to the specification with a violation frequency closer to the benchmark. Applying these criteria, the selected models at the 95% level are GJR-$sk\mathcal{N}$ (Argentina and Peru), GARCH-$\mathcal{GH}sk\mathcal{S}$ (Brazil), GARCH-$sk\mathcal{N}$ (Chile and Colombia), and GJR-$sk\mathcal{NRIG}$ (Mexico). At the 99% level, the selected models are GARCH-$sk\mathcal{S}$ (Argentina), GARCH-$sk\mathcal{NRIG}$ (Brazil), GJR-$sk\mathcal{NIG}$ (Chile and Colombia), GJR-$sk\mathcal{NRIG}$ (Mexico), and GARCH-$\mathcal{GH}sk\mathcal{S}$ (Peru). These results indicate that, at the 95% level, capturing distributional asymmetry is essential, as all selected specifications exhibit this feature. At the 99% level, all selected distributions are heavy-tailed, indicating that, in addition to asymmetry, tail thickness must also be modeled.

Regarding volatility specification, there is no conclusive evidence in Latam markets in favor of either GARCH or GJR, as both are selected with similar frequency. At the 99% level, however, two clear patterns emerge. In Brazil, Mexico, and Peru, the selected models and their close alternatives exhibit very similar violation rates and p-values; in these cases, they share the same volatility specification but differ in the return distribution, a pattern also observed for HI markets. By contrast, in Argentina, Chile, and Colombia, models share the same distribution but differ in volatility specification. This result is consistent with García-Jorcano (2018), Angelidis and Degiannakis (2007b), and Braione and Scholtes (2016), who emphasize that the choice of distribution is more important than volatility specification for VaR forecasting.

Figure 4 plots VaR forecasts from the selected models alongside realized returns for Latam countries. Returns are larger in absolute value than in HI markets, and high-volatility episodes are more frequent, which helps explain the greater difficulty of passing backtesting tests. VaR forecasts display patterns similar to those observed for HI markets when the confidence level increases from 95% to 99%. At the 95% level, differences between the selected models (yellow line) and the benchmark (red line) are negligible. At the 99% level, however, larger discrepancies emerge in both low- and high-volatility periods. Argentina provides a representative example: the selected model tracks return dynamics more closely, particularly during 2019–20, and therefore delivers more accurate risk forecasts. Similar patterns are observed for Brazil (2017 and 2020), Chile (2020 and 2021), Colombia (2020), Mexico (2018 and 2020), and Peru (2016 and 2020).

4.4 ES Backtesting

Tables 6 (HI) and 7 (Latam) report the ES backtesting results. The first row corresponds to the GARCH-$\mathcal{N}$ benchmark, the second to the selected model, and the remaining rows to specifications with comparable performance. Within each test, blue entries denote the highest p-value, while red entries indicate rejected models.

Model selection follows a two-stage evaluation procedure. In the first stage, one-sided tests (*ER 1s*, *Sim 1s*, *Gen 1s*, and *Int 1s*), are applied. Models must achieve p-values above 0.05 to proceed, thereby excluding specifications that underestimate ES. In the second stage, two-sided tests (*ER 2s*, *Sim 2s*, *Gen 2s*, *Str*, *Aux*, and *Int 2s*) are used to identify correctly specified models, that is, those that neither underestimate nor overestimate ES. Among the models that pass all tests simultaneously, the preferred specification is the one with the highest p-values, with priority given to the *Str* and *Aux* tests of Bayer and Dimitriadis (2022), given their superior size and power properties.

Table 6 reports the results for HI markets. In the first stage, a clear pattern emerges. At the 95% level, only models with asymmetric heavy-tailed distributions pass all one-sided tests across all countries. The sole exception is Australia, where no specification satisfies all four tests; in this case, models that pass three of the four evaluations are allowed to proceed. At the 99% level, asymmetric heavy-tailed distributions again dominate, passing all one-sided tests in Canada, Norway, Denmark, Australia, the United Kingdom, Switzerland, and Europe. In Japan, some symmetric heavy-tailed specifications also satisfy the criterion.

In the second stage, all models that pass the first stage also pass the six two-sided tests, so discrimination relies on comparing p-values. At the 95% level, no model consistently attains the highest p-values across all tests. However, the *Str* and *Aux* tests—those with the greatest power—indicate superior performance for models with the $\mathcal{GH}sk\mathcal{S}$ distribution in most cases. Under this criterion, GARCH-$\mathcal{GH}sk\mathcal{S}$ is selected for Norway, Switzerland, and Japan, while GJR-$\mathcal{GH}sk\mathcal{S}$ is chosen for the United States, Australia, the United Kingdom, and Europe. In the remaining markets, the evidence favors GARCH-$sk\mathcal{NIG}$. At the 99% level, $\mathcal{GH}sk\mathcal{S}$ specifications are selected for all countries, as they simultaneously deliver the highest p-values in the *Str* and *Aux* tests. Specifically, GARCH-$\mathcal{GH}sk\mathcal{S}$ is chosen for the United States, Denmark, the United Kingdom, and Europe, while GJR-$\mathcal{GH}sk\mathcal{S}$ is selected for Canada, Norway, Australia, Switzerland, and Japan.

Across the selected HI models, no volatility specification clearly dominates, in contrast to the VaR results where GJR often prevails. At the 95% level, GARCH is selected in five countries and GJR in four; at the 99% level, both specifications appear in nearly equal proportions. These findings indicate that, for ES, the choice between GARCH and GJR is more balanced and strongly conditioned on the assumed return distribution.

Table 7 presents the results for Latam markets. In the first stage, patterns are more heterogeneous than in HI countries. At the 95% level, in Argentina and Peru several heavy-tailed specifications—both symmetric and asymmetric—pass all four one-sided tests, whereas in Brazil and Mexico only asymmetric heavy-tailed models satisfy this criterion. In the remaining two countries, no specification passes all four tests, and a more flexible rule is applied. For Chile, only GARCH-$\mathcal{GH}sk\mathcal{S}$ advances, as it passes the *Sim 1s* and *Gen 1s* tests and attains p-values close to 0.05 in the *ER 1s* and *Int 1s* tests. For Colombia, models that pass three tests are allowed to proceed. At the 99% level, heavy-tailed specifications (symmetric and asymmetric) pass all four tests in Argentina and Peru, while in Chile, Colombia, and Mexico only asymmetric heavy-tailed models advance. In Brazil, no model passes the *ER 1s* test, so only those that pass the remaining tests are retained.

In the second stage, performance is compared using the two-sided tests. Since no model attains the highest p-value across all tests, the *Str* and *Au* tests are again prioritized. At the 95% level, these tests agree on the best model for Brazil, Chile, and Colombia, while in Argentina, Mexico, and Peru selection is based on the *Str* test alone. Under this criterion, the selected models are GARCH-$\mathcal{GH}sk\mathcal{S}$ (Brazil and Chile), GJR-$\mathcal{GH}sk\mathcal{S}$ (Colombia and Mexico), and GARCH-$sk\mathcal{NIG}$ (Argentina and Peru). At the 99% level, *Str* and *Aux* again coincide for Brazil, Chile, and Colombia, while in Argentina, Mexico, and Peru the decision relies on the *Str* test. The selected models are GARCH-$\mathcal{GH}sk\mathcal{S}$ (Brazil, Chile, and Colombia), GJR-$\mathcal{GH}sk\mathcal{S}$ (Mexico), GJR-$sk\mathcal{S}$ (Argentina), and GARCH-$\mathcal{NRIG}$ (Peru). These results show that, unlike in HI countries $\mathcal{GH}sk\mathcal{S}$ is not selected in all cases; nevertheless, in Argentina and Peru, models with this distribution perform very close to the chosen specification.

As in the HI group, no volatility specification clearly dominates among the selected Latam models. At the 95% level, GARCH is selected in four markets (Argentina, Brazil, Chile, and Peru) and GJR in two (Colombia and Mexico). At the 99% level, the two specifications are again selected in similar proportions: GARCH in four countries (Brazil, Chile, Colombia, and Peru) and GJR in two (Argentina and Mexico). Overall, these results indicate that, for Latam markets, ES forecasts are driven primarily by the assumed return distribution, as in HI countries.

In summary, the results show that the $\mathcal{GH}sk\mathcal{S}$ distribution delivers superior performance in ES estimation, consistent with Aas and Haff (2006), who highlight its ability to fit empirical densities with pronounced tail asymmetry, particularly when the left tail is substantially heavier. A comparison of VaR and ES suggests that differences in selected models reflect the fact that VaR is more strongly influenced by volatility dynamics, whereas ES is more sensitive to the shape and asymmetry of the lower tail. Accordingly, when the objective is to assess the severity of extreme losses—as in regulatory or stress-testing contexts—ES provides a more informative and robust criterion for model selection.

4.5 Model Comparison: Model Confidence Set

In addition to backtesting procedures, the Model Confidence Set (MCS) of Hansen et al. (2011) is implemented to evaluate VaR forecasts using $T = 2,000$ out-of-sample observations. The MCS procedure conducts a sequence of hypothesis tests to assess the predictive ability of an initial set of m models, $\mathcal{M}^0$, until obtaining a subset $\hat{\mathcal{M}}^*_{1-\eta}$ that contains the superior models at confidence level $1-\eta$, where η is 10% . At each iteration, the worst-performing model is eliminated until the null hypothesis of equal predictive ability cannot be rejected for any remaining model.

Formally, a loss function $l_{i,t}$ is defined for each model $i \in \mathcal{M}^0$ to evaluate the null hypothesis. In this context, $l_{i,t}$ corresponds to the asymmetric loss function (ALF) of González-Rivera et al. (2004), given by $l_{i,t}(r_t, VaR^\alpha_{i,t}) = (p - I\{r_t < VaR^\alpha_{i,t}\})(r_t - VaR^\alpha_{i,t})$, where $p = 1-\alpha$, r_t denotes asset returns, and $VaR^\alpha_{i,t}$ is the VaR forecast for period t at confidence level α obtained from model i. Next, the loss differential between models i and j is defined as $\bar{d}_{ij} = m^{-1}\sum_{t=1}^{T}(l_{i,t} - l_{j,t}),\;\; i,j \in \mathcal{M}^0$. Based on these differentials, the test statistic is computed as $T_{max} = \max_{i \in M} t_{i.} \quad i = 1, ..., m$, where $t_{i.} = \frac{\bar{d}_{i,.}}{\sqrt{\hat{var}(d_{i,.})}}$ and $\bar{d}_{i,.} = (m-1)^{-1}\sum_{j \in M}\bar{d}_{ij}$ represents the average loss of model i relative to the remaining models in the set. At each iteration, a model set $M \subset \mathcal{M}^0$ is considered and the statistic T_{max} is used to test the null hypothesis. If the null hypothesis is not rejected, the resulting set defines $\hat{\mathcal{M}}^*_{1-\eta} = M$. Otherwise, the elimination rule $e_{max} = \arg\max_{i \in M} t_{i.}$ is applied to remove the worst-performing model.

Tables 8 (HI) and 9 (Latam) report the MCS results. In most cases, a large number of models belong to the MCS with p-value = 1.000. For this reason, ALF values are also examined to further discriminate among competing specifications.

For HI countries, GJR models with asymmetric distributions belong to the superior set in all markets and at both confidence levels. In most countries, specifications with p-value = 1.000 are predominantly GJR models, although in the United States, the United Kingdom, and Europe some GARCH specifications also satisfy this criterion. An examination of ALF values indicates that the largest differences stem from changes in volatility specification rather than from the choice of return distribution, suggesting that conditional volatility dynamics play a more prominent role than distributional assumptions in these markets. At the 99% level, models with minimum ALF always employ heavy-tailed distributions, whereas at the 95% level some specifications with $\mathcal{N}$ or $sk\mathcal{N}$ distributions also appear among the best-performing models.

For Latam countries, the MCS results show that, in most markets, only GJR models attain a p-value = 1.000 at both confidence levels, with the exception of Mexico, where the best-performing specifications correspond to GARCH models. Analysis of ALF values reveals that Argentina, Colombia, and Peru are more sensitive to the choice of return distribution than to volatility specification. Moreover, at the 95% level, models with $\mathcal{N}$ or $sk\mathcal{N}$ distributions occasionally achieve minimum ALF values, while at the 99% level the best-performing models consistently rely on heavy-tailed distributions. These patterns are consistent with the backtesting results and underscore that, for Latam markets, return distributional assumptions play a more central role than volatility structure in VaR predictive performance.

5 Application to Foreign Exchange Markets

This section provides a brief analysis of VaR and ES forecasts for HI and Latam foreign exchange (FX) markets.[4]

Relative to equity markets, descriptive statistics for FX returns indicate smaller magnitudes (in absolute value) and lower standard deviations. Skewness coefficients take both positive and negative values, but their magnitudes are smaller than those observed in equity markets. All kurtosis coefficients exceed 3, indicating that empirical return distributions are leptokurtic.

Regarding the estimated parameters, the values of $\hat{\nu}$ and $\hat{\alpha}$ are small, pointing to the presence of heavy tails in return distributions. As for asymmetry, the coefficients $\hat{\xi}$ and $\hat{\beta}$ provide evidence of positive skewness in some cases and negative skewness in others, although their magnitudes are lower than in equity markets. In addition, the estimated values of $\hat{\gamma}$ are small in absolute terms, suggesting that incorporating a GJR specification may not be necessary to estimate VaR and ES in these markets.

For VaR forecasts, models with asymmetric distributions perform similarly to those with symmetric distributions in terms of violation rates and p-values. This contrasts with equity markets, where asymmetric distributions tend to dominate. The difference reflects the lower degree of asymmetry in empirical FX return distributions. Moreover, at the 95% confidence level—and for Chile and Mexico also at the 99% level—models with $\mathcal{N}$ or $sk\mathcal{N}$ distributions are selected. This outcome can be attributed to the smaller magnitude of FX returns, which makes these distributions adequate without requiring heavy tails.

Regarding volatility specification in VaR backtesting, GARCH models are selected more frequently than GJR models in FX markets. At the 95% level, GARCH is selected in 6 of the 13 countries, while at the 99% level this number increases to 8 out of 13. A plausible explanation is that the estimated values of $\hat{\gamma}$ are small in absolute terms, implying that explicitly modeling leverage effects does not materially improve VaR predictive performance. Consequently, GARCH models are sufficient to capture volatility dynamics in a substantial number of cases.

For ES forecasts, the results indicate that, in general, the selected models rely on heavy-tailed distributions at both confidence levels (95% and 99%), although in the case of Mexico an $\mathcal{N}$ distribution is selected. Across both country groups, the selected specifications include a mix of symmetric and asymmetric distributions, in contrast to equity markets where asymmetric distributions predominate. This difference again reflects the lower degree of asymmetry in FX return distributions. In addition, GARCH models perform as well as or better than GJR models in most countries, indicating that modeling leverage effects is not essential for ES forecasting in FX markets.

The MCS results show that, for both country groups, the number of models included in the confidence set is larger than in equity markets. Among the models with p-value = 1.000, GARCH and GJR specifications appear in similar proportions, indicating comparable predictive ability. An examination of ALF values yields two main findings. First, switching between volatility specifications generates smaller changes in ALF values than in equity markets, suggesting that prediction errors from GARCH and GJR models do not differ substantially. Second, models with the lowest ALF values employ $\mathcal{N}$ or $sk\mathcal{N}$ distributions in some countries, even at the 99% confidence level.

[4] The Tables and Figures corresponding to the FX market analysis are provided in an Appendix available upon request.

These results are consistent with the backtesting evidence and indicate that modeling leverage effects is not critical for VaR forecasting in FX markets. Moreover, in some cases, heavy-tailed distributions are not required.

6 Conclusions

This study evaluates market risk in equity markets of HI and Latam countries using VaR and ES as risk measures. To this end, GARCH and GJR models are estimated and compared under a broad set of heavy-tailed and asymmetric return distributions, and the best-performing specifications are selected using backtesting procedures and the Model Confidence Set of Hansen et al. (2011).

Backtesting results for VaR can be summarized as follows. At the 95% level are GJR-$sk\mathcal{N}$ (Canada, the United States, and Denmark), GJR-$sk\mathcal{NRIG}$ (Norway and the United Kingdom), GARCH-$sk\mathcal{NRIG}$ (Australia), GJR-$\mathcal{GH}sk\mathcal{S}$ (Switzerland), and GJR-$sk\mathcal{S}$ (Japan and Europe). At the 99% level, the selected models are GJR-$sk\mathcal{NRIG}$ (Canada, Australia, and Switzerland), GJR-$\mathcal{GH}sk\mathcal{S}$ (the United States and Japan), GJR-$sk\mathcal{S}$ (Denmark and the United Kingdom), GARCH-$\mathcal{GH}sk\mathcal{S}$ (Norway), and GARCH-$sk\mathcal{NIG}$ (Europe).

Backtesting results for ES can be summarized as follows. At the 95% level are GJR-$sk\mathcal{N}$ (Argentina and Peru), GARCH-$\mathcal{GH}sk\mathcal{S}$ (Brazil), GARCH-$sk\mathcal{N}$ (Chile and Colombia), and GJR-$sk\mathcal{NRIG}$ (Mexico). At the 99% level, the selected models are GARCH-$sk\mathcal{S}$ (Argentina), GARCH-$sk\mathcal{NRIG}$ (Brazil), GJR-$sk\mathcal{NIG}$ (Chile and Colombia), GJR-$sk\mathcal{NRIG}$ (Mexico), and GARCH-$\mathcal{GH}sk\mathcal{S}$ (Peru). These results indicate that, at the 95% level, capturing distributional asymmetry is essential, as all selected specifications exhibit this feature. At the 99% level, all selected distributions are heavy-tailed, indicating that, in addition to asymmetry, tail thickness must also be modeled.

With respect to volatility modeling, incorporating leverage effects is important for most HI countries and for about half of the Latam countries. Moreover, in HI markets, volatility specification exerts a stronger influence on VaR forecasts than the choice of return distribution. While this pattern also holds for roughly half of the Latam markets, in the remaining cases the evidence suggests that return distributional assumptions play a more prominent role than volatility modeling.

For ES forecasts, the selected models consistently employ asymmetric heavy-tailed distributions, with $\mathcal{GH}sk\mathcal{S}$ standing out due to its ability to fit densities with pronounced left-tail asymmetry. In contrast to VaR, volatility specification plays a secondary role for ES, as GARCH models are sufficient in most HI and Latam markets. Taken together, these findings indicate that while VaR is more sensitive to volatility dynamics, ES depends primarily on the shape of the lower tail of the return distribution, making it a more suitable measure for assessing extreme losses.

The study also compares VaR forecasts using the MCS approach of Hansen et al. (2011). In equity markets, GJR models exhibit superior predictive ability at both confidence levels across all markets, with the exception of Mexico. In addition, at the 99% confidence level, the best-performing VaR models rely on heavy-tailed distributions in all cases.

A natural extension of this work would be to evaluate alternative members of the GARCH family for forecasting both risk measures. For instance, Markov-switching GARCH models, which allow for multiple volatility regimes, could be considered (Ataurima Arellano and Rodríguez, 2020).

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Table 1. Value-at-Risk (VaR) Backtesting Statistics

VaR Backtest Statistic	Notation	Null Hypothesis
Binomial Test: $Z_{bin} = \frac{x-pT}{\sqrt{p(1-p)T}} \xrightarrow{d} \mathcal{N}(0,1)$	• T. Number of observations • p: Probability of observing a failure if the model is correct • x: Number of failures	The proportion of VaR violations is not significantly different from $1-\alpha$ across the days.
POF Test: $LR_{POF} = -2\ln\left(\frac{p^x(1-p)^{T-x}}{(\frac{x}{T})^x(1-\frac{x}{T})^{T-x}}\right) \xrightarrow{d} \chi_1^2$	• T: Number of observations • p: Probability of observing a failure if the model is correct • x: Number of failures	The proportion of VaR violations is not significantly different from $1-\alpha$ across the days.
Traffic Light Test: $P(X \leq x \mid T,p) = F(x \mid T,p)$	• T: Number of observations • p: Probability of observing a failure if the model is correct • $F(x \mid T,p)$: Binomial cumulative distribution with parameters T and p. A model is considered valid when $F(x \mid T,p) \leq 0.95$ (green zone). If $0.95 < F(x \mid T,p) \leq 0.9999$, the model is in the yellow zone. Finally, when $0.9999 < F(x \mid T,p)$, the model is in the red zone.	The proportion of VaR violations is not significantly higher than $1-\alpha$ across the days.
CCI Test: $LR_{CCI} =$ $-2\log\left(\frac{(1-\pi)^{T_{00}+T_{10}}\pi^{T_{01}+T_{11}}}{(1-\pi_{01})^{T_{00}}\pi_{01}^{T_{01}}(1-\pi_{11})^{T_{10}}\pi_{11}^{T_{11}}}\right) \xrightarrow{d} \chi_1^2$	• $T_{ij}, i,j = 0,1$: Number of observations with a j succeeding an i • $\pi = \frac{T_{01}+T_{11}}{T_{00}+T_{01}+T_{10}+T_{11}}$: Probability of having a failure on period t • $\pi_{01} = \frac{T_{01}}{T_{00}+T_{01}}$: Number of periods with no failures followed by a period with failures • $\pi_{11} = \frac{T_{11}}{T_{10}+T_{11}}$: Number of periods with failures followed by a period with failures	The VaR violations are uncorrelated with past violations.
CC Test: $LR_{CC} = LR_{CCI} + LR_{POF} \xrightarrow{d} \chi_2^2$	• LR_{CCI}: CCI Test Statistic • LR_{POF}: POF Test Statistic	The probability of having a violation is $1-\alpha$ for every period t, given all the information available up to period $t-1$.
DQ Test: $DQ = \frac{\hat{\delta}' Z' Z \hat{\delta}}{p(1-p)} \xrightarrow{d} \chi_{M+2}^2$, where $\hat{\delta} \equiv (\hat{\delta}_0, ..., \hat{\delta}_{M+1})$ are the OLS parameter estimates of the following regression: $Hit_t^\alpha = \delta_0 + \sum_{m=1}^{M} \delta_m Hit_{t-m}^\alpha + \delta_{M+1} VaR_{t-1}^\alpha + \epsilon_t$	• p: Probability of observing a failure if the model is correct • $Hit_t^\alpha \equiv I_t^\alpha - p$: Response variable of the regression • Z: Data matrix with the observations for the $M+2$ explanatory variables of the regression • VaR_{t-1}^α: VaR predictions at a risk level α with a lag of one period	The probability of having a violation is $1-\alpha$ for every period t, given all the information available up to period $t-1$.

Table 2. Expected Shortfall (ES) Backtesting Statistics

ES Backtest Statistic	Notation	Null Hypothesis
One-Sided and Two-Sided Exceedance Residuals (ER 1s and ER 2s) Tests: $T_{ER} = \frac{1}{m} \sum_{t=1}^{T} \frac{r_t - ES_t^{\alpha}}{\hat{h}_t} I\{r_t > VaR_t^{\alpha}\}$	• T: Number of observations • x: Number of failures • ES_t^{α}: ES prediction at a risk level α for the time t • VaR_t^{α}: VaR prediction at a risk level α for the time t • $\hat{h}_t$: Variance prediction for time t	$H_0^{2s} : \mathbb{E}\left[\frac{r_t - ES_t^{\alpha}}{h_t}\right] = 0$ $H_0^{1s} : \mathbb{E}\left[\frac{r_t - ES_t^{\alpha}}{h_t}\right] \leqslant 0$
Two-Sided Simple (Simple 2s) CC Test: $T_{CC-2S} = T(\frac{1}{T} \sum_{t=1}^{T} g_t V_t)' \hat{\Omega}_T^{-1} (\frac{1}{T} \sum_{t=1}^{T} g_t V_t) \xrightarrow{d} \chi_2^2$, where $g_t = 1$ is a test function.	• T: Number of observations • $V_t = \begin{pmatrix} 1 - \alpha - I\{r_t > VaR_t^{\alpha}\} \\ VaR_t^{\alpha} - ES_t^{\alpha} - \frac{1}{1-\alpha} I\{r_t > VaR_t^{\alpha}\}(VaR_t^{\alpha} - r_t) \end{pmatrix}$ • ES_t^{α}: ES prediction at a risk level α for the time t • VaR_t^{α}: VaR prediction at a risk level α for the time t • $\hat{h}_t$: Variance prediction for time t • $\hat{\Omega}_T$: Consistent estimator of the variance of the vector $g_t V_t$	$H_0^{2s} : \mathbb{E}[g_t' V_t] = 0$
Two-Sided General (General 2s) CC Test: $T_{CC-2S} = T(\frac{1}{T} \sum_{t=1}^{T} g_t V_t)' \hat{\Omega}_T^{-1} (\frac{1}{T} \sum_{t=1}^{T} g_t V_t) \xrightarrow{d} \chi_1^2$, where $g_t = \hat{h_t}^{-1} \left(\frac{ESt^{\alpha} - VaRt^{\alpha}}{1-\alpha}, \quad 1\right)$ is a test function.		$H_0^{2s} : \mathbb{E}[g_t' V_t] = 0$
One-Sided Simple CC (Simple 1s) Test: $T_{CC-1S} = \sqrt{T}^{-1} \hat{\Omega}_T^{-1/2} \sum_{t=1}^{T} g_t V_t \xrightarrow{d} \mathcal{N}(0, I_2)$, where $g_t = 1$ is a test function.		$H_0^{1s} : \mathbb{E}\left[V_{i,t} g_{i,t,l}\right] \geq 0$, for all $t, i = 1, 2, l = 1, 2$
One-Sided General CC (General 2s) Test: $T_{CC-1S} = \sqrt{T}^{-1} \hat{\Omega}_T^{-1/2} \sum_{t=1}^{T} g_t V_t \xrightarrow{d} \mathcal{N}(0, I_4)$, where $g_t = \begin{pmatrix} 1 & \lvert VaR_t^{\alpha} \rvert & 0 & 0 \\ 0 & 0 & 1 & \hat{h_t}^{-1} \end{pmatrix}'$ is a test function.		$H_0^{1s} : \mathbb{E}\left[V_{i,t} g_{i,t,l}\right] \geq 0$, for all $t, i = 1, 2, l = 1, 2$
Two-Sided Strict ESR Test: $T_{S-ESR} = T(\hat{\gamma}_T - (0,1)) \hat{\Omega}_{\gamma}^{-1} (\hat{\gamma}_T - (0,1))' \xrightarrow{d} \chi_2^2$, where $\hat{\gamma}_T = (\hat{\gamma}_1, \hat{\gamma}_2)$ are the parameter estimates of the following joint regression: $r_t = \beta_1 + \beta_2 ES_t^{\alpha} + u_t$ $r_t = \gamma_1 + \gamma_2 ES_t^{\alpha} + v_t$	• T: Number of observations • $\hat{\Omega}_T$: Covariance estimator of the vector $\hat{\gamma}_T$ • ES_t^{α}: ES prediction at a risk level α for the time t	$H_0^{2s} : (\gamma_1, \gamma_2) = (0, 1)$
Two-Sided Auxiliary ESR Test: $T_{S-ESR} = T(\hat{\gamma}_T - (0,1)) \hat{\Omega}_{\gamma}^{-1} (\hat{\gamma}_T - (0,1))' \xrightarrow{d} \chi_2^2$, where $\hat{\gamma}_T = (\hat{\gamma}_1, \hat{\gamma}_2)$ are the parameter estimates of the following joint regression: $r_t = \beta_1 + \beta_2 VaR_t^{\alpha} + u_t$ $r_t = \gamma_1 + \gamma_2 ES_t^{\alpha} + v_t$	• T: Number of observations • $\hat{\Omega}_T$: Covariance estimator of the vector $\hat{\gamma}_T$ • ES_t^{α}: ES prediction at a risk level α for the time t • VaR_t^{α}: VaR prediction at a risk level α for the time t	$H_0^{2s} : (\gamma_1, \gamma_2) = (0, 1)$
One-Sided and Two-Sided Intercept ESR (Int 1s and Int 2s) Tests: $T_{I-ESR} = T(\hat{\gamma}_1 - \gamma_1^0) \hat{\Omega}_{\gamma_1}^{-1} (\hat{\gamma}_1 - \gamma_1^0)' \xrightarrow{d} \chi_1^2$, where $\hat{\gamma}_1$ is the estimated parameter of the following joint regression: $r_t - ES_t^{\alpha} = \beta_1 + \beta_2 ES_t^{\alpha} + u_t$ $r_t - ES_t^{\alpha} = \gamma_1 + v_t$	• T: Number of observations • $\hat{\Omega}_{\gamma_1}$: Covariance estimator of $\hat{\gamma}_1$ • ES_t^{α}: ES prediction at a risk level α for the time t • VaR_t^{α}: VaR prediction at a risk level α for the time t	$H_0^{2s} : \gamma_1 = 0$ $H_0^{1s} : \gamma_1 \leqslant 0$

This Table shows the statistics for the Exceedance Residuals Test of McNeil and Frey (2000); Simple and General Tests of Nolde and Ziegel (2017); Strict, Auxiliary and Intercept Tests of Bayer and Dimitriadis (2020) for both one-sided and two-sided hypoteses. The hypoteses (H_0) for the one-sided tests have a superscript $1s$ and the hypotheses for the two-sided tests have a superscript $2s$.

Table 3. Descriptive Statistics for Stock Markets Returns

Country	Security ID	Start Date	End Date	Obs.	Std.	Min	Max	Skew	Kurt	VaR		ES	
										99%	95%	99%	95%
(a) Stock High Income Markets													
Canada	SPTSX	11/20/2014	02/28/2023	2000	1.02	-13.19	11.28	-1.65	42.42	-2.67	-1.43	-4.91	-2.46
USA	SPX	11/25/2014	02/28/2023	2000	1.18	-12.08	8.70	-0.94	17.31	-3.44	-1.86	-5.15	-3.03
Denmark	KFX	12/15/2014	02/28/2023	2000	1.19	-7.85	5.12	-0.44	5.58	-3.25	-1.95	-4.35	-2.83
Norway	OSEBX	02/05/2015	02/28/2023	2000	1.11	-9.20	5.43	-0.85	9.33	-3.21	-1.69	-4.64	-2.74
Australia	AS51	01/30/2015	02/28/2023	2000	1.01	-10.01	6.49	-1.18	15.40	-2.81	-1.59	-4.57	-2.54
Switzerland	SMI	01/12/2015	02/28/2023	2000	1.01	-10.15	6.76	-1.07	14.70	-2.92	-1.53	-4.42	-2.47
UK	UKX	01/21/2015	02/28/2023	2000	1.05	-11.52	8.66	-0.90	16.20	-3.24	-1.56	-4.45	-2.62
Japan	NKY	07/14/2014	02/28/2023	2000	1.25	-8.24	7.75	-0.06	8.13	-3.60	-1.98	-4.68	-2.96
Europe	SX5E	04/24/2015	02/28/2023	2000	1.27	-13.25	8.82	-0.87	14.30	-3.82	-1.96	-5.37	-3.16
(b) Stock Latam Markets													
Argentina	MERVAL	07/15/2014	02/28/2023	2000	2.59	-47.81	9.65	-3.45	62.58	-6.53	-3.70	-11.14	-5.94
Brazil	IBOV	09/16/2014	02/28/2023	2000	1.64	-16.18	12.83	-1.00	17.12	-4.00	-2.60	-7.07	-3.91
Chile	IPSA	10/17/2014	02/28/2023	2000	1.23	-15.27	9.20	-1.39	28.26	-3.05	-1.76	-5.59	-2.88
Colombia	COLCAP	05/29/2014	02/28/2023	2000	1.21	-16.34	12.42	-1.44	37.72	-3.07	-1.72	-5.65	-2.98
Mexico	MEXBOL	12/09/2014	02/28/2023	2000	0.99	-6.68	4.71	-0.44	6.86	-2.54	-1.62	-3.82	-2.37
Peru	SPBLPGPT	12/12/2014	02/28/2023	2000	1.13	-11.13	8.14	-0.64	13.85	-3.27	-1.67	-5.20	-2.79

Note: This Table presents the descriptive statistics for the 2000 out-of-sample observations.

Table 4. Failure Rate and VaR Backtesting for High Income Stock Markets

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
Canada (SPTSX)								
GARCH-$\mathcal{N}$	95%	6.200%	0.014	yellow	0.017	0.060	0.010	0.002
GJR-$sk\mathcal{N}$	95%	5.350%	0.473	green	0.477	0.586	0.670	0.175
GARCH-$sk\mathcal{N}$	95%	5.450%	0.356	green	0.362	0.214	0.305	0.102
GARCH-$sk\mathcal{NRIG}$	95%	5.350%	0.473	green	0.477	0.084	0.175	0.059
GJR-$sk\mathcal{S}$	95%	5.500%	0.305	green	0.312	0.422	0.435	0.054
GARCH-$\mathcal{N}$	99%	2.400%	0.000	red	0.000	0.139	0.000	0.000
GJR-$sk\mathcal{NRIG}$	99%	1.050%	0.822	green	0.824	0.217	0.455	0.335
GARCH-$sk\mathcal{S}$	99%	1.300%	0.178	green	0.197	0.348	0.281	0.254
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.292	0.393	0.252
GARCH-$sk\mathcal{NIG}$	99%	1.250%	0.261	green	0.279	0.320	0.340	0.249
GARCH-$sk\mathcal{NRIG}$	99%	1.300%	0.178	green	0.197	0.348	0.281	0.248
GJR-$sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.266	0.434	0.390
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.217	0.455	0.334
GJR-$sk\mathcal{NIG}$	99%	1.050%	0.822	green	0.824	0.217	0.455	0.335
USA (SPX)								
GARCH-$\mathcal{N}$	95%	5.600%	0.218	green	0.227	0.011	0.020	0.007
GJR-$sk\mathcal{N}$	95%	5.050%	0.918	green	0.918	0.097	0.252	0.490
GJR-$sk\mathcal{S}$	95%	5.150%	0.758	green	0.759	0.053	0.146	0.247
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	5.150%	0.758	green	0.759	0.053	0.146	0.254
GJR-$sk\mathcal{NIG}$	95%	4.950%	0.918	green	0.918	0.078	0.210	0.237
GJR-$sk\mathcal{NRIG}$	95%	4.850%	0.758	green	0.757	0.061	0.166	0.136
GARCH-$\mathcal{N}$	99%	2.700%	0.000	red	0.000	0.248	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.266	0.434	0.245
GJR-$sk\mathcal{S}$	99%	1.300%	0.178	green	0.197	0.348	0.281	0.278
GJR-$sk\mathcal{NIG}$	99%	1.150%	0.500	green	0.510	0.266	0.434	0.244
GJR-$sk\mathcal{NRIG}$	99%	1.250%	0.261	green	0.279	0.320	0.340	0.284
Denmark (KFX)								
GARCH-$\mathcal{N}$	95%	5.650%	0.182	green	0.191	0.150	0.151	0.592
GJR-$sk\mathcal{N}$	95%	5.000%	1.000	green	1.000	0.626	0.888	0.835
GARCH-$\mathcal{S}$	95%	5.700%	0.151	green	0.160	0.166	0.142	0.530
GARCH-$\mathcal{NIG}$	95%	5.700%	0.151	green	0.160	0.166	0.142	0.530
GARCH-$\mathcal{NRIG}$	95%	5.650%	0.182	green	0.191	0.150	0.151	0.553
GJR-$\mathcal{N}$	95%	5.700%	0.151	green	0.160	0.837	0.364	0.683

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 4. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GJR-$\mathcal{S}$	95%	5.700%	0.151	green	0.160	0.517	0.302	0.506
GJR-$\mathcal{NIG}$	95%	5.650%	0.182	green	0.191	0.286	0.240	0.463
GJR-$\mathcal{NRIG}$	95%	5.650%	0.182	green	0.191	0.286	0.240	0.462
GARCH-$sk\mathcal{N}$	95%	5.100%	0.837	green	0.838	0.218	0.459	0.786
GARCH-$sk\mathcal{S}$	95%	5.400%	0.412	green	0.418	0.354	0.468	0.901
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	5.450%	0.356	green	0.362	0.205	0.296	0.775
GARCH-$sk\mathcal{NIG}$	95%	5.200%	0.682	green	0.683	0.259	0.486	0.819
GARCH-$sk\mathcal{NRIG}$	95%	5.200%	0.682	green	0.683	0.259	0.486	0.821
GJR-$sk\mathcal{S}$	95%	5.300%	0.538	green	0.542	0.449	0.623	0.783
GJR-$\mathcal{GH}$-$sk\mathcal{S}$	95%	5.300%	0.538	green	0.542	0.449	0.623	0.784
GJR-$sk\mathcal{NIG}$	95%	5.250%	0.608	green	0.611	0.476	0.681	0.819
GJR-$sk\mathcal{NRIG}$	95%	5.250%	0.608	green	0.611	0.476	0.681	0.818
GARCH-$\mathcal{N}$	99%	1.900%	0.000	red	0.000	0.753	0.001	0.000
GJR-$sk\mathcal{S}$	99%	1.000%	1.000	green	1.000	0.525	0.817	0.974
GJR-$\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.581
GJR-$\mathcal{NIG}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.608
GJR-$\mathcal{NRIG}$	99%	1.200%	0.369	green	0.383	0.292	0.393	0.352
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.950%	0.822	green	0.821	0.546	0.812	0.987
GJR-$sk\mathcal{NIG}$	99%	0.950%	0.822	green	0.821	0.546	0.812	0.987
GJR-$sk\mathcal{NRIG}$	99%	0.950%	0.822	green	0.821	0.546	0.812	0.988
Norway (OSEBX)								
GARCH-$\mathcal{N}$	95%	6.100%	0.024	yellow	0.029	0.555	0.077	0.145
GJR-$sk\mathcal{NRIG}$	95%	5.400%	0.412	green	0.418	0.709	0.671	0.777
GARCH-$sk\mathcal{N}$	95%	5.650%	0.182	green	0.191	0.299	0.248	0.222
GARCH-$sk\mathcal{NIG}$	95%	5.600%	0.218	green	0.227	0.276	0.266	0.231
GARCH-$sk\mathcal{NRIG}$	95%	5.600%	0.218	green	0.227	0.276	0.266	0.232
GJR-$sk\mathcal{N}$	95%	5.450%	0.356	green	0.362	0.980	0.660	0.768
GJR-$sk\mathcal{S}$	95%	5.500%	0.305	green	0.312	0.642	0.539	0.681
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	5.600%	0.218	green	0.227	0.907	0.478	0.722
GJR-$sk\mathcal{NIG}$	95%	5.450%	0.356	green	0.362	0.675	0.605	0.727
GARCH-$\mathcal{N}$	99%	2.300%	0.000	red	0.000	0.111	0.000	0.000
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.030
GARCH-$sk\mathcal{NIG}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.015
GARCH-$sk\mathcal{NRIG}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.015

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 4. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GJR-$\mathcal{GH}$-sk$\mathcal{S}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.027
GJR-sk$\mathcal{NIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.027
GJR-sk$\mathcal{NRIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.026
Australia (AS51)								
GARCH-$\mathcal{N}$	95%	5.650%	0.182	green	0.191	0.033	0.044	0.173
GARCH-sk$\mathcal{NRIG}$	95%	5.150%	0.758	green	0.759	0.021	0.066	0.252
GARCH-sk$\mathcal{N}$	95%	5.150%	0.758	green	0.759	0.021	0.066	0.222
GARCH-sk$\mathcal{S}$	95%	5.200%	0.682	green	0.683	0.024	0.072	0.223
GARCH-sk$\mathcal{NIG}$	95%	5.200%	0.682	green	0.683	0.024	0.072	0.223
GJR-sk$\mathcal{N}$	95%	5.400%	0.412	green	0.418	0.041	0.090	0.341
GJR-sk$\mathcal{S}$	95%	5.450%	0.356	green	0.362	0.047	0.092	0.335
GJR-$\mathcal{GH}$sk$\mathcal{S}$	95%	5.450%	0.356	green	0.362	0.047	0.092	0.329
GJR-sk$\mathcal{NIG}$	95%	5.350%	0.473	green	0.477	0.036	0.087	0.315
GJR-sk$\mathcal{NRIG}$	95%	5.350%	0.473	green	0.477	0.036	0.087	0.316
GARCH-$\mathcal{N}$	99%	2.800%	0.000	red	0.000	0.293	0.000	0.000
GJR-sk$\mathcal{NRIG}$	99%	1.350%	0.116	green	0.135	0.053	0.050	0.012
Switzerland (SMI)								
GARCH-$\mathcal{N}$	95%	5.050%	0.918	green	0.918	0.038	0.116	0.017
GJR-$\mathcal{GH}$sk$\mathcal{S}$	95%	4.950%	0.918	green	0.918	0.596	0.864	0.592
GARCH-$\mathcal{S}$	95%	5.450%	0.356	green	0.362	0.100	0.170	0.105
GARCH-$\mathcal{NIG}$	95%	5.300%	0.538	green	0.542	0.071	0.163	0.086
GARCH-$\mathcal{NRIG}$	95%	5.300%	0.538	green	0.542	0.071	0.163	0.086
GJR-$\mathcal{N}$	95%	5.250%	0.608	green	0.611	0.811	0.854	0.532
GJR-$\mathcal{S}$	95%	5.500%	0.305	green	0.312	0.999	0.600	0.359
GJR-$\mathcal{NIG}$	95%	5.350%	0.473	green	0.477	0.886	0.769	0.481
GJR-$\mathcal{NRIG}$	95%	5.350%	0.473	green	0.477	0.886	0.769	0.480
GJR-sk$\mathcal{N}$	95%	4.500%	0.305	green	0.297	0.633	0.518	0.584
GJR-sk$\mathcal{S}$	95%	4.950%	0.918	green	0.918	0.596	0.864	0.590
GJR-sk$\mathcal{NIG}$	95%	4.800%	0.682	green	0.680	0.514	0.742	0.626
GJR-sk$\mathcal{NRIG}$	95%	4.750%	0.608	green	0.605	0.483	0.684	0.632
GARCH-$\mathcal{N}$	99%	2.050%	0.000	red	0.000	0.863	0.000	0.001
GJR-sk$\mathcal{NRIG}$	99%	0.950%	0.822	green	0.821	0.546	0.812	0.990
GARCH-$\mathcal{S}$	99%	1.350%	0.116	green	0.135	0.390	0.227	0.556
GARCH-$\mathcal{NIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.771
GARCH-$\mathcal{NRIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.771

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 4. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GARCH-$sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.963
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.100%	0.653	green	0.658	0.484	0.710	0.982
GARCH-$sk\mathcal{NIG}$	99%	1.100%	0.653	green	0.658	0.484	0.710	0.982
GARCH-$sk\mathcal{NRIG}$	99%	1.100%	0.653	green	0.658	0.484	0.710	0.982
GJR-$sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.989
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.850%	0.500	green	0.489	0.589	0.680	0.974
GJR-$sk\mathcal{NIG}$	99%	0.900%	0.653	green	0.648	0.567	0.765	0.985
UK (UKX)								
GARCH-$\mathcal{N}$	95%	5.450%	0.356	green	0.362	0.002	0.007	0.004
GJR-$sk\mathcal{NRIG}$	95%	5.150%	0.758	green	0.759	0.248	0.490	0.646
GJR-$sk\mathcal{N}$	95%	5.200%	0.682	green	0.683	0.133	0.298	0.458
GJR-$sk\mathcal{S}$	95%	5.350%	0.473	green	0.477	0.084	0.175	0.251
GJR-$sk\mathcal{NIG}$	95%	5.200%	0.682	green	0.683	0.133	0.298	0.452
GARCH-$\mathcal{N}$	99%	2.500%	0.000	red	0.000	0.812	0.000	0.000
GJR-$sk\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.669
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.250%	0.261	green	0.279	0.320	0.340	0.392
GARCH-$sk\mathcal{NIG}$	99%	1.250%	0.261	green	0.279	0.320	0.340	0.393
GARCH-$sk\mathcal{NRIG}$	99%	1.250%	0.261	green	0.279	0.320	0.340	0.394
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.666
GJR-$sk\mathcal{NIG}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.667
GJR-$sk\mathcal{NRIG}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.669
Japan (NKY)								
GARCH-$\mathcal{N}$	95%	5.200%	0.682	green	0.683	0.133	0.298	0.074
GJR-$sk\mathcal{S}$	95%	5.000%	1.000	green	1.000	0.372	0.672	0.806
GJR-$\mathcal{N}$	95%	5.300%	0.538	green	0.542	0.075	0.171	0.260
GJR-$\mathcal{S}$	95%	5.600%	0.218	green	0.227	0.143	0.165	0.249
GJR-$\mathcal{NIG}$	95%	5.550%	0.259	green	0.267	0.130	0.171	0.251
GJR-$\mathcal{NRIG}$	95%	5.550%	0.259	green	0.267	0.130	0.171	0.256
GARCH-$sk\mathcal{N}$	95%	4.800%	0.682	green	0.680	0.054	0.144	0.100
GARCH-$sk\mathcal{S}$	95%	4.950%	0.918	green	0.918	0.078	0.210	0.149
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	4.950%	0.918	green	0.918	0.078	0.210	0.154
GJR-$sk\mathcal{N}$	95%	4.650%	0.473	green	0.468	0.423	0.557	0.777
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	4.800%	0.682	green	0.680	0.274	0.505	0.739
GJR-$sk\mathcal{NIG}$	95%	4.650%	0.473	green	0.468	0.740	0.727	0.918
GJR-$sk\mathcal{NRIG}$	95%	4.600%	0.412	green	0.406	0.704	0.658	0.807

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 4. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GARCH-$\mathcal{N}$	99%	2.400%	0.000	red	0.000	0.462	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	1.000%	1.000	green	1.000	0.525	0.817	0.925
GARCH-$\mathcal{NIG}$	99%	1.350%	0.116	green	0.135	0.053	0.050	0.014
GARCH-$\mathcal{NRIG}$	99%	1.300%	0.178	green	0.197	0.348	0.281	0.374
GARCH-$sk\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.292	0.393	0.446
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.100%	0.653	green	0.658	0.241	0.456	0.565
GARCH-$sk\mathcal{NIG}$	99%	1.200%	0.369	green	0.383	0.292	0.393	0.448
GARCH-$sk\mathcal{NRIG}$	99%	1.200%	0.369	green	0.383	0.292	0.393	0.456
GJR-$sk\mathcal{S}$	99%	1.250%	0.261	green	0.279	0.426	0.406	0.639
GJR-$sk\mathcal{NIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.887
GJR-$sk\mathcal{NRIG}$	99%	1.100%	0.653	green	0.658	0.484	0.710	0.847
Europe (SX5E)								
GARCH-$\mathcal{N}$	95%	5.600%	0.218	green	0.227	0.068	0.091	0.022
GJR-$sk\mathcal{S}$	95%	4.950%	0.918	green	0.918	0.963	0.994	0.530
GJR-$\mathcal{N}$	95%	5.300%	0.538	green	0.542	0.553	0.696	0.096
GJR-$\mathcal{S}$	95%	5.750%	0.124	green	0.132	0.579	0.276	0.245
GJR-$\mathcal{NIG}$	95%	5.600%	0.218	green	0.227	0.763	0.460	0.225
GJR-$\mathcal{NRIG}$	95%	5.450%	0.356	green	0.362	0.655	0.598	0.213
GARCH-$sk\mathcal{NIG}$	95%	5.200%	0.682	green	0.683	0.489	0.724	0.072
GARCH-$sk\mathcal{NRIG}$	95%	5.200%	0.682	green	0.683	0.489	0.724	0.072
GJR-$sk\mathcal{N}$	95%	4.600%	0.412	green	0.406	0.395	0.493	0.088
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	5.100%	0.837	green	0.838	0.719	0.918	0.467
GJR-$sk\mathcal{NIG}$	95%	4.650%	0.473	green	0.468	0.481	0.599	0.203
GJR-$sk\mathcal{NRIG}$	95%	4.650%	0.473	green	0.468	0.481	0.599	0.201
GARCH-$\mathcal{N}$	99%	2.250%	0.000	red	0.000	0.150	0.000	0.000
GARCH-$sk\mathcal{NIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.687
GARCH-$sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.611
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.636
GARCH-$sk\mathcal{NRIG}$	99%	1.100%	0.653	green	0.658	0.484	0.710	0.661
GJR-$sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.006
GJR-$\mathcal{GH}$-$sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.523
GJR-$sk\mathcal{NIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.529
GJR-$sk\mathcal{NRIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.539

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 5. Failure Rate and VaR Backtesting for Latam Stock Markets

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
Argentina (MERVAL)								
GARCH-$\mathcal{N}$	95%	5.250%	0.608	green	0.611	0.292	0.504	0.492
GJR-$sk\mathcal{N}$	95%	4.950%	0.918	green	0.918	0.346	0.638	0.682
GARCH-$\mathcal{S}$	95%	5.750%	0.124	green	0.132	0.095	0.080	0.052
GARCH-$\mathcal{NIG}$	95%	5.650%	0.182	green	0.191	0.076	0.088	0.062
GJR-$\mathcal{N}$	95%	5.400%	0.412	green	0.418	0.367	0.479	0.430
GJR-$\mathcal{NIG}$	95%	5.550%	0.259	green	0.267	0.254	0.282	0.410
GJR-$\mathcal{NRIG}$	95%	5.500%	0.305	green	0.312	0.233	0.295	0.420
GARCH-$sk\mathcal{N}$	95%	5.050%	0.918	green	0.918	0.209	0.451	0.436
GJR-$sk\mathcal{S}$	95%	5.400%	0.412	green	0.418	0.196	0.312	0.414
GJR$\mathcal{GH}sk\mathcal{S}$	95%	5.400%	0.412	green	0.418	0.094	0.178	0.271
GJR-$sk\mathcal{NIG}$	95%	5.050%	0.918	green	0.918	0.209	0.451	0.519
GJR-$sk\mathcal{NRIG}$	95%	5.100%	0.837	green	0.838	0.228	0.473	0.539
GARCH-$\mathcal{N}$	99%	1.800%	0.000	yellow	0.001	0.251	0.003	0.013
GARCH-$sk\mathcal{S}$	99%	1.000%	1.000	green	1.000	0.525	0.817	0.990
GARCH-$\mathcal{S}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.558
GARCH-$\mathcal{NIG}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.678
GARCH-$\mathcal{NRIG}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.710
GJR-$\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.708
GJR-$\mathcal{NIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.983
GJR-$\mathcal{NRIG}$	99%	1.050%	0.822	green	0.824	0.504	0.781	0.983
GARCH$\mathcal{GH}sk\mathcal{S}$	99%	0.800%	0.369	green	0.352	0.611	0.570	0.973
GARCH-$sk\mathcal{NIG}$	99%	0.750%	0.261	green	0.240	0.634	0.447	0.944
GARCH-$sk\mathcal{NRIG}$	99%	0.800%	0.369	green	0.352	0.611	0.570	0.971
GJR-$sk\mathcal{S}$	99%	1.000%	1.000	green	1.000	0.525	0.817	0.984
GJR$\mathcal{GH}$-$sk\mathcal{S}$	99%	0.800%	0.369	green	0.352	0.611	0.570	0.975
GJR-$sk\mathcal{NIG}$	99%	0.800%	0.369	green	0.352	0.611	0.570	0.975
GJR-$sk\mathcal{NRIG}$	99%	0.850%	0.500	green	0.489	0.589	0.680	0.970
Brazil (IBOV)								
GARCH-$\mathcal{N}$	95%	4.950%	0.918	green	0.918	0.963	0.994	0.668
GARCH$\mathcal{GH}sk\mathcal{S}$	95%	5.000%	1.000	green	1.000	0.999	1.000	0.373
GARCH-$\mathcal{S}$	95%	5.350%	0.473	green	0.477	0.743	0.736	0.397
GARCH-$\mathcal{NIG}$	95%	5.350%	0.473	green	0.477	0.743	0.736	0.398
GARCH-$\mathcal{NRIG}$	95%	5.250%	0.608	green	0.611	0.814	0.855	0.346

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 5. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GJR-$\mathcal{N}$	95%	5.250%	0.608	green	0.611	0.476	0.681	0.683
GJR-$\mathcal{S}$	95%	5.400%	0.412	green	0.418	0.709	0.671	0.546
GJR-$\mathcal{NIG}$	95%	5.350%	0.473	green	0.477	0.743	0.736	0.557
GJR-$\mathcal{NRIG}$	95%	5.350%	0.473	green	0.477	0.743	0.736	0.557
GARCH-$sk\mathcal{N}$	95%	4.400%	0.218	green	0.209	0.947	0.454	0.450
GARCH-$sk\mathcal{S}$	95%	5.050%	0.918	green	0.918	0.962	0.994	0.408
GARCH-$sk\mathcal{NIG}$	95%	4.950%	0.918	green	0.918	0.963	0.994	0.337
GARCH-$sk\mathcal{NRIG}$	95%	4.950%	0.918	green	0.918	0.963	0.994	0.337
GJR-$sk\mathcal{N}$	95%	4.800%	0.682	green	0.680	0.403	0.647	0.823
GJR-$sk\mathcal{S}$	95%	5.200%	0.682	green	0.683	0.851	0.904	0.612
GJR$\mathcal{GH}sk\mathcal{S}$	95%	5.200%	0.682	green	0.683	0.851	0.904	0.612
GJR-$sk\mathcal{NIG}$	95%	5.000%	1.000	green	1.000	0.626	0.888	0.559
GJR-$sk\mathcal{NRIG}$	95%	5.000%	1.000	green	1.000	0.626	0.888	0.560
GARCH-$\mathcal{N}$	99%	1.500%	0.025	yellow	0.036	0.082	0.025	0.001
GARCH-$sk\mathcal{NRIG}$	99%	0.900%	0.653	green	0.648	0.567	0.765	0.185
GJR-$\mathcal{S}$	99%	1.250%	0.261	green	0.279	0.426	0.406	0.336
GJR-$\mathcal{NIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.382
GJR-$\mathcal{NRIG}$	99%	1.300%	0.178	green	0.197	0.408	0.309	0.382
GARCH$\mathcal{GH}sk\mathcal{S}$	99%	0.900%	0.653	green	0.648	0.567	0.765	0.183
GARCH-$sk\mathcal{NIG}$	99%	0.900%	0.653	green	0.648	0.567	0.765	0.184
GJR-$sk\mathcal{S}$	99%	1.200%	0.369	green	0.383	0.445	0.511	0.326
GJR$\mathcal{GH}sk\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.312
GJR-$sk\mathcal{NIG}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.315
GJR-$sk\mathcal{NRIG}$	99%	1.150%	0.500	green	0.510	0.464	0.616	0.318
Chile (IPSA)								
GARCH-$\mathcal{N}$	95%	5.700%	0.151	green	0.160	0.837	0.364	0.041
GARCH-$sk\mathcal{N}$	95%	5.350%	0.473	green	0.477	0.905	0.771	0.084
GJR-$\mathcal{N}$	95%	5.600%	0.218	green	0.227	0.482	0.376	0.118
GARCH-$sk\mathcal{NRIG}$	95%	5.700%	0.151	green	0.160	0.837	0.364	0.021
GJR-$sk\mathcal{N}$	95%	5.300%	0.538	green	0.542	0.316	0.502	0.046
GARCH-$\mathcal{N}$	99%	2.000%	0.000	red	0.000	0.826	0.000	0.000
GJR-$sk\mathcal{NIG}$	99%	1.450%	0.043	yellow	0.058	0.355	0.108	0.249
GARCH-$sk\mathcal{NIG}$	99%	1.450%	0.043	yellow	0.058	0.440	0.123	0.000

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 5. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
Colombia (COLCAP)								
GARCH-$\mathcal{N}$	95%	6.200%	0.014	yellow	0.017	0.011	0.002	0.005
GARCH-$sk\mathcal{N}$	95%	5.700%	0.151	green	0.160	0.014	0.019	0.047
GARCH-$\mathcal{N}$	99%	2.650%	0.000	red	0.000	0.063	0.000	0.000
GJR-$sk\mathcal{NIG}$	99%	1.500%	0.025	yellow	0.036	0.339	0.071	0.236
GARCH-$sk\mathcal{S}$	99%	1.550%	0.013	yellow	0.022	0.505	0.059	0.122
GARCH$\mathcal{GH}sk\mathcal{S}$	99%	1.550%	0.013	yellow	0.022	0.505	0.059	0.121
GARCH-$sk\mathcal{NIG}$	99%	1.500%	0.025	yellow	0.036	0.339	0.071	0.181
Mexico (MEXBOL)								
GARCH-$\mathcal{N}$	95%	5.750%	0.124	green	0.132	0.502	0.257	0.605
GJR-$sk\mathcal{NRIG}$	95%	5.500%	0.305	green	0.312	0.150	0.212	0.598
GARCH-$sk\mathcal{N}$	95%	5.500%	0.305	green	0.312	0.150	0.212	0.592
GARCH-$sk\mathcal{S}$	95%	5.750%	0.124	green	0.132	0.488	0.254	0.608
GARCH$\mathcal{GH}sk\mathcal{S}$	95%	5.750%	0.124	green	0.132	0.257	0.170	0.434
GARCH-$sk\mathcal{NIG}$	95%	5.550%	0.259	green	0.267	0.138	0.180	0.492
GARCH-$sk\mathcal{NRIG}$	95%	5.550%	0.259	green	0.267	0.138	0.180	0.490
GJR-$sk\mathcal{N}$	95%	5.600%	0.218	green	0.227	0.127	0.150	0.464
GJR-$sk\mathcal{S}$	95%	5.650%	0.182	green	0.191	0.117	0.124	0.426
GJR$\mathcal{GH}sk\mathcal{S}$	95%	5.700%	0.151	green	0.160	0.107	0.102	0.351
GJR-$sk\mathcal{NIG}$	95%	5.550%	0.259	green	0.267	0.138	0.180	0.579
GARCH-$\mathcal{N}$	99%	2.200%	0.000	red	0.000	0.159	0.000	0.000
GJR-$sk\mathcal{NRIG}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.131
GARCH-$sk\mathcal{S}$	99%	1.350%	0.116	green	0.135	0.390	0.227	0.089
GARCH$\mathcal{GH}sk\mathcal{S}$	99%	1.350%	0.116	green	0.135	0.390	0.227	0.084
GARCH-$sk\mathcal{NIG}$	99%	1.350%	0.116	green	0.135	0.390	0.227	0.093
GARCH-$sk\mathcal{NRIG}$	99%	1.350%	0.116	green	0.135	0.390	0.227	0.095
GJR-$sk\mathcal{S}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.127
GJR$\mathcal{GH}sk\mathcal{S}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.121
GJR-$sk\mathcal{NIG}$	99%	1.350%	0.116	green	0.135	0.378	0.222	0.130
Peru (SPBLPGPT)								
GARCH-$\mathcal{N}$	95%	5.450%	0.356	green	0.362	0.007	0.018	0.025
GJR-$sk\mathcal{N}$	95%	5.250%	0.608	green	0.611	0.028	0.078	0.123
GJR-$\mathcal{N}$	95%	5.300%	0.538	green	0.542	0.032	0.083	0.145

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 5. (Continued)

Model	Level	Failure Rate	Binomial	Traffic Light	POF	CCI	CC	DQ
GARCH-$\mathcal{N}$	99%	1.600%	0.007	yellow	0.013	0.014	0.002	0.000
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.018	0.060	0.003
GARCH-$\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.027	0.070	0.005
GARCH-$\mathcal{NIG}$	99%	1.100%	0.653	green	0.658	0.022	0.067	0.004
GARCH-$\mathcal{NRIG}$	99%	1.100%	0.653	green	0.658	0.022	0.067	0.004
GJR-$\mathcal{S}$	99%	1.150%	0.500	green	0.510	0.027	0.070	0.006
GJR-$\mathcal{NIG}$	99%	1.100%	0.653	green	0.658	0.022	0.067	0.004
GJR-$\mathcal{NRIG}$	99%	1.100%	0.653	green	0.658	0.022	0.067	0.004
GARCH-$sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.018	0.060	0.003
GJR-$sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.018	0.060	0.002
GJR$\mathcal{GH}sk\mathcal{S}$	99%	1.050%	0.822	green	0.824	0.018	0.060	0.002

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best models for each confidence level. See Table 1 for details about the test statistics and null hypotheses.

Table 6. ES Backtesting for High Income Stock Markets

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
Canada (SPTSX)											
GARCH-$\mathcal{N}$	95%	0.000	0.002	0.000	0.000	0.000	0.005	0.000	0.000	0.001	0.000
GARCH-$sk\mathcal{NIG}$	95%	0.263	0.547	0.760	0.118	0.501	0.610	0.511	0.656	0.603	0.236
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.533	0.667	1.000	0.187	0.985	0.490	0.983	0.625	0.702	0.374
GARCH-$sk\mathcal{S}$	95%	0.208	0.354	0.492	0.056	0.414	0.300	0.437	0.410	0.429	0.111
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.547	0.592	0.822	0.135	0.969	0.348	0.966	0.652	0.663	0.269
GARCH-$sk\mathcal{NRIG}$	95%	0.174	0.516	0.716	0.110	0.350	0.615	0.398	0.635	0.575	0.220
GJR-$sk\mathcal{S}$	95%	0.150	0.415	0.577	0.084	0.294	0.480	0.352	0.388	0.448	0.168
GJR-$sk\mathcal{NIG}$	95%	0.331	0.618	0.891	0.169	0.624	0.641	0.619	0.578	0.648	0.338
GJR-$sk\mathcal{NRIG}$	95%	0.269	0.590	0.823	0.170	0.513	0.656	0.541	0.566	0.614	0.339
GARCH-$\mathcal{N}$	99%	0.001	0.011	0.001	0.000	0.001	0.000	0.003	0.101	0.027	0.001
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.314	0.576	1.000	0.236	0.536	0.672	0.510	0.963	0.999	0.471
GARCH-$sk\mathcal{S}$	99%	0.228	0.354	0.495	0.167	0.420	0.379	0.439	0.868	0.895	0.334
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.561	0.590	1.000	0.236	0.913	0.613	0.867	0.897	0.954	0.472
GARCH-$sk\mathcal{NIG}$	99%	0.358	0.442	0.792	0.149	0.615	0.478	0.587	0.916	0.959	0.298
GARCH-$sk\mathcal{NRIG}$	99%	0.351	0.409	0.677	0.151	0.606	0.400	0.566	0.915	0.956	0.302
GJR-$sk\mathcal{NIG}$	99%	0.094	0.437	0.782	0.262	0.204	0.548	0.290	0.952	0.962	0.524
GJR-$sk\mathcal{NRIG}$	99%	0.053	0.417	0.717	0.236	0.138	0.528	0.242	0.940	0.943	0.472
USA (SPX)											
GARCH-$\mathcal{N}$	95%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.240	0.909	1.000	0.294	0.449	0.875	0.515	0.792	0.826	0.588
GARCH-$\mathcal{GH}$-$sk\mathcal{S}$	95%	0.250	0.689	1.000	0.206	0.486	0.760	0.506	0.710	0.807	0.411
GARCH-$sk\mathcal{NIG}$	95%	0.075	0.688	0.959	0.207	0.157	0.570	0.198	0.702	0.775	0.414
GJR-$sk\mathcal{NIG}$	95%	0.085	0.688	1.000	0.253	0.184	0.687	0.236	0.659	0.694	0.507
GARCH-$\mathcal{N}$	99%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.511	0.748	1.000	0.260	0.939	0.602	0.938	0.541	0.840	0.520
GARCH-$sk\mathcal{S}$	99%	0.123	0.260	0.444	0.178	0.249	0.343	0.303	0.310	0.371	0.356
GARCH-$sk\mathcal{NIG}$	99%	0.060	0.294	0.515	0.182	0.142	0.431	0.184	0.296	0.412	0.364
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.115	0.599	0.832	0.115	0.237	0.700	0.291	0.450	0.497	0.230
Denmark (KFX)											
GARCH-$\mathcal{N}$	95%	0.000	0.001	0.002	0.001	0.000	0.002	0.000	0.003	0.003	0.001
GARCH-$sk\mathcal{NIG}$	95%	0.186	0.708	0.984	0.236	0.364	0.768	0.420	0.669	0.688	0.472
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.230	0.713	0.991	0.256	0.448	0.766	0.456	0.575	0.589	0.512
GARCH-$sk\mathcal{S}$	95%	0.145	0.468	0.650	0.162	0.298	0.592	0.362	0.498	0.509	0.324

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 6. (Continued)

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.368	0.721	1.000	0.233	0.676	0.666	0.681	0.657	0.695	0.466
GARCH-$sk\mathcal{NRIG}$	95%	0.149	0.630	0.875	0.211	0.292	0.711	0.355	0.615	0.640	0.422
GJR-$sk\mathcal{S}$	95%	0.087	0.466	0.647	0.169	0.196	0.594	0.232	0.388	0.394	0.338
GJR-$sk\mathcal{NIG}$	95%	0.188	0.698	0.970	0.251	0.376	0.766	0.406	0.567	0.567	0.501
GJR-$sk\mathcal{NRIG}$	95%	0.149	0.632	0.878	0.221	0.296	0.723	0.341	0.514	0.507	0.441
GARCH-$\mathcal{N}$	99%	0.000	0.001	0.003	0.001	0.000	0.003	0.005	0.004	0.006	0.001
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.272	0.655	0.910	0.230	0.480	0.689	0.520	0.666	0.682	0.459
GARCH-$sk\mathcal{S}$	99%	0.064	0.370	0.521	0.169	0.152	0.510	0.230	0.485	0.495	0.338
Norway (OSEBX)											
GARCH-$\mathcal{N}$	95%	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.265	0.234	0.325	0.083	0.514	0.237	0.536	0.367	0.342	0.167
GARCH-$sk\mathcal{NIG}$	95%	0.088	0.223	0.310	0.079	0.186	0.336	0.246	0.318	0.323	0.158
GARCH-$sk\mathcal{NRIG}$	95%	0.075	0.208	0.288	0.072	0.164	0.320	0.223	0.296	0.302	0.144
GJR-$\mathcal{GH}$-$sk\mathcal{S}$	95%	0.077	0.220	0.305	0.073	0.167	0.336	0.220	0.292	0.301	0.146
GJR-$sk\mathcal{NIG}$	95%	0.051	0.211	0.294	0.072	0.108	0.335	0.151	0.306	0.290	0.144
GARCH-$\mathcal{N}$	99%	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.435	0.439	0.876	0.287	0.790	0.470	0.777	0.680	0.738	0.574
GARCH-$sk\mathcal{S}$	99%	0.059	0.109	0.179	0.126	0.136	0.189	0.196	0.265	0.295	0.252
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.311	0.277	0.520	0.208	0.578	0.393	0.588	0.455	0.592	0.415
GARCH-$sk\mathcal{NIG}$	99%	0.103	0.158	0.274	0.152	0.218	0.258	0.254	0.316	0.395	0.304
GARCH-$sk\mathcal{NRIG}$	99%	0.067	0.131	0.221	0.134	0.152	0.228	0.196	0.282	0.362	0.269
GJR-$sk\mathcal{S}$	99%	0.413	0.205	0.397	0.230	0.755	0.173	0.744	0.593	0.434	0.459
GJR-$sk\mathcal{NIG}$	99%	0.240	0.307	0.599	0.252	0.447	0.408	0.489	0.651	0.584	0.505
GJR-$sk\mathcal{NRIG}$	99%	0.186	0.280	0.548	0.247	0.351	0.389	0.415	0.644	0.547	0.493
Australia (AS51)											
GARCH-$\mathcal{N}$	95%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.086	0.199	0.293	0.048	0.184	0.322	0.223	0.287	0.330	0.097
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.015	0.178	0.247	0.050	0.039	0.264	0.070	0.235	0.235	0.100
GARCH-$\mathcal{N}$	99%	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.377	0.420	0.917	0.265	0.687	0.454	0.679	0.421	0.594	0.531
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.117	0.188	0.285	0.098	0.251	0.247	0.301	0.249	0.289	0.196
GJR-$sk\mathcal{S}$	99%	0.360	0.198	0.345	0.163	0.644	0.166	0.647	0.359	0.304	0.325
GJR-$sk\mathcal{NIG}$	99%	0.134	0.243	0.449	0.203	0.277	0.352	0.336	0.349	0.352	0.406
GJR-$sk\mathcal{NRIG}$	99%	0.152	0.215	0.383	0.178	0.309	0.295	0.343	0.289	0.318	0.355

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 6. (Continued)

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
Switzerland (SMI)											
GARCH-$\mathcal{N}$	95%	0.000	0.006	0.004	0.001	0.000	0.003	0.000	0.005	0.004	0.002
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.220	0.688	1.000	0.297	0.437	0.801	0.473	0.843	0.864	0.594
GJR-$sk\mathcal{NIG}$	95%	0.213	0.706	0.963	0.461	0.399	0.862	0.451	0.792	0.803	0.923
GARCH-$sk\mathcal{NIG}$	95%	0.130	0.566	0.786	0.292	0.277	0.655	0.337	0.842	0.863	0.583
GARCH-$sk\mathcal{NRIG}$	95%	0.070	0.507	0.704	0.274	0.158	0.537	0.237	0.809	0.821	0.547
GJR-$sk\mathcal{S}$	95%	0.101	0.688	0.929	0.342	0.235	0.844	0.337	0.694	0.697	0.684
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.340	0.690	0.958	0.454	0.593	0.978	0.600	0.829	0.837	0.908
GJR-$sk\mathcal{NRIG}$	95%	0.127	0.653	0.867	0.444	0.280	0.759	0.355	0.787	0.781	0.889
GARCH-$\mathcal{N}$	99%	0.000	0.008	0.005	0.000	0.000	0.002	0.006	0.002	0.002	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.123	0.448	0.660	0.298	0.282	0.398	0.347	0.854	0.872	0.596
GJR-$\mathcal{S}$	99%	0.097	0.232	0.323	0.093	0.256	0.196	0.339	0.331	0.340	0.186
GARCH-$sk\mathcal{S}$	99%	0.225	0.497	0.802	0.114	0.415	0.621	0.468	0.587	0.586	0.228
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.592	0.875	1.000	0.255	0.954	0.851	0.937	0.756	0.737	0.511
GARCH-$sk\mathcal{NIG}$	99%	0.210	0.525	0.847	0.137	0.398	0.642	0.440	0.650	0.630	0.275
GARCH-$sk\mathcal{NRIG}$	99%	0.080	0.437	0.669	0.105	0.199	0.562	0.306	0.579	0.557	0.209
GJR-$sk\mathcal{S}$	99%	0.125	0.594	0.856	0.205	0.282	0.673	0.356	0.648	0.685	0.410
UK (UKX)											
GARCH-$\mathcal{N}$	95%	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.245	0.347	0.482	0.131	0.466	0.390	0.472	0.466	0.456	0.262
GARCH-N	99%	0.000	0.000	0.000	0.033	0.000	0.000	0.002	0.001	0.035	0.067
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.281	0.415	0.731	0.078	0.538	0.510	0.526	0.526	0.552	0.156
Japan (NKY)											
GARCH-$\mathcal{N}$	95%	0.000	0.001	0.001	0.002	0.000	0.000	0.000	0.000	0.000	0.003
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.309	0.768	1.000	0.547	0.583	0.993	0.615	0.521	0.415	0.905
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.377	0.977	1.000	0.617	0.715	0.907	0.720	0.483	0.426	0.766
GARCH-$sk\mathcal{NIG}$	95%	0.075	0.691	0.863	0.514	0.175	0.698	0.226	0.381	0.315	0.972
GJR-$sk\mathcal{S}$	95%	0.204	0.750	1.000	0.425	0.396	0.965	0.424	0.386	0.343	0.849
GJR-$sk\mathcal{NIG}$	95%	0.181	0.686	0.953	0.590	0.347	0.718	0.380	0.384	0.330	0.820
GJR-$sk\mathcal{NRIG}$	95%	0.095	0.590	0.725	0.536	0.213	0.559	0.238	0.366	0.310	0.928
GARCH-$\mathcal{N}$	99%	0.000	0.001	0.001	0.005	0.000	0.000	0.003	0.001	0.001	0.010
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.738	1.000	1.000	0.667	0.609	0.799	0.528	0.889	0.961	0.666
GARCH-$\mathcal{S}$	99%	0.324	0.349	0.485	0.106	0.588	0.307	0.583	0.294	0.332	0.212
GARCH-$\mathcal{NIG}$	99%	0.124	0.238	0.330	0.097	0.246	0.321	0.298	0.229	0.225	0.193

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 6. (Continued)

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
GJR-$\mathcal{S}$	99%	0.261	0.348	0.483	0.243	0.486	0.306	0.481	0.345	0.350	0.487
GJR-$\mathcal{NIG}$	99%	0.110	0.260	0.361	0.213	0.237	0.281	0.249	0.205	0.231	0.425
GJR-$\mathcal{NRIG}$	99%	0.081	0.211	0.293	0.190	0.183	0.260	0.197	0.175	0.211	0.380
GARCH-$sk\mathcal{S}$	99%	0.655	1.000	1.000	0.385	0.778	0.677	0.723	0.719	0.719	0.770
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.839	1.000	1.000	0.660	0.339	0.517	0.246	0.803	0.863	0.680
GARCH-$sk\mathcal{NIG}$	99%	0.700	1.000	1.000	0.447	0.686	0.649	0.636	0.759	0.758	0.893
GARCH-$sk\mathcal{NRIG}$	99%	0.537	0.955	1.000	0.349	0.962	0.712	0.961	0.669	0.626	0.697
GJR-$sk\mathcal{S}$	99%	0.651	1.000	1.000	0.453	0.779	0.552	0.727	0.687	0.698	0.906
GJR-$sk\mathcal{NIG}$	99%	0.431	0.880	1.000	0.498	0.782	0.976	0.784	0.715	0.742	0.996
GJR-$sk\mathcal{NRIG}$	99%	0.369	0.999	1.000	0.427	0.675	0.896	0.658	0.598	0.671	0.854
Europe (SX5E)											
GARCH-$\mathcal{N}$	95%	0.000	0.001	0.000	0.000	0.000	0.002	0.000	0.001	0.001	0.001
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.287	0.871	1.000	0.423	0.554	0.969	0.594	0.749	0.753	0.845
GARCH-$sk\mathcal{S}$	95%	0.191	0.482	0.669	0.194	0.378	0.507	0.432	0.456	0.434	0.388
GARCH-$\mathcal{GH}$-$sk\mathcal{S}$	95%	0.434	0.667	0.927	0.260	0.804	0.504	0.799	0.614	0.601	0.519
GARCH-$sk\mathcal{NIG}$	95%	0.141	0.670	0.931	0.264	0.289	0.747	0.315	0.592	0.577	0.527
GARCH-$sk\mathcal{NRIG}$	95%	0.090	0.557	0.773	0.226	0.192	0.663	0.227	0.522	0.526	0.452
GJR-$sk\mathcal{S}$	95%	0.102	0.688	0.956	0.345	0.208	0.810	0.254	0.607	0.593	0.691
GARCH-$\mathcal{N}$	99%	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.000
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.193	0.670	1.000	0.334	0.361	0.691	0.381	0.905	0.743	0.667
GARCH-$\mathcal{S}$	99%	0.211	0.114	0.158	0.174	0.415	0.066	0.434	0.246	0.190	0.347
GARCH-$sk\mathcal{S}$	99%	0.093	0.389	0.580	0.306	0.205	0.523	0.272	0.674	0.517	0.613
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.050	0.495	0.688	0.160	0.125	0.495	0.196	0.569	0.611	0.320

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 7. ES Backtesting for Latam Stock Markets

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
Argentina (MERVAL)											
GARCH-$\mathcal{N}$	95%	0.003	0.094	0.090	0.040	0.003	0.173	0.015	0.108	0.113	0.079
GARCH-$sk\mathcal{NIG}$	95%	0.292	0.745	1.000	0.253	0.513	0.793	0.517	0.809	0.786	0.505
GJR-$sk\mathcal{NIG}$	95%	0.334	0.811	0.956	0.313	0.560	0.836	0.536	0.806	0.786	0.626
GARCH-$sk\mathcal{S}$	95%	0.194	0.492	0.683	0.178	0.370	0.560	0.417	0.645	0.582	0.357
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.411	0.659	0.915	0.250	0.686	0.643	0.646	0.794	0.738	0.501
GARCH-$sk\mathcal{NRIG}$	95%	0.230	0.679	0.943	0.233	0.424	0.754	0.458	0.765	0.754	0.466
GJR-$sk\mathcal{S}$	95%	0.328	0.609	0.845	0.230	0.568	0.617	0.513	0.645	0.620	0.460
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.475	0.748	1.000	0.268	0.774	0.671	0.652	0.729	0.722	0.537
GJR-$sk\mathcal{NRIG}$	95%	0.307	0.750	1.000	0.282	0.527	0.796	0.492	0.760	0.748	0.565
GARCH-$\mathcal{N}$	99%	0.016	0.158	0.220	0.027	0.017	0.020	0.123	0.127	0.262	0.055
GJR-$sk\mathcal{S}$	99%	0.323	0.632	0.951	0.245	0.560	0.691	0.427	0.746	0.735	0.491
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.168	0.370	0.564	0.306	0.353	0.308	0.375	0.728	0.833	0.612
GARCH-$\mathcal{S}$	99%	0.466	0.515	0.760	0.145	0.713	0.418	0.539	0.444	0.459	0.291
GARCH-$\mathcal{NIG}$	99%	0.313	0.501	0.730	0.167	0.537	0.546	0.445	0.573	0.471	0.334
GARCH-$\mathcal{NRIG}$	99%	0.179	0.474	0.675	0.125	0.351	0.578	0.373	0.447	0.439	0.249
GJR-$\mathcal{S}$	99%	0.169	0.493	0.699	0.198	0.357	0.593	0.376	0.692	0.582	0.395
GJR-$\mathcal{NIG}$	99%	0.093	0.480	0.675	0.173	0.230	0.603	0.321	0.582	0.552	0.347
GJR-$\mathcal{NRIG}$	99%	0.063	0.453	0.633	0.157	0.178	0.579	0.297	0.577	0.514	0.315
GARCH-$sk\mathcal{S}$	99%	0.352	0.668	1.000	0.209	0.580	0.718	0.483	0.562	0.538	0.417
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.258	0.398	0.705	0.288	0.470	0.333	0.429	0.575	0.614	0.577
GARCH-$sk\mathcal{NIG}$	99%	0.116	0.293	0.407	0.195	0.268	0.175	0.334	0.586	0.579	0.390
GARCH-$sk\mathcal{NRIG}$	99%	0.105	0.315	0.505	0.201	0.250	0.267	0.328	0.577	0.539	0.402
GJR-$sk\mathcal{NIG}$	99%	0.106	0.325	0.481	0.231	0.254	0.276	0.340	0.744	0.734	0.462
GJR-$sk\mathcal{NRIG}$	99%	0.097	0.349	0.646	0.218	0.250	0.370	0.325	0.692	0.677	0.436
Brazil (IBOV)											
GARCH-$\mathcal{N}$	95%	0.000	0.041	0.057	0.015	0.000	0.028	0.005	0.064	0.064	0.031
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.077	0.443	0.822	0.139	0.182	0.467	0.266	0.467	0.520	0.278
GARCH-$sk\mathcal{NIG}$	95%	0.050	0.429	0.798	0.136	0.136	0.416	0.216	0.465	0.496	0.271
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.092	0.369	0.570	0.131	0.205	0.493	0.267	0.460	0.480	0.261
GARCH-$\mathcal{N}$	99%	0.000	0.038	0.053	0.030	0.004	0.062	0.037	0.077	0.077	0.061
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.005	0.327	0.579	0.113	0.023	0.144	0.117	0.450	0.502	0.226
GARCH-$\mathcal{S}$	99%	0.009	0.203	0.328	0.059	0.054	0.318	0.148	0.314	0.349	0.118
GARCH-$\mathcal{NRIG}$	99%	0.005	0.182	0.290	0.051	0.035	0.290	0.124	0.287	0.325	0.101

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 7. (Continued)

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
GJR-$\mathcal{S}$	99%	0.003	0.144	0.223	0.052	0.027	0.251	0.119	0.252	0.296	0.104
GJR-$\mathcal{NIG}$	99%	0.008	0.128	0.195	0.074	0.036	0.220	0.121	0.229	0.260	0.147
GJR-$\mathcal{NRIG}$	99%	0.007	0.122	0.185	0.073	0.032	0.213	0.115	0.218	0.236	0.145
GARCH-$sk\mathcal{N}$	99%	0.003	0.064	0.089	0.053	0.003	0.128	0.048	0.110	0.123	0.106
GARCH-$sk\mathcal{S}$	99%	0.010	0.259	0.440	0.095	0.051	0.314	0.144	0.397	0.421	0.190
GARCH-$sk\mathcal{NIG}$	99%	0.000	0.269	0.464	0.101	0.007	0.111	0.088	0.419	0.443	0.201
GARCH-$sk\mathcal{NRIG}$	99%	0.001	0.254	0.434	0.099	0.008	0.102	0.086	0.411	0.423	0.198
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.009	0.239	0.407	0.054	0.036	0.360	0.128	0.302	0.409	0.108
Chile (IPSA)											
GARCH-$\mathcal{N}$	95%	0.000	0.012	0.006	0.003	0.000	0.030	0.001	0.012	0.011	0.005
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.046	0.103	0.143	0.041	0.122	0.098	0.187	0.227	0.149	0.082
GARCH-$\mathcal{N}$	99%	0.000	0.019	0.018	0.009	0.000	0.003	0.016	0.019	0.032	0.019
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.103	0.175	0.244	0.070	0.203	0.136	0.273	0.248	0.253	0.140
GJR-$\mathcal{S}$	99%	0.071	0.127	0.177	0.085	0.157	0.074	0.235	0.187	0.187	0.170
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.110	0.191	0.276	0.059	0.231	0.142	0.319	0.227	0.235	0.118
GJR-$sk\mathcal{S}$	99%	0.077	0.145	0.201	0.063	0.178	0.100	0.253	0.215	0.215	0.126
Colombia (COLCAP)											
GARCH-$\mathcal{N}$	95%	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.116	0.060	0.076	0.012	0.228	0.034	0.294	0.069	0.072	0.025
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.144	0.057	0.063	0.016	0.291	0.020	0.330	0.061	0.065	0.031
GARCH-$sk\mathcal{NIG}$	95%	0.071	0.061	0.067	0.016	0.155	0.033	0.208	0.062	0.062	0.032
GARCH-N	99%	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.016	0.016	0.001
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.578	0.392	0.545	0.168	0.997	0.136	0.992	0.903	0.826	0.337
GARCH-$sk\mathcal{S}$	99%	0.327	0.286	0.397	0.092	0.583	0.130	0.599	0.802	0.740	0.184
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.641	0.394	0.547	0.060	0.887	0.136	0.845	0.841	0.797	0.121
Mexico (MEXBOL)											
GARCH-$\mathcal{N}$	95%	0.000	0.001	0.001	0.000	0.000	0.002	0.000	0.000	0.000	0.000
GJR-$\mathcal{GH}$-$sk\mathcal{S}$	95%	0.121	0.269	0.373	0.108	0.249	0.336	0.276	0.316	0.249	0.217
GARCH-$\mathcal{GH}$-$sk\mathcal{S}$	95%	0.108	0.236	0.328	0.076	0.225	0.294	0.241	0.266	0.282	0.153
GARCH-$\mathcal{N}$	99%	0.000	0.002	0.001	0.005	0.000	0.001	0.002	0.002	0.006	0.010
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.424	0.500	0.694	0.159	0.789	0.396	0.797	0.706	0.796	0.318
GARCH-$sk\mathcal{S}$	99%	0.253	0.344	0.477	0.098	0.490	0.368	0.494	0.542	0.605	0.195
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.534	0.545	0.757	0.153	0.991	0.399	0.986	0.692	0.813	0.306
GARCH-$sk\mathcal{NIG}$	99%	0.219	0.350	0.486	0.091	0.425	0.368	0.430	0.515	0.622	0.182

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 7. (Continued)

Model	ES Level	ER 1s	Sim 1s	Gen 1s	Int 1s	ER 2s	Sim 2s	Gen 2s	Str	Aux	Int 2s
GARCH-$sk\mathcal{NRIG}$	99%	0.169	0.299	0.416	0.079	0.324	0.349	0.333	0.477	0.543	0.157
GJR-$sk\mathcal{S}$	99%	0.142	0.310	0.430	0.103	0.296	0.355	0.349	0.520	0.613	0.206
GJR-$sk\mathcal{NIG}$	99%	0.159	0.323	0.449	0.109	0.318	0.358	0.365	0.494	0.625	0.219
GJR-$sk\mathcal{NRIG}$	99%	0.109	0.277	0.385	0.092	0.235	0.338	0.291	0.417	0.555	0.183
Peru (SPBLPGPT)											
GARCH-$\mathcal{N}$	95%	0.000	0.014	0.008	0.002	0.000	0.028	0.000	0.015	0.013	0.004
GARCH-$sk\mathcal{NRIG}$	95%	0.603	0.697	1.000	0.217	0.864	0.502	0.851	0.809	0.788	0.434
GARCH-$\mathcal{S}$	95%	0.651	0.378	0.646	0.100	0.779	0.144	0.778	0.588	0.627	0.200
GARCH-$\mathcal{NIG}$	95%	0.598	0.421	0.700	0.118	0.872	0.198	0.868	0.633	0.639	0.236
GARCH-$\mathcal{NRIG}$	95%	0.512	0.420	0.674	0.128	0.977	0.296	0.980	0.665	0.654	0.256
GJR-$\mathcal{S}$	95%	0.648	0.411	0.741	0.106	0.746	0.169	0.749	0.617	0.700	0.212
GJR-$\mathcal{NIG}$	95%	0.507	0.451	0.798	0.137	0.986	0.337	0.988	0.711	0.691	0.275
GJR-$\mathcal{NRIG}$	95%	0.480	0.453	0.779	0.141	0.910	0.413	0.888	0.705	0.674	0.281
GARCH-$sk\mathcal{S}$	95%	0.772	0.595	1.000	0.173	0.506	0.167	0.455	0.723	0.736	0.346
GARCH-$\mathcal{GH}sk\mathcal{S}$	95%	0.795	0.622	1.000	0.184	0.444	0.166	0.398	0.733	0.747	0.368
GARCH-$sk\mathcal{NIG}$	95%	0.645	0.706	1.000	0.216	0.760	0.452	0.719	0.798	0.790	0.432
GJR-$sk\mathcal{S}$	95%	0.685	0.586	1.000	0.176	0.662	0.310	0.624	0.736	0.747	0.352
GJR-$\mathcal{GH}sk\mathcal{S}$	95%	0.692	0.603	1.000	0.177	0.651	0.310	0.615	0.722	0.761	0.355
GJR-$sk\mathcal{NIG}$	95%	0.591	0.696	1.000	0.203	0.877	0.552	0.886	0.800	0.821	0.407
GJR-$sk\mathcal{NRIG}$	95%	0.480	0.666	1.000	0.208	0.893	0.674	0.898	0.793	0.791	0.416
GARCH-$\mathcal{N}$	99%	0.000	0.006	0.005	0.004	0.000	0.015	0.001	0.005	0.006	0.008
GARCH-$\mathcal{NIG}$	99%	0.102	0.269	0.660	0.121	0.193	0.346	0.239	0.373	0.411	0.242
GARCH-$\mathcal{GH}sk\mathcal{S}$	99%	0.418	0.628	1.000	0.192	0.744	0.667	0.740	0.343	0.310	0.383
GARCH-$\mathcal{S}$	99%	0.323	0.422	1.000	0.128	0.623	0.558	0.620	0.336	0.359	0.256
GARCH-$\mathcal{NRIG}$	99%	0.068	0.215	0.490	0.105	0.132	0.274	0.172	0.358	0.415	0.210
GJR-$\mathcal{S}$	99%	0.256	0.342	0.951	0.063	0.505	0.474	0.540	0.205	0.199	0.126
GJR-$\mathcal{NIG}$	99%	0.070	0.217	0.592	0.063	0.133	0.270	0.179	0.227	0.258	0.127
GARCH-$sk\mathcal{S}$	99%	0.326	0.559	1.000	0.185	0.586	0.603	0.607	0.348	0.336	0.369
GARCH-$sk\mathcal{NIG}$	99%	0.066	0.391	1.000	0.158	0.148	0.223	0.192	0.355	0.414	0.315
GJR-$sk\mathcal{S}$	99%	0.263	0.445	1.000	0.130	0.488	0.476	0.484	0.233	0.231	0.261
GJR-$\mathcal{GH}sk\mathcal{S}$	99%	0.315	0.487	1.000	0.119	0.587	0.523	0.590	0.226	0.235	0.237

The values in blue represent the highest p-values for each test, while the values in red indicate that the model is rejected. The models highlighted in light blue are the best for each confidence level. See Table 2 for details about the test statistics and null hypotheses.

Table 8. Model Confidence Set Results for High Income Stock Markets

Model	Canada (SPTSX)		USA (SPX)		Denmark (KFX)	
	95%	99%	95%	99%	95%	99%
GARCH-$\mathcal{N}$	-	-	0.1263 (0.084)	-	0.1349 (0.215)	0.0420 (0.175)
GARCH-$\mathcal{S}$	-	0.0323 (0.172)	0.1267 (0.149)	0.0404 (0.802)	0.1348 (0.270)	0.0408 (0.340)
GARCH-$\mathcal{NIG}$	-	0.0322 (0.205)	0.1265 (0.151)	0.0402 (0.916)	0.1348 (0.239)	0.0407 (0.421)
GARCH-$\mathcal{NRIG}$	-	0.0322 (0.220)	0.1265 (0.124)	0.0401 (0.960)	0.1348 (0.221)	0.0407 (0.443)
GJR-$\mathcal{N}$	0.0984 (1.000)	0.0313 (1.000)	0.1232 (1.000)	0.0406 (0.711)	0.1329 (1.000)	0.0402 (1.000)
GJR-$\mathcal{S}$	0.0990 (0.650)	0.0309 (1.000)	0.1233 (1.000)	0.0394 (1.000)	0.1331 (1.000)	0.0394 (1.000)
GJR-$\mathcal{NIG}$	0.0989 (0.844)	0.0308 (1.000)	0.1232 (1.000)	0.0393 (1.000)	0.1330 (1.000)	0.0393 (1.000)
GJR-$\mathcal{NRIG}$	0.0989 (0.849)	0.0308 (1.000)	0.1233 (1.000)	0.0393 (1.000)	0.1330 (1.000)	0.0393 (1.000)
GARCH-$\mathcal{S}$	-	0.0320 (0.166)	0.1260 (0.101)	0.0406 (0.671)	0.1345 (0.268)	0.0411 (0.310)
GARCH-$\mathcal{S}$	-	0.0317 (0.589)	0.1262 (0.188)	0.0398 (1.000)	0.1345 (0.334)	0.0406 (0.535)
GARCH-$\mathcal{GH}sk\mathcal{S}$	-	0.0318 (0.621)	0.1264 (0.183)	0.0399 (1.000)	0.1345 (0.355)	0.0406 (0.630)
GARCH-$\mathcal{S}$	-	0.0317 (0.684)	0.1263 (0.181)	0.0398 (1.000)	0.1345 (0.302)	0.0405 (0.799)
GARCH-$\mathcal{S}$	-	0.0317 (0.730)	0.1262 (0.148)	0.0397 (1.000)	0.1345 (0.301)	0.0404 (0.834)
GJR-$sk\mathcal{N}$	0.0985 (1.000)	0.0307 (1.000)	0.1229 (1.000)	0.0393 (1.000)	0.1325 (1.000)	0.0395 (1.000)
GJR-$sk\mathcal{S}$	0.0984 (1.000)	0.0307 (1.000)	0.1229 (1.000)	0.0394 (1.000)	0.1328 (1.000)	0.0394 (1.000)
GJR-$\mathcal{GH}sk\mathcal{S}$	0.0989 (0.797)	0.0309 (1.000)	0.1230 (1.000)	0.0395 (1.000)	0.1328 (1.000)	0.0395 (1.000)
GJR-$sk\mathcal{NIG}$	0.0987 (1.000)	0.0309 (1.000)	0.1231 (1.000)	0.0395 (1.000)	0.1328 (1.000)	0.0395 (1.000)
GJR-$sk\mathcal{NRIG}$	0.0987 (1.000)	0.0308 (1.000)	0.1231 (1.000)	0.0394 (1.000)	0.1328 (1.000)	0.0394 (1.000)

Values of the ALF are reported in the main entries and the MCS p-values are in the parentheses. When the model has a p-value = 1,000, the value is in blue font. In addition, the value is underlined when the model has the minimum ALF, however, this cannot be seen in the Table because not all decimals are shown. The value is omitted when the model does not belong to the Model Confidence Set.

Table 8. (Continued)

Model	Norway (OSEBX)		Australia (AS51)		Switzerland (SMI)	
	95%	99%	95%	99%	95%	99%
GARCH$-\mathcal{N}$	-	-	0.1129 (0.483)	-	0.1125 (0.065)	0.0369 (0.114)
GARCH$-\mathcal{S}$	-	-	0.1136 (0.400)	-	0.1127 (0.142)	0.0356 (0.434)
GARCH$-\mathcal{NIG}$	-	-	0.1135 (0.345)	-	0.1126 (0.125)	0.0356 (0.427)
GARCH$-\mathcal{NRIG}$	-	-	0.1134 (0.353)	-	0.1125 (0.119)	0.0356 (0.443)
GJR$-\mathcal{N}$	0.1234 (1.000)	-	0.1116 (1.000)	-	0.1093 (1.000)	0.0352 (0.951)
GJR$-\mathcal{S}$	0.1236 (0.837)	-	0.1120 (1.000)	-	0.1088 (1.000)	0.0342 (1.000)
GJR$-\mathcal{NIG}$	0.1235 (0.976)	0.0360 (0.237)	0.1119 (1.000)	-	0.1088 (1.000)	0.0341 (1.000)
GJR$-\mathcal{NRIG}$	0.1235 (0.994)	0.0359 (0.283)	0.1119 (1.000)	-	0.1088 (1.000)	0.0341 (1.000)
GARCH-$\mathcal{S}$	-	-	0.1128 (0.592)	-	0.1125 (0.026)	0.0358 (0.309)
GARCH-$\mathcal{S}$	-	-	0.1131 (0.444)	-	0.1124 (0.048)	0.0353 (0.743)
GARCH$-\mathcal{GH}sk\mathcal{S}$	-	-	0.1133 (0.552)	-	0.1124 (0.065)	0.0352 (0.877)
GARCH-$\mathcal{S}$	-	-	0.1131 (0.476)	-	0.1125 (0.024)	0.0352 (0.847)
GARCH-$\mathcal{S}$	-	-	0.1130 (0.445)	-	-	0.0352 (0.848)
GJR$-sk\mathcal{N}$	0.1234 (1.000)	0.0361 (0.062)	0.1115 (1.000)	0.0338 (0.093)	0.1093 (1.000)	0.0343 (1.000)
GJR$-sk\mathcal{S}$	0.1233 (1.000)	0.0354 (1.000)	0.1116 (1.000)	0.0328 (1.000)	0.1087 (1.000)	0.0340 (1.000)
GJR$-\mathcal{GH}sk\mathcal{S}$	0.1235 (1.000)	0.0354 (1.000)	0.1117 (1.000)	0.0329 (1.000)	0.1087 (1.000)	0.0341 (1.000)
GJR$-sk\mathcal{NIG}$	0.1234 (1.000)	0.0353 (1.000)	0.1116 (1.000)	0.0327 (1.000)	0.1088 (1.000)	0.0341 (1.000)
GJR$-sk\mathcal{NRIG}$	0.1234 (1.000)	0.0353 (1.000)	0.1116 (1.000)	0.0327 (1.000)	0.1088 (1.000)	0.0340 (1.000)

Values of the ALF are reported in the main entries and the MCS p-values are in the parentheses. When the model has a p-value = 1,000, the value is in blue font. In addition, the value is underlined when the model has the minimum ALF, however, this cannot be seen in the Table because not all decimals are shown. The value is omitted when the model does not belong to the Model Confidence Set.

Table 8. (Continued)

Model	UK (UKX)		Japan (NKY)		Europe (SX5E)	
	95%	99%	95%	99%	95%	99%
GARCH$-\mathcal{N}$	0.1174 (0.119)	-	0.1460 (0.076)	-	0.1453 (0.036)	-
GARCH$-\mathcal{S}$	0.1175 (0.203)	0.0374 (0.458)	0.1462 (0.177)	0.0435 (0.591)	0.1458 (0.037)	0.0470 (0.925)
GARCH$-\mathcal{NIG}$	0.1174 (0.203)	0.0373 (0.527)	0.1461 (0.154)	0.0435 (0.548)	0.1454 (0.044)	0.0469 (0.976)
GARCH$-\mathcal{NRIG}$	0.1173 (0.207)	0.0373 (0.492)	0.1460 (0.155)	0.0435 (0.560)	0.1453 (0.047)	0.0469 (0.972)
GJR$-\mathcal{N}$	0.1146 (1.000)	-	0.1423 (1.000)	-	0.1403 (1.000)	0.0475 (0.551)
GJR$-\mathcal{S}$	0.1150 (1.000)	0.0362 (1.000)	0.1421 (1.000)	0.0425 (1.000)	0.1402 (1.000)	0.0461 (1.000)
GJR$-\mathcal{NIG}$	0.1149 (1.000)	0.0362 (1.000)	0.1422 (1.000)	0.0424 (1.000)	0.1400 (1.000)	0.0461 (1.000)
GJR$-\mathcal{NRIG}$	0.1149 (1.000)	0.0362 (1.000)	0.1422 (1.000)	0.0425 (1.000)	0.1400 (1.000)	0.0461 (1.000)
GARCH-$\mathcal{S}$	0.1173 (0.097)	0.0378 (0.199)	-	0.0441 (0.259)	0.1450 (0.048)	0.0479 (0.583)
GARCH-$\mathcal{S}$	0.1173 (0.188)	0.0369 (0.955)	0.1460 (0.078)	0.0432 (0.799)	0.1450 (0.057)	0.0466 (1.000)
GARCH$-\mathcal{GH}sk\mathcal{S}$	0.1172 (0.240)	0.0368 (0.979)	0.1459 (0.087)	0.0430 (0.986)	0.1449 (0.079)	0.0466 (1.000)
GARCH-$\mathcal{S}$	0.1173 (0.179)	0.0367 (0.999)	0.1462 (0.028)	0.0431 (0.938)	0.1448 (0.056)	0.0465 (1.000)
GARCH-$\mathcal{S}$	0.1172 (0.186)	0.0367 (1.000)	0.1462 (0.018)	0.0431 (0.911)	0.1449 (0.044)	0.0464 (1.000)
GJR$-sk\mathcal{N}$	0.1144 (1.000)	0.0363 (1.000)	0.1425 (1.000)	0.0427 (1.000)	0.1402 (1.000)	0.0465 (1.000)
GJR$-sk\mathcal{S}$	0.1146 (1.000)	0.0362 (1.000)	0.1421 (1.000)	0.0421 (1.000)	0.1398 (1.000)	0.0462 (1.000)
GJR$-\mathcal{GH}sk\mathcal{S}$	0.1146 (1.000)	0.0363 (1.000)	0.1421 (1.000)	0.0419 (1.000)	0.1398 (1.000)	0.0464 (1.000)
GJR$-sk\mathcal{NIG}$	0.1146 (1.000)	0.0362 (1.000)	0.1423 (1.000)	0.0421 (1.000)	0.1399 (1.000)	0.0463 (1.000)
GJR$-sk\mathcal{NRIG}$	0.1146 (1.000)	0.0362 (1.000)	0.1424 (1.000)	0.0421 (1.000)	0.1400 (1.000)	0.0462 (1.000)

Values of the ALF are reported in the main entries and the MCS p-values are in the parentheses. When the model has a p-value = 1,000, the value is in blue font. In addition, the value is underlined when the model has the minimum ALF, however, this cannot be seen in the Table because not all decimals are shown. The value is omitted when the model does not belong to the Model Confidence Set.

Table 9. Model Confidence Set Results for Latam Stock Markets

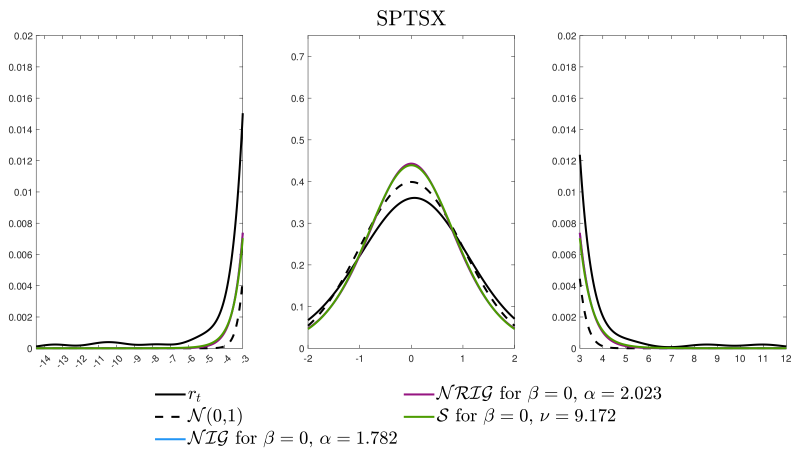
Model	Argentina (MERVAL)		Brazil (IBOV)		Chile (IPSA)	
	95%	99%	95%	99%	95%	99%
GARCH $-\mathcal{N}$	0.2831 (0.923)	0.0954 (0.664)	0.1709 (1.000)	0.0536 (0.915)	0.1257 (1.000)	0.0425 (0.843)
GARCH $-\mathcal{S}$	0.2845 (0.428)	0.0934 (1.000)	0.1714 (0.862)	0.0529 (1.000)	0.1268 (0.404)	0.0424 (0.942)
GARCH $-\mathcal{NIG}$	0.2832 (0.676)	0.0930 (1.000)	0.1712 (0.918)	-	0.1265 (0.518)	0.0420 (1.000)
GARCH $-\mathcal{NRIG}$	0.2831 (0.776)	0.0930 (1.000)	0.1711 (0.959)	0.0528 (1.000)	0.1264 (0.612)	0.0420 (1.000)
GJR $-\mathcal{N}$	0.2821 (1.000)	0.0961 (0.466)	0.1706 (1.000)	0.0538 (0.445)	0.1253 (1.000)	0.0428 (0.306)
GJR $-\mathcal{S}$	0.2811 (1.000)	0.0948 (0.839)	0.1707 (1.000)	0.0527 (1.000)	0.1256 (1.000)	0.0418 (1.000)
GJR $-\mathcal{NIG}$	0.2807 (1.000)	0.0944 (0.942)	0.1707 (1.000)	0.0527 (1.000)	0.1252 (1.000)	0.0417 (1.000)
GJR $-\mathcal{NRIG}$	0.2805 (1.000)	0.0943 (0.993)	0.1707 (1.000)	0.0527 (1.000)	0.1252 (1.000)	0.0417 (1.000)
GARCH- $\mathcal{S}$	0.2832 (0.893)	0.0938 (1.000)	0.1711 (0.897)	0.0533 (0.988)	0.1257 (1.000)	0.0420 (1.000)
GARCH- $\mathcal{S}$	0.2841 (0.340)	0.0928 (1.000)	0.1712 (0.908)	0.0530 (1.000)	0.1268 (0.377)	0.0425 (0.914)
GARCH $-\mathcal{GH}sk\mathcal{S}$	0.2842 (0.394)	0.0932 (1.000)	0.1712 (0.915)	-	0.1267 (0.465)	0.0424 (0.945)
GARCH- $\mathcal{S}$	0.2834 (0.541)	0.0930 (1.000)	0.1712 (0.935)	0.0531 (1.000)	0.1264 (0.584)	0.0420 (1.000)
GARCH- $\mathcal{S}$	0.2833 (0.567)	0.0928 (1.000)	0.1712 (0.914)	0.0530 (1.000)	0.1264 (0.536)	0.0419 (1.000)
GJR $-sk\mathcal{N}$	0.2818 (1.000)	0.0944 (0.981)	0.1706 (1.000)	0.0536 (0.522)	0.1252 (1.000)	0.0422 (0.991)
GJR $-sk\mathcal{S}$	0.2801 (1.000)	0.0940 (1.000)	0.1704 (1.000)	0.0529 (1.000)	0.1254 (1.000)	0.0419 (1.000)
GJR $-\mathcal{GH}sk\mathcal{S}$	0.2809 (1.000)	0.0944 (0.960)	0.1705 (1.000)	0.0530 (1.000)	0.1253 (1.000)	0.0418 (1.000)
GJR $-sk\mathcal{NIG}$	0.2807 (1.000)	0.0942 (0.997)	0.1704 (1.000)	-	0.1251 (1.000)	0.0416 (1.000)
GJR $-sk\mathcal{NRIG}$	0.2802 (1.000)	0.0938 (1.000)	0.1704 (1.000)	0.0529 (1.000)	0.1250 (1.000)	0.0416 (1.000)

Values of the ALF are reported in the main entries and the MCS p-values are in the parentheses. When the model has a p-value = 1,000, the value is in blue font. In addition, the value is underlined when the model has the minimum ALF, however, this cannot be seen in the Table because not all decimals are shown. The value is omitted when the model does not belong to the Model Confidence Set.

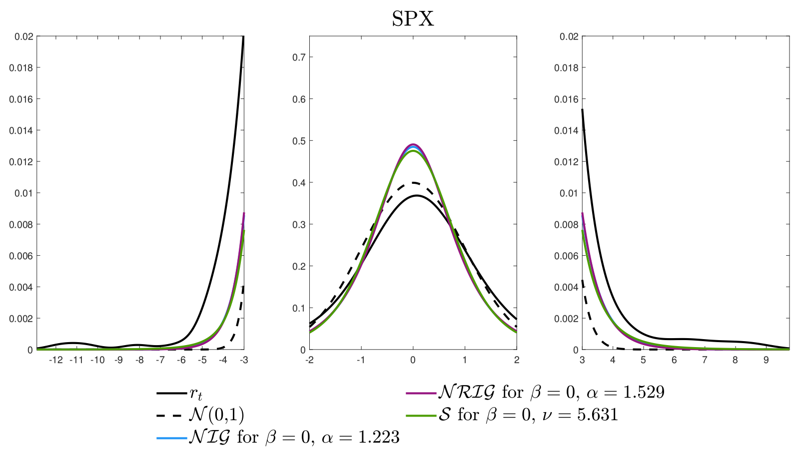
Table 9. (Continued)

Model	Colombia (COLCAP)		Mexico (MEXBOL)		Peru (SPBLPGPT)	
	95%	99%	95%	99%	95%	99%
GARCH$-\mathcal{N}$	0.1271 (0.899)	-	0.1088 (1.000)	0.0331 (0.228)	0.1232 (0.961)	0.0408 (0.993)
GARCH$-\mathcal{S}$	0.1283 (0.108)	0.0402 (0.237)	0.1091 (0.978)	0.0320 (1.000)	0.1237 (0.608)	0.0408 (0.982)
GARCH$-\mathcal{NIG}$	0.1280 (0.136)	0.0401 (0.270)	0.1090 (1.000)	0.0320 (1.000)	0.1235 (0.607)	0.0406 (1.000)
GARCH$-\mathcal{NRIG}$	0.1278 (0.201)	0.0401 (0.241)	0.1090 (1.000)	0.0320 (1.000)	0.1235 (0.380)	0.0404 (1.000)
GJR$-\mathcal{N}$	0.1255 (1.000)	-	0.1089 (1.000)	0.0331 (0.159)	0.1224 (1.000)	0.0408 (0.995)
GJR$-\mathcal{S}$	0.1269 (1.000)	0.0397 (1.000)	0.1091 (0.928)	0.0321 (1.000)	0.1224 (1.000)	0.0409 (0.840)
GJR$-\mathcal{NIG}$	0.1264 (1.000)	0.0396 (1.000)	0.1091 (0.993)	0.0320 (1.000)	0.1223 (1.000)	0.0407 (0.995)
GJR$-\mathcal{NRIG}$	0.1263 (1.000)	0.0396 (1.000)	0.1090 (0.995)	0.0320 (1.000)	0.1223 (1.000)	0.0406 (1.000)
GARCH-$\mathcal{S}$	0.1268 (1.000)	-	<u>0.1087 (1.000)</u>	0.0324 (0.890)	0.1235 (0.749)	0.0406 (1.000)
GARCH-$\mathcal{S}$	0.1279 (0.171)	0.0400 (0.410)	0.1090 (1.000)	0.0319 (1.000)	0.1236 (0.567)	0.0406 (1.000)
GARCH$-\mathcal{GH}sk\mathcal{S}$	0.1279 (0.198)	0.0399 (0.729)	0.1090 (1.000)	0.0319 (1.000)	0.1237 (0.532)	0.0406 (1.000)
GARCH-$\mathcal{S}$	0.1275 (0.407)	0.0398 (0.939)	0.1089 (1.000)	<u>0.0318 (1.000)</u>	0.1235 (0.420)	0.0404 (1.000)
GARCH-$\mathcal{S}$	0.1274 (0.534)	0.0398 (0.881)	0.1089 (1.000)	0.0318 (1.000)	0.1234 (0.374)	<u>0.0403 (1.000)</u>
GJR$-sk\mathcal{N}$	<u>0.1253 (1.000)</u>	-	0.1088 (1.000)	0.0325 (0.605)	0.1227 (1.000)	0.0408 (0.999)
GJR$-sk\mathcal{S}$	0.1265 (1.000)	0.0395 (1.000)	0.1091 (0.976)	0.0320 (1.000)	0.1225 (1.000)	0.0407 (0.995)
GJR$-\mathcal{GH}sk\mathcal{S}$	0.1267 (1.000)	0.0394 (1.000)	0.1091 (0.979)	0.0319 (1.000)	0.1225 (1.000)	0.0407 (0.989)
GJR$-sk\mathcal{NIG}$	0.1261 (1.000)	<u>0.0393 (1.000)</u>	0.1090 (1.000)	0.0319 (1.000)	<u>0.1223 (1.000)</u>	0.0406 (1.000)
GJR$-sk\mathcal{NRIG}$	0.1260 (1.000)	0.0393 (1.000)	0.1090 (1.000)	0.0319 (1.000)	0.1224 (1.000)	0.0405 (1.000)

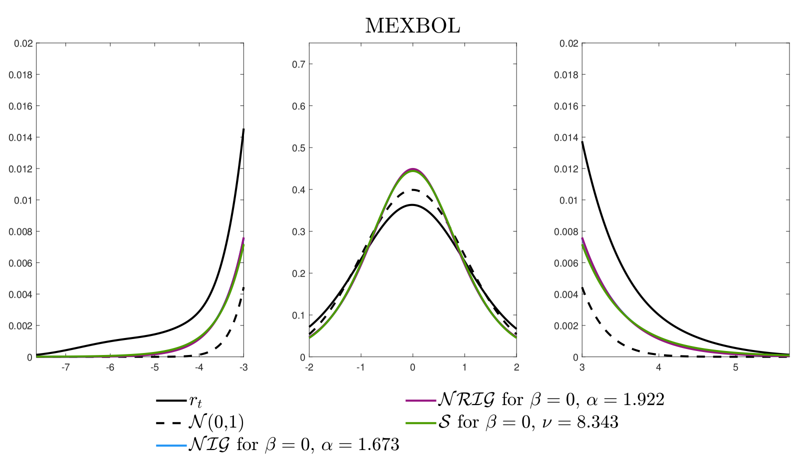
Values of the ALF are reported in the main entries and the MCS p-values are in the parentheses. When the model has a p-value = 1,000, the value is in blue font. In addition, the value is underlined when the model has the minimum ALF, however, this cannot be seen in the Table because not all decimals are shown. The value is omitted when the model does not belong to the Model Confidence Set.



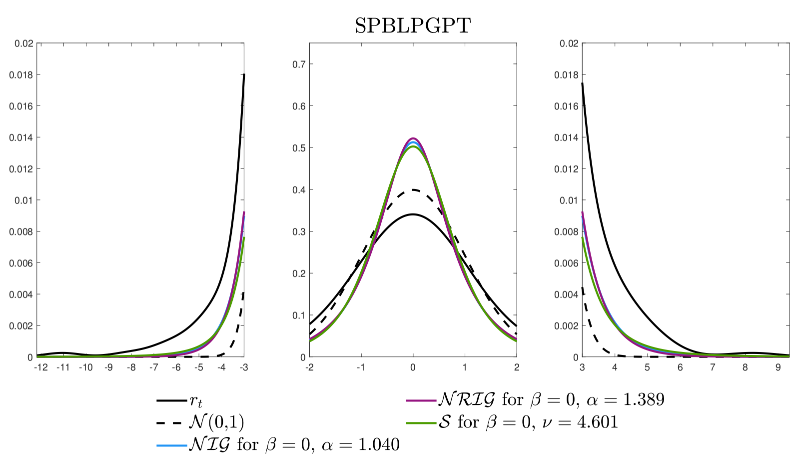
(a) Canada



(b) USA

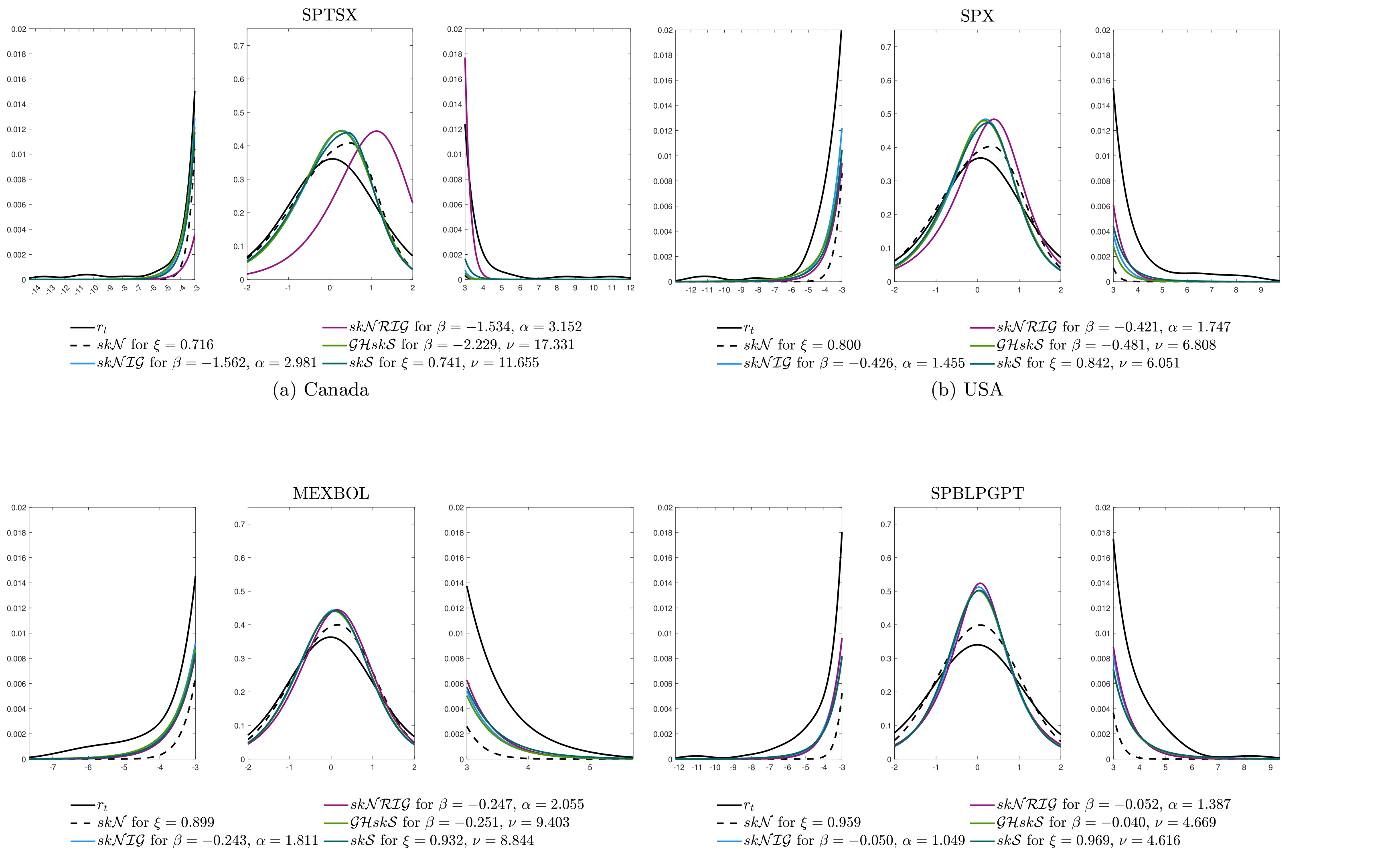


(c) Mexico



(d) Peru

Figure 1. Empirical density and densities of the symmetric $\mathcal{N}(0,1)$, $\mathcal{NIG}$, $\mathcal{NRIG}$ and $\mathcal{S}$ distributions using estimated parameters. Since there are 2000 out-of-sample observations and the models are re-estimated every 10 observations, 200 sets of parameters are obtained. Therefore, the parameter values used in this figure are set as the median of the 200 estimates for each market. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)



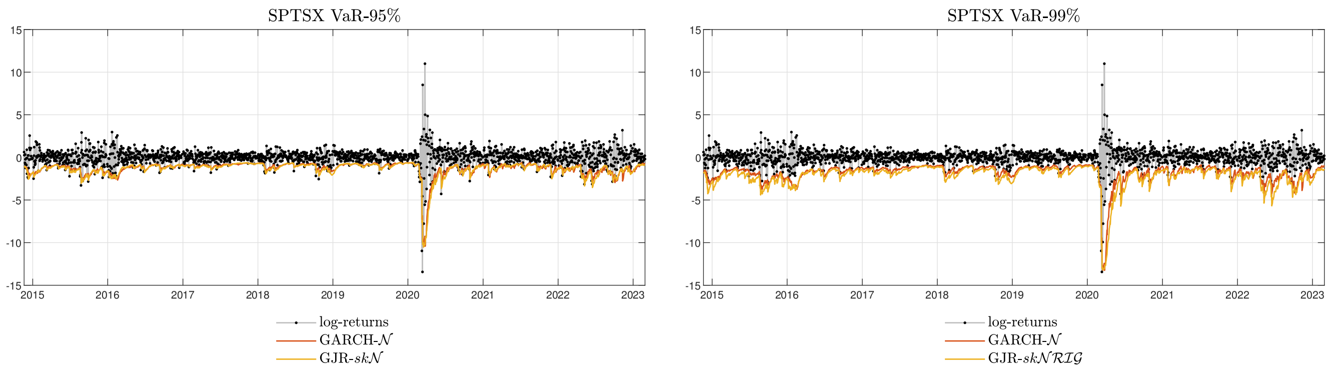
(a) Canada

(b) USA

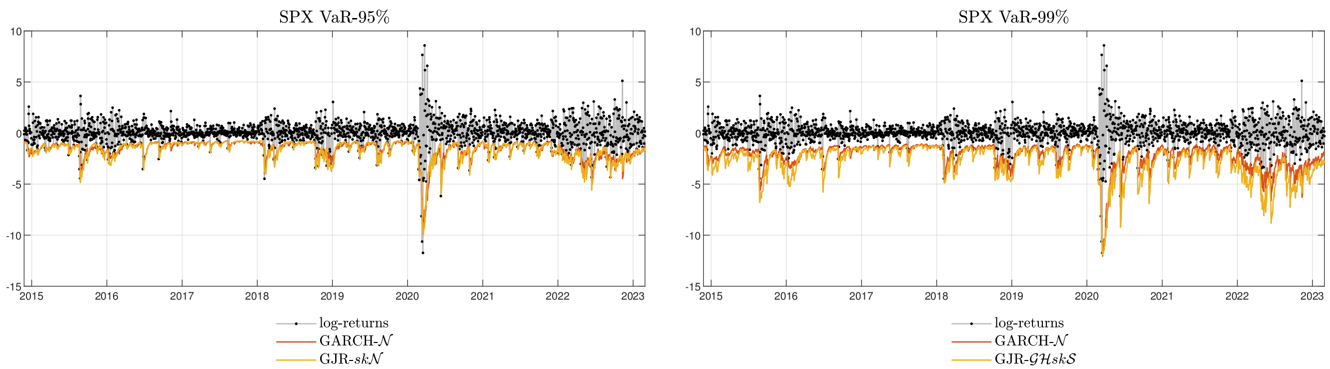
(c) Mexico

(d) Peru

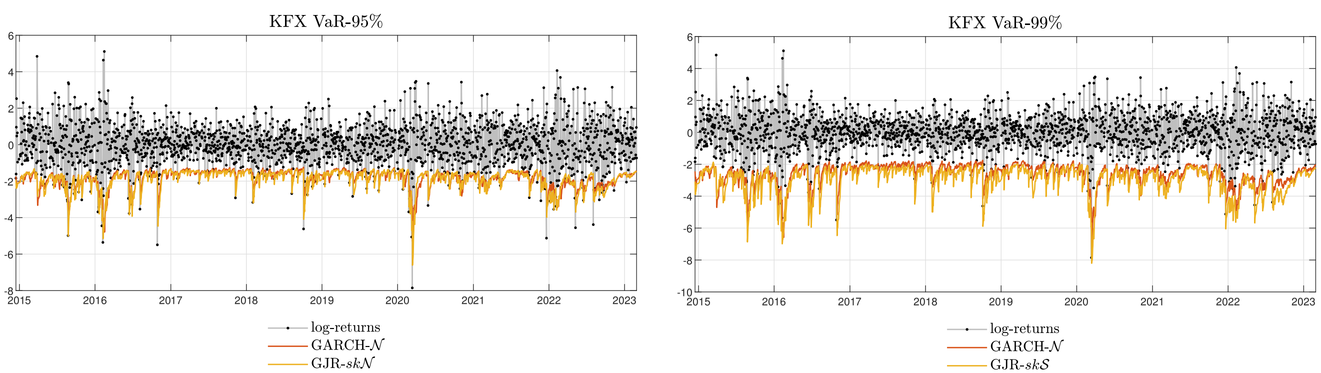
Figure 2. Empirical density and densities of the $\mathcal{N}(0,1)$, $sk\mathcal{NIG}$, $sk\mathcal{NRIG}$, $\mathcal{GH} - sk\mathcal{S}$ and $sk\mathcal{S}$ distributions using the estimated parameter values. Since there are 2000 out-of-sample observations and the models are re-estimated every 10 observations, 200 sets of parameters are obtained. Therefore, the parameter values used in this figure are set as the median of the 200 estimates for each market. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)



(a) Canada

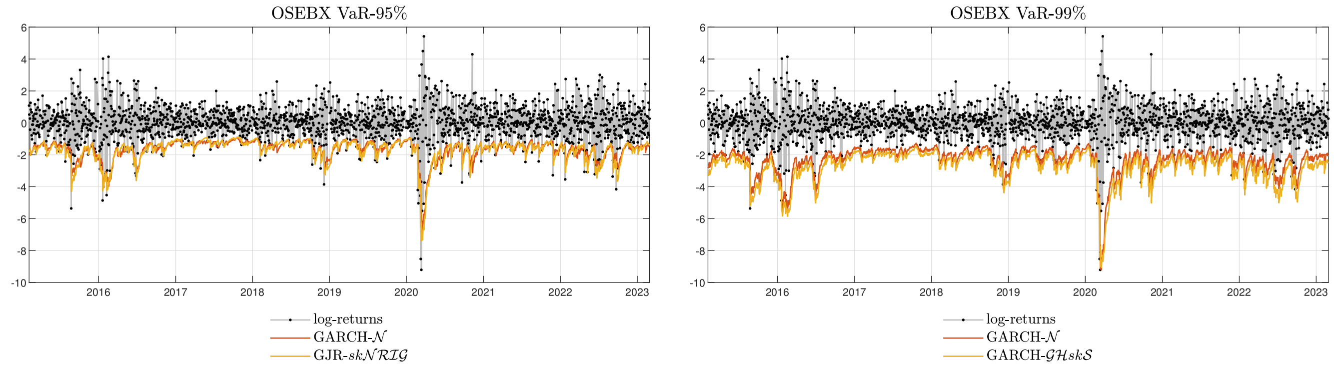


(b) USA

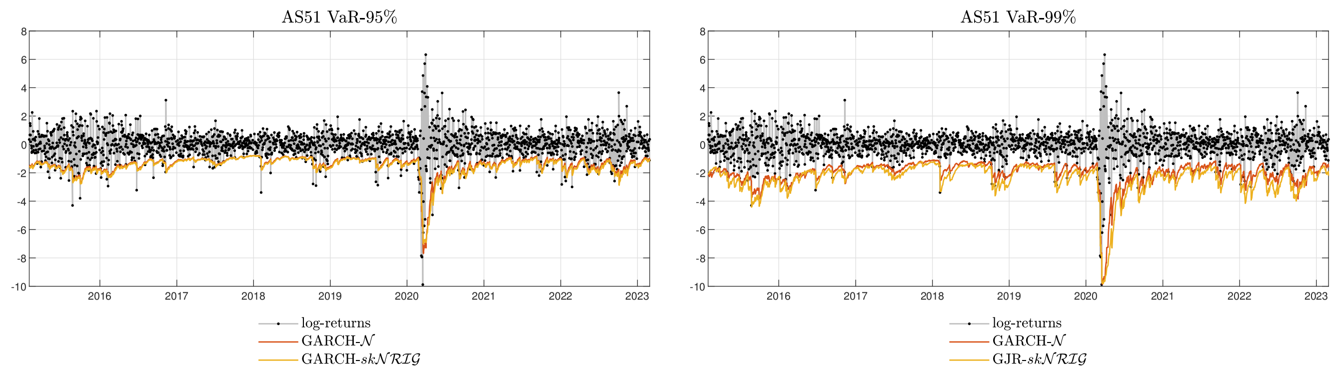


(c) Denmark

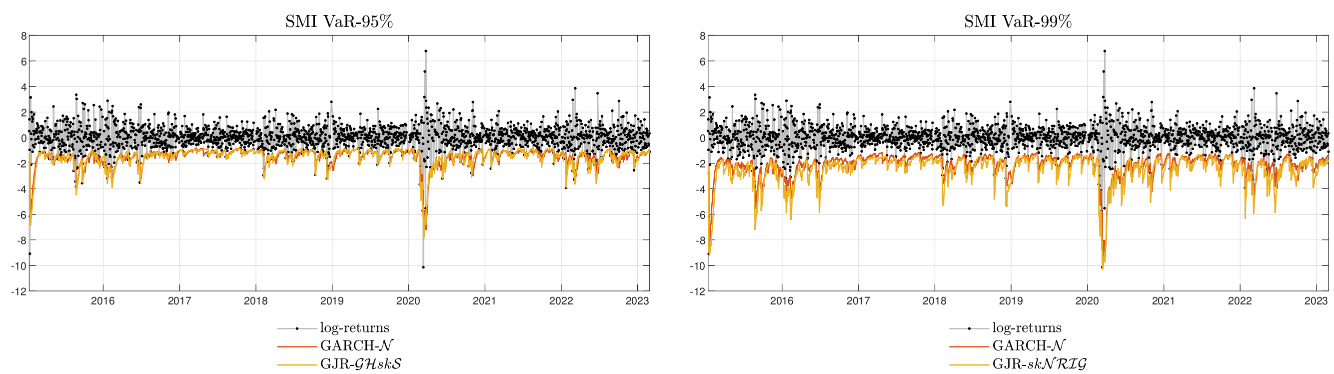
Figure 3. VaR Forecasts for the Selected Models and Daily Log-Returns for High Income Stock Markets. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)



(d) Norway

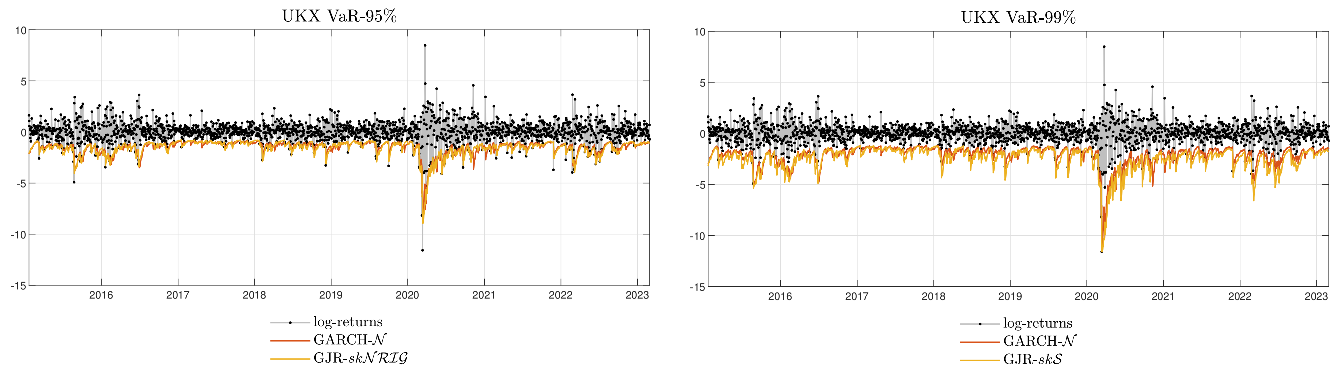


(e) Australia

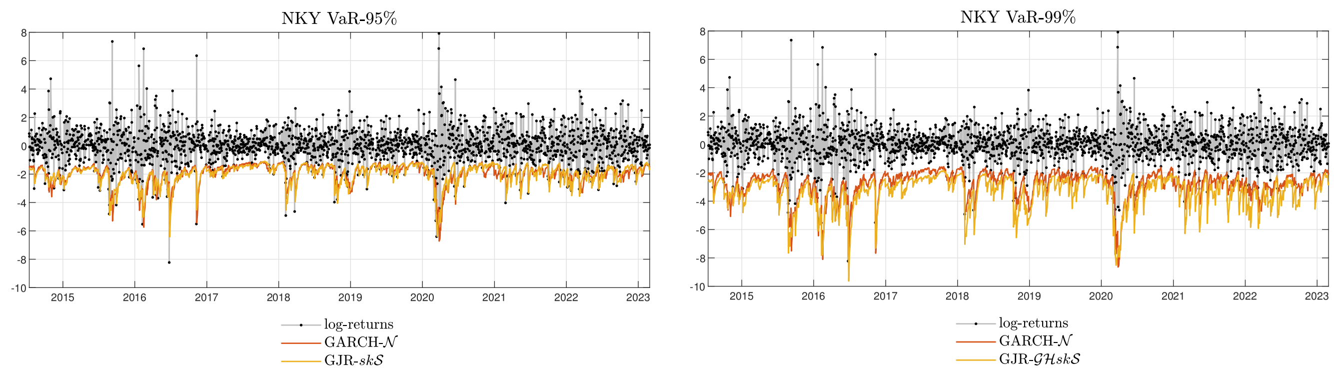


(f) Switzerland

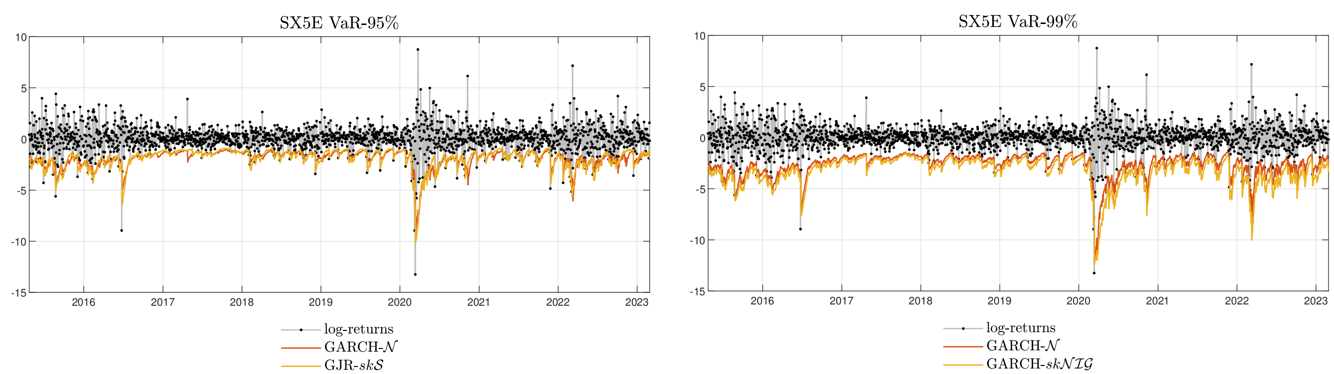
Figure 3. (Continued)



(g) United Kingdom

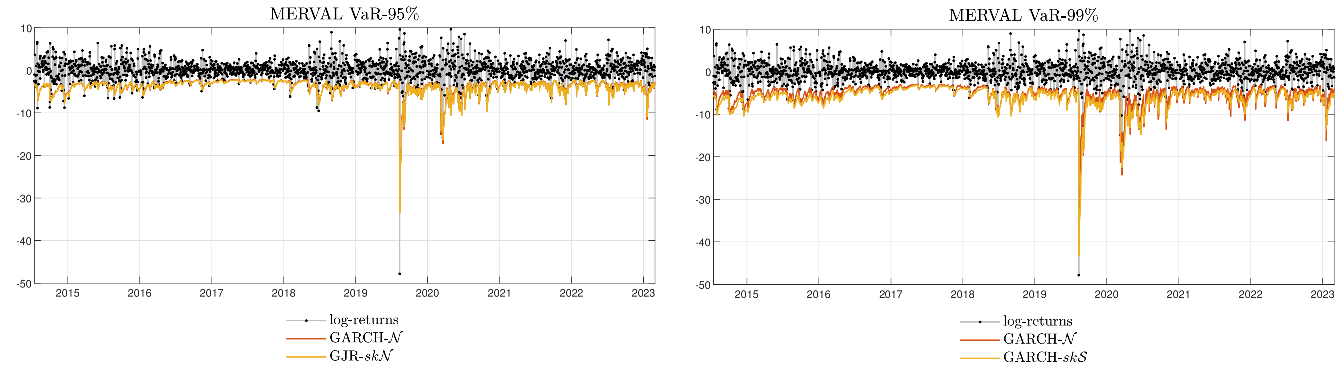


(h) Japan

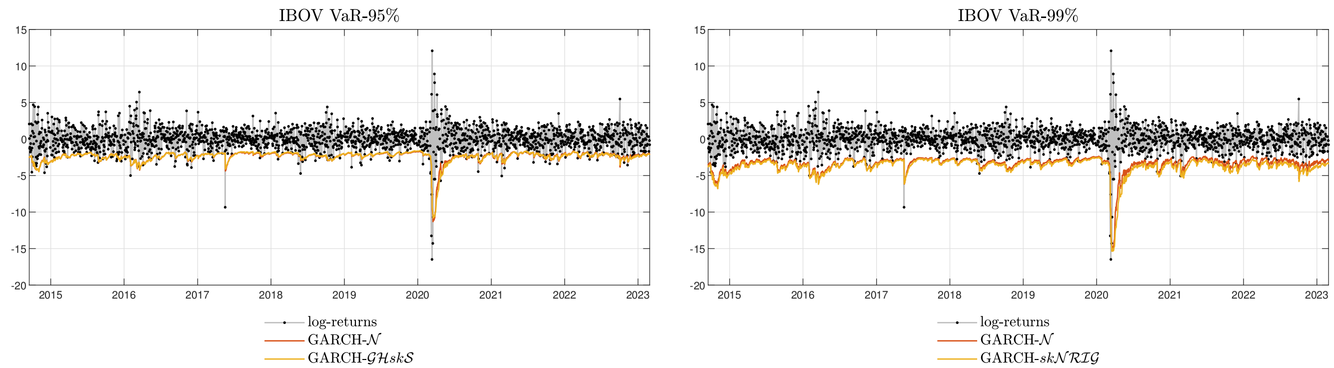


(i) Europe

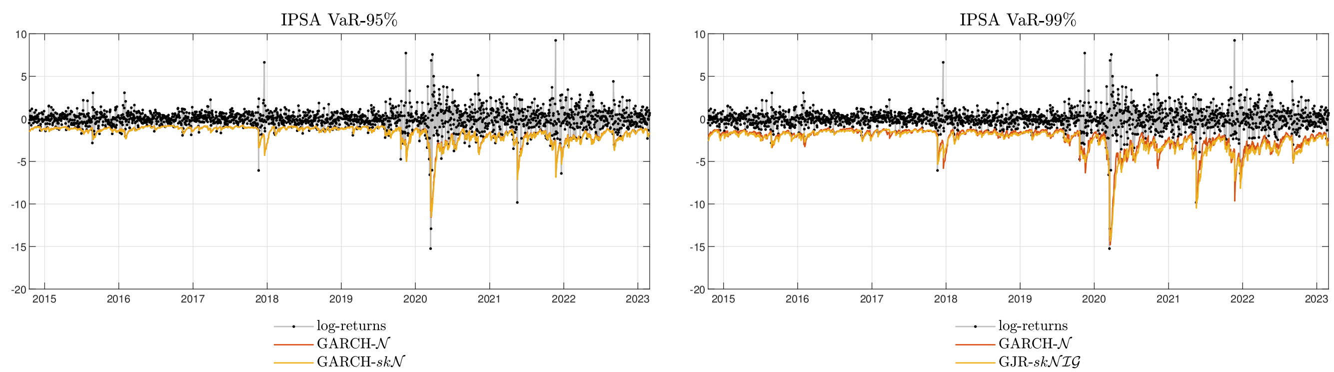
Figure 3. (Continued)



(a) Argentina

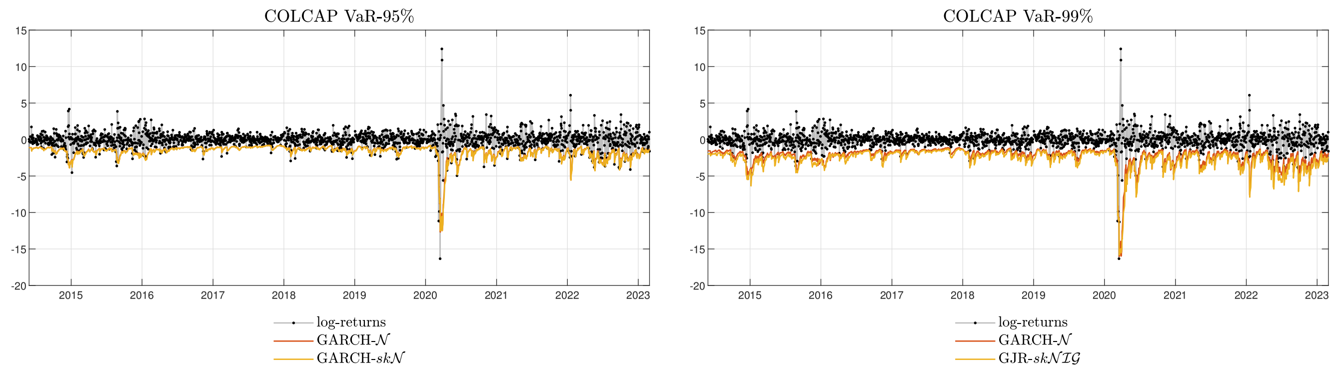


(b) Brazil

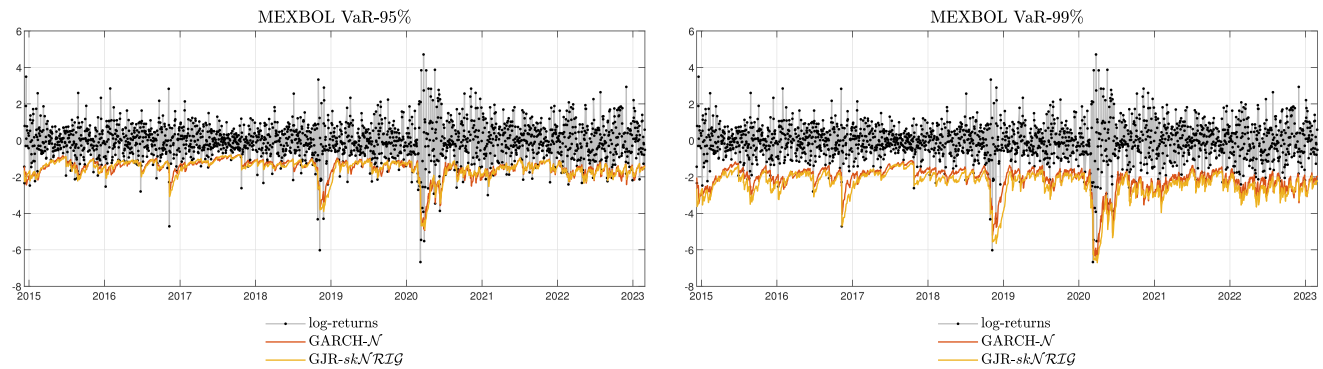


(c) Chile

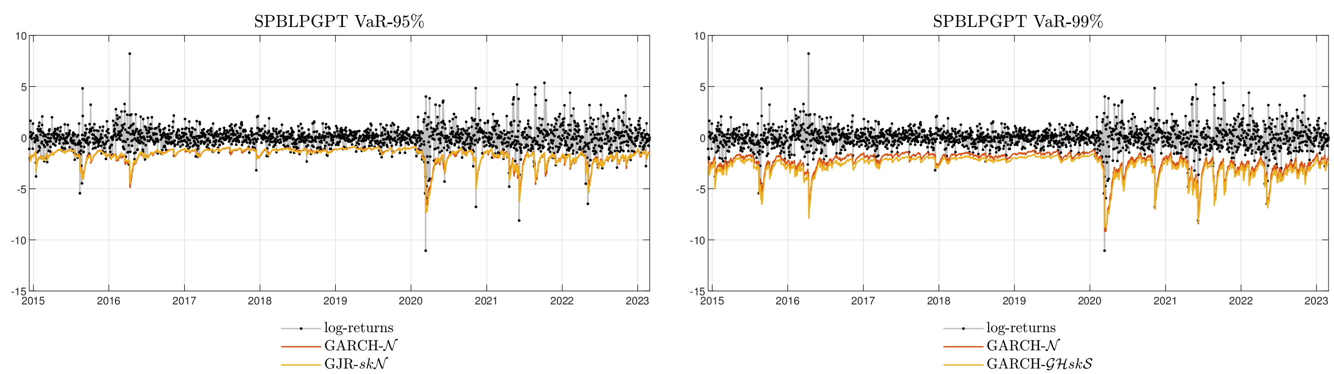
Figure 4. VaR Forecasts for the Selected Models and Daily Log-Returns for Latam Stock Markets. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)



(d) Colombia



(e) Mexico



(f) Peru

Figure 4. (Continued)

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