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A NOTE ABOUT DETECTION OF ADDITIVE OUTLIERS WITH FRACTIONAL ERRORS

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Abstract

Perron and Rodríguez (2003) claimed that their procedure to detect for additive outliers (τ_d) is powerful even when we have departures from the unit root case. In this note, we use Monte-Carlo simulations to show that τ_d is powerful when we have $ARFIMA(p, d, q)$ errors. Using simulations, we calculate the expected number of additive outliers found in this context and the number of times that the approach τ_d identifies the true location of the additive outliers. The results indicate that the power of the procedure τ_d depends of the size of the additive outliers. When we have a DGP with big sized additive outliers the percentage of time that τ_d detects correctly the location of the additive outliers is 100.0%. A comparison between τ_d and the procedure TRAMO-SEATS is also included.

KeyWords: Additive Outliers, ARFIMA Errors, Detection of Additive Outliers

JEL: C2, C3, C5

Resumen

Perron y Rodríguez (2003) argumentan que su procedimiento para detectar outliers aditivos (τ_d) es potente aún cuando hay desviaciones del caso de raíz unitaria. En esta nota usamos simulaciones de Monte Carlo para mostrar que τ_d es potente cuando los errores son de tipo $ARFIMA(p, d, q)$. Usando dichas simulaciones, calculamos el número esperado de outliers aditivos hallados en este contexto y el número de veces que el método τ_d identifica la verdadera localización de los outliers aditivos. Los resultados muestran que la potencia del procedimiento τ_d depende del tamaño de los outliers. Cuando tenemos un PGD con outliers de gran tamaño, el porcentaje de veces que τ_d detecta correctamente la posición de los outliers es 100%. Una comparación entre τ_d y le procedimiento TRAMO-SEATS es incluído como ilustración.

Palabras Claves: Outliers Aditivos, Errores ARFIMA, Detección de Outliers Aditivos.

Classificación JEL: C2, C3, C5

A Note about Detection of Additive Outliers with Fractional Errors[1]

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1 Introduction

Perron and Rodríguez (2003) proposed a procedure to identify for additive outliers using the first differences of the data. This procedure, denoted by τ_d, assumes the existence of a unit root in the levels of the variables and is a powerful modification of other procedures; see Vogelsang (1999). Perron and Rodríguez (2003) claim that the procedure τ_d is also powerful respect departures of the unit root case and it is not dependent of the autocorrelation and heteroskedasticity structure[3].

Of course, there are many other forms to detect for additive outliers in the context of stationary (I(0)) variables; see, among others, Tsay (1986), Chang, Tiao and Chen (1988), Shin, Sharkar and Lee (1996), Chen and Liu (1993), Gómez and Maravall (1992), Chang et al. (1988), and Chan (1992, 1995). There are also some procedures named robust to the presence of outliers which are not interested in the location of the outliers as Chareka et al. (2006) and Fajardo et al. (2009). See also Lucas (1995a, 1995b), and Rodríguez (2004), among others and the references cited there in.

In this note, we are interested to show the performance in terms of size and power of the statistic τ_d. We calculate power to detect discrete additive outliers using a data generating process similar to Perron and Rodríguez (2003), that is, allowing until four additive outliers with different combinations related to their size. Also, we verify power properties observing the average number of additive outliers found. The correct number of addi-

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[3] However, departures from Normality affect severely the statistic τ_d as mentioned by Perron and Rodríguez (2003). Burridge and Taylor (2006) note this issue and fix it using extreme value theory.

tive outliers found (named matches) is also important to be analyzed. All this analysis is performed using $ARFIMA(1,d,0)$ and $ARFIMA(0,d,1)$ processes for the errorrs. The results show, as expected, that more corrected properties are found when d is close to unity. Type of correlation affects the properties slightly when $\theta = -0.80$ or $\rho = -0.80$.

This note is organized as follows. Section 2 presents the model, discusses the issue of outlier detection and briefly revises the principal method proposed by Perron and Rodríguez (2003) to detect for additive outliers. In section 3, we describe the Monte-Carlo experiments as well the discussion of the results. Section 4 presents results of the empirical analysis. Section 5 concludes.

2 The Issue of Outlier Detection and Testing for Unit Roots with Additive Outliers

The issue of outlier detection in the unit root framework is the approach taken by Perron and Rodríguez (2003) which is based on Vogelsang (1999)[4]. The data-generating process entertained is of the following general form:

$$y_t = d_t + \sum_{j=1}^{m} \delta_j D(T_{ao,j})_t + u_t \quad , \tag{1}$$

where $D(T_{ao,j})_t = 1$ if $t = T_{ao,j}$ and 0 otherwise. This permits the presence of m additive outliers occurring at dates $T_{ao,j}$ $(j = 1, ..., m)$. The term d_t specifies the deterministic components. In most cases, $d_t = \mu$ if the series is non-trending or $d_t = \mu + \beta t$ if the series is trending. The noise function is integrated of order one, i.e, $u_t = u_{t-1} + v_t$; where v_t is a stationary process. While in Perron and Rodríguez (2003), they use an $ARMA(p,q)$ for the process v_t, in this note, we assume that v_t is an $ARFIMA(p,d,q)$ process.

As shown in Perron and Rodríguez (2003), the original procedure of Vogelsang (1999) has severe size distortions when applied in an iterative

[4] Let $t_{\widehat{\delta}}(T_{ao})$ denote the t-statistic for testing $\delta = 0$ in (1). Following Chen and Liu (1993), the presence of an additive outlier can be tested using $\tau = \sup_{T_{ao}} \mid t_{\widehat{\delta}}(T_{ao})$. Assuming that $\lambda = T_{ao}/T$ remains fixed as T grows, Vogelsang (1999) showed that as $T \to \infty$, the limiting distribution of $t_{\widehat{\delta}}(T_{ao})$ is non-standard. More precisely, $t_{\widehat{\delta}}(T_{ao}) \Rightarrow H(\lambda) = W^*(\lambda)/(\int_0^1 W^*(r)^2 dr)^{1/2}$, where $W^*(\lambda)$ denotes a demeaned standard Wiener process. If (1) also includes a time trend, $W^*(\lambda)$ will denote a detrended Wiener process. Furthermore, from the continuous mapping theorem it follows that, $\tau \Rightarrow \sup_{\lambda \in (0,1)} |H(\lambda)| \equiv H^*$. This distribution is invariant with respect to any nuisance parameters, including the correlation structure of the noise function.

fashion to search for additive outliers. The reason for this is that the limiting distribution of the statistic is only valid in the first step of the iteration as specified in Theorem 1 of Perron and Rodríguez (2003). In subsequent steps, the asymptotic critical values used need to be modified.

Perron and Rodríguez (2003) have proposed a more powerful iterative strategy based on the first-differences of the data. Consider data generated by (1) with $d_t = \mu$, and a single outlier occurring at date T_{ao} with magnitude δ. Then,

$$\Delta y_t = \delta[D(T_{ao})_t - D(T_{ao})_{t-1}] + v_t, \tag{2}$$

where $D(T_{ao})_t = 1$, if $t = T_{ao}$ (0, otherwise) and $D(T_{ao})_{t-1} = 1$, if $t = T_{ao} - 1$ (0, otherwise). If the data are trending, a constant should be included. In this case we are interested in $\tau_d = \sup_{T_{ao}} |t_{\widehat{\delta}}(T_{ao})|$, where $t_{\widehat{\delta}}(T_{ao}) = \hat{\delta}/(2(\hat{R}_u(0) - \hat{R}_u(1))$ and $R_u(j)$ is the autocovariance function of v_t at delay j.[5]

To detect multiple outliers, we can follow a strategy similar to that suggested by Vogelsang (1999), by dropping the observation labelled as an outlier before proceeding to the next step. The important feature is that, unlike for the case of tests based on levels, the limit distribution of the test τ_d is the same as each step of the iterations when dealing with multiple outliers. The disadvantage of this procedure, compared to that based on the level of the data, is that the limiting distribution depends on the specific distribution of the errors v_t, though not on the presence of serial correlation and heteroskedasticity[6]. This problem is exactly the same as that for finding outliers in stationary time series.

In this note, we analyze the performance of the statistic τ_d in terms of size and power when errors are $ARFIMA(0, d, 1)$ and $ARFIMA(1, d, 0)$ processes, respectively. More exactly, we verify the exact size of τ_d when there are no outliers and when there are medium and big sized additive outliers. After it, we observe the average number of additive outliers found by the approach τ_d when there are no additive outliers and when there are big sized additive outliers. Finally, we also analyze the number of times that the procedure τ_d locate or detect exactly the position of the additive outliers. we named it the number of matches[7].

[5] Where $\hat{R}_u(j) = T^{-1}\sum_{t=1}^{T-j} \hat{v}_t\hat{v}_{t-j}$ with $\hat{v}_t$ the least-squares residuals obtained from regression (2). Then, $\hat{R}_u(j)$ is a consistent estimate of $R_u(j)$.

[6] The dependence of the distribution of the error v_t is shown in Burridge and Taylor (2006).

[7] I do it because a procedure may have high power but the exact identification of the additive outliers could be very bad.

3 Monte Carlo Results

In order to analyze the size, power and matches of the statistic τ_d, we consider the following experiment. Let y_t follow (1) where $u_t = u_{t-1} + v_t$ and v_t is an $ARFIMA(p, d, q)$ process. More exactly, we consider $p = 1$ and $q = 0$ and $p = 0$ and $q = 1$, respectively. The fractional parameter $d = -0.48$ to 0.48 with a step of 0.12. In order to save space, we only present selected Tables[8]. The number of replications is 10,000 and the tabulated critical values for $T = 100$ and $T = 200$ are obtained from Perron and Rodríguez (2003). In all Tables, the total iterative procedure is applied, that is, we search for all outliers and procedure finish when no outliers are found. In each Table, three cases are presented. In the first case no outliers are in the process, that is, $\delta_i = 0$ for $i = 1, 2, 3, 4$. In the second case, we consider medium sized additive outliers: $\delta_i = 5, 3, 2, 2$. The final case is for big sized additive outliers, that is, $\delta_i = 10, 5, 5, 5$. In summary, the design of the experiment follow closely Perron and Rodríguez (2003)[9].

We include only results for $d \geq 0$ because results for $d < 0$ are very similar. In Table 1, we show power of τ_d when there are $ARFIMA(0, d, 0)$. Power is measured as the probability to find the four outliers. When $\delta_i = 0$ (no outliers exist), exact size is very close to 5.0%. There is no differences between the different values of d. The results are better for $T = 200$ even when for $T = 100$ are also acceptable. When there are medium sized additive outliers, the probability to find the first outlier (the biggest one) is almost unity. Probabilities to find second until fourth outlier appears to be sensitive to the value of d. When $d = 0$, the probability to find second outlier (sized three) is around 67% and 71.4% for $T = 100$ and $T = 200$, respectively.

[8] A number of other extensive Tables are available upon request.

[9] It is worth to mention that I also simulated data using another DGP. Let $\{y_t\}$, $t \in \mathbb{Z}$ be a weakly stationary process. Let $\{z_t\}$, $t \in \mathbb{Z}$ be a process contaminated by additive outliers, which is described by

$$z_t = y_t + \sum_{j=1}^{m} \omega_j X_{j,t} \qquad , \tag{3}$$

where m is the maximum number of outliers and the unknown parameter ω_j indicates the magnitude of the j_{th} outlier. The $X_{j,t}$ is a random variable with probability distribution $\Pr(X_j = -1) = \Pr(X_j = 1) = p_j/2$ and $\Pr(X_j = 0) = 1 - p_j$. Therefore, X_j is the product of Bernouilli (p_j) and Rademacher random variables; the latter equals 1 or -1, both with probability 1/2. Furthermore, y_t and X_j are independent random variables. The model (3) is based on the parametric models proposed by Fox (1972). In order to save space, results from this model are not included but they indicate similar conclusions as in the previous DGP. Such Tables are available upon request.

These values increase when d is higher. When $d = 0.48$ (near to the limit of stationarity) these probabilities are 89.2% and 93.5%, for $T = 100$ nd $T = 200$, respectively. Similar observation is obtained for the third and fourth outlier. In summary, probability to find the second until fourth outlier are not close to unity (100%) because the size of the additive outliers is reduced. Power increases when d increases. When we have big sized additive outliers, power is 100% for finding each one of the four outliers.

In Tables 2a-2c, we have fractional moving average correlation, that is, $ARFIMA(0, d, 1)$ errors. When there is no additive outliers, the exact size is close to the nominal 5% except for θ close to unity where the test is undersized. When we have medium sized additive outliers, power is high when $\theta \geq -0.4$. However, as before, power is limited when finding the second, third and fourth additive outliers. When we have big sized outliers, power is high although for $\theta \geq -0.4$ we have small power. Table 2b shows similar results as the previous Table but in general, power is higher compared to the case where $d = 0$ (Table 2a). In Table 2c ($d = 0.48$), power is almost 100% even for $\theta = -0.8$ and also for the third and fourth outliers. In conclusion, more persistent or more memory the time series have ($d = 0.48$), more power we found. This is coherent because τ_d works under the assumption that there is a unit root in the process. Therefore, more d is close to 1, more power we find.

In Tables 3a-3c, we have fractional autoregressive correlation, that is $ARFIMA(1, d, 0)$ errors. In Table 3a, we have $d = 0.0$ and size is very close to 5% in particular for $\rho \in [-0.4, 0.4]$. In the other cases, the statistic is undersized but with smaller distortions. For medium sized additive outliers, power is low when $\rho = -0.80$ even for the first additive outlier. This is fixed when $\rho \geq -0.4$. For the second, third and fourth additive outlier, power is better or higher when $\rho \geq 0$ or $\rho \geq 0.4$. When we have big sized additive outliers, power increase when ρ goes to unity. In Table 3b ($d = 0.24$), the exact size is similar as in the previous Table. Similar conclusions are obtained for medium sized additive outliers. In the case where there are big sized additive outliers, power is high when $\rho \geq -0.4$. In Table 3c ($d = 0.48$) the evidence is very similar as observed in Tables 3a-3b.

Table 4 shows the average number of additive outliers found using τ_d as well the exact number of matches found when we have $ARFIMA(0, d, 0)$ errors. The results are shown for the case where there is no additive outliers and when there are big sized additive outliers. When there are no additive outliers, the average number of additive outliers find by the procedure τ_d confirms the above results related to the exact size. The correct number of matches is, as expected, zero. Both results are very similar for all values of the fractional parameter d. When there are big sized additive outliers, the average number of additive outliers detected is very close to 4 which is the correct number of additive outliers considered in the experiments. The number of correct matches indicates that the procedure performs very well. There are almost 100% of times that the procedure locates correctly the additive outliers. The exception is when $d = -0.48$. Furthermore, we observe that more d is higher, more good are the results both in terms of the average number of additive outliers and the number of correct matches.

Tables 5a-5c are similar to Table 4 but now we consider fractional moving average correlation, that is, $ARFIMA(0, d, 1)$ errors. Both the average number of additive outliers found and the correct number of matches is very good except when $\theta = -0.80$. This situation is better when $d = 0.24$ (Table 5b). This issue is completely fixed when we have a fractional parameter $d = 0.48$. More memory has the process, more we have a correct average number of additive outliers and correct number of matches.

Tables 6a-6c are similar to previous Tables but we now consider fractional autoregressive correlation, that is, $ARFIMA(1, d, 0)$ errors. Now, the average number of found outliers is less than 5% when $\rho = -0.80$, which is opposite to the case where we have moving average autocorrelation. When we have big sized outliers, the average number of found outliers is close to 2 for $\rho = -0.80$. In other cases, this value is close to 4. Of course, the correct number of matches is very low when $\rho = -0.80$ but it is fixed for values of $\rho \geq -0.40$. Similar conclusions are observed in Table 6b ($d = 0.24$). The problem is relatively fixed when $d = 0.48$ (Table 6c). One more time, higher is the fractional parameter, results are better which is consistent with the assumptions of the model and the procedure τ_d.

4 Empirical Application

The Latin-American inflation series offer a good example of the strong presence of big sized additive outliers in a possible nonstationary time series. Figure 1 shows quarterly inflation rates for eight Latin-American countries (Argentina, Bolivia, Chile, Colombia, Ecuador, Peru, Uruguay and Venezuela). Notice that Argentina, Bolivia, Chile and Peru show big sized outliers (see vertical axis). These countries were analyzed by Rodríguez (2004) but using different sample and different frequency. The other countries show presence of outliers but their magnitude is very small compared to the four above mentioned countries. This evidence suggests that the procedure τ_d will find more additive outliers in the countries with large or big sized outliers in comparison with the other countries.

Many or all these countries have experimented with different stabilization programs to stop high inflation episodes. Intervention of this kind, in most of these cases, has introduced additive outliers in the evolution of their inflation series. For example, the periods of high inflation in Argentina and Peru were located between 1985 and 1990, where the most important stabilization programs were applied. For example, in the case of Argentina, the most known governmental plans were the *Austral Program* (June 1985),the program of February of 1987, the *Austral II Program* (October 1987), The *Spring Program* (August 1988), the *BB Program* (1989), The *Bonex Program* (January 1990) and the *Cavallo's Program* (March 1991). The dates in parenthesis correspond to the start date of the programs. In the Peruvian case, we can mention two principal stabilization programs. These are the *Salinas' Program* (September 1988) and the *Fujimori's Program* (July-August 1990). In the Bolivian case, the episode of high inflation was in the middle of 1980. Many small stabilization programs were applied during the period between 1982 and 1984 but it was the program applied in August 1985 which stopped the high inflation. Finally, high inflation in Chile was located around 1975. Diverse programs were applied between 1975 and 1977 until the shock plan applied at the end of 1977 until 1979. A related research to this note is Rodríguez (2004) where four Latin-American countries were analyzed. In this note, we add more countries and more observations. For more details related to the inflationary process in some of these countries, see Rodríguez (2004).

In order to compare the empirical performance of the τ_d with another procedure, we use the approach suggested by Gómez and Maravall (1992) which is used in many government institutions under the name TRAMO (Time Series Regression with ARIMA Noise, Missing Observations, and Outliers) and SEATS (Signal Extraction in ARIMA Time Series). It is worth to mention that the software TRAMO-SEATS also identifies innovative outliers (IO), Level shifts (LS) and temporary change (TC). We applied this approach to our eight Latin-American inflation series and the results are observed in Table 7. In order to preclude an excessive number of additive outliers detected using the τ_d, we use a critical value of 1.0%.

The Table 7 shows outliers identified using the procedure TRAMO-SEATS and τ_d, respectively. In the column of TRAMO-SEATS, the first value indicates the number of all atypical observations which include additive outliers (AO), innovative outliers (IO), level shifts (LS) and temporary changes (TC). In the following column and in parenthesis appear the number of additive outliers which is the number we are interested in. In the column of τ_d, the first value indicates the total number of additive outliers identified while in parenthesis appears the number of additive outliers that coincide with those obtained using the procedure TRAMO-SEATS.

For Argentina, the automatized procedure TRAMO-SEATS found 10 atypical observations from which 6 are additive outliers. The procedure τ_d identifies only 3 additive outliers which are also identified by TRAMO-SEATS. In the case of Bolivia, TRAMO-SEATS identifies 7 additive outliers (from 17 atypical observations) and τ_d locates 16 additive outliers. In Chile, TRAMO-SEATS identifies 8 additive outliers while the procedure τ_d identifies 14 from which four are the same as identified by the automatic procedure. In Colombia the procedure of Gómez and Maravall (1992) identifies 4 additive outliers while the procedure τ_d locates 3 which are the same as these located by the automatic procedure. In Ecuador both procedure identifies 2 additive outliers but dates are not the same. In Peruvian inflation the automatic procedure finds 3 additive outliers while τ_d locates 19 outliers. In Uruguay both procedures find 2 and 3 additive outliers, respectively. Finally, in Venezuela the approach of Gómez and Maravall (1992) finds 3 additive outliers and τ_d locates 5 additive outliers where only one is a common date.

Some comments are worth to be mentioned respect to the number of additive outliers and their location using both procedures. The first comment is the fact that for countries where additive outliers are big sized, the procedure τ_d find many outliers (except for the case of Argentina) which is consistent with the results obtained in the simulations. In the other countries where the size of the additive outliers is significantly low, the procedure detects less additive outliers. The second comment is that the procedure TRAMO-SEATS identifies also many atypical observations but the number of additive outliers is very similar to those obtained using the τ_d except for Bolivia, Chile and Peru. The third comment is that, in most of cases, locations of the additive outliers using τ_d is the same as those located by TRAMO-SEATS. The exception is Ecuador where locations are very different but visual inspection support the findings of τ_d. The fourth comment is that TRAMO-SEATS finds less additive outliers compared to those found by τ_d exceptions are Argentina and Colombia. Accounting LS, TC and IO, the procedure TRAMO-SEATS finds more atypical observations compared to the case of τ_d.

Visual inspection indicated that 4 countries show big sized additive outliers: Argentina, Bolivia, Chile and Peru. In the other cases the presence of additive outliers appears to be limited or less clear. Consequently, it is normal to expect to find more additive outliers in the first group of countries.

It is also supported by the fact that the first four countries above mentioned are these with episodes of high inflation and even hyperinflation.

The Figure 2 shows two issues. The first issue (see left axis) shows original inflation series (dotted line) and the respective inflation series excluding the additive outliers detected by the procedure τ_d (solid line). The second issue (see right axis) shows the original inflation series (dotted line) and the same series excluding the additive outliers detected by the program TRAMO-SEATS (solid line). We may appreciate some important differences between both approaches. For example, in the case of Ecuador, TRAMO-SEATS detects an additive outlier close to the beginning of the sample (1974:2) where inflation rate is less than 10.0%. The procedure τ_d does not qualify this observation as an additive outlier. However, the procedure τ_d detects the big additive outliers in 2000:1 and 2000:2 which are related to around 20.0% on inflation. These two additive outliers are not detected by the procedure TRAMO-SEATS which is surprising.

Another example is Uruguay where the additive outlier of 1972:2 (almost 24.0% of inflation) is not detected by the program TRAMO-SEATS. Venezuela is also another good example. The procedure TRAMO-SEATS detects the observation 1974:3 as an additive outliers which means an inflation rate of around 10.0%. The procedure τ_d does not detect this observation as an additive outlier. However, this procedure (τ_d) detects as additive outliers the observations 1994:3, 1996:1 and 1996:2 (around or more than 20.0%) and surprisingly, TRAMO-SEATS does not select these observations which may be interpreted as a bad performance of this procedure.

Let us take as another example the Peruvian inflation series where many additive outliers have been found. In principle, the number of additive outliers detected may appear excessive. However, the procedure τ_d identifies as additive outliers all quarters related to the large period of high inflation lived by this country until arrive to the episode of hyperinflation. In the Figure 2, we observe that the procedure τ_d locates as additive outliers all quarters covering 1987:4-1992:4 (see solid line compared to dotted line) which are observations of around 200% to 650% of inflation. The procedure TRAMO-SEATS does not detect as additive outliers the observation in the period 1988-1989 where was applied the *Salinas Program* even when these observations are related to level of inflation of around 200.0%.[10]

[10] Of course another explanation to the high number of additive outliers found in the case of Peruvian inflation series is that errors are very far away from a Normal distribution as suggested for Perron and Rodríguez (2003) and analyzed and fixed by Burridge and Taylor (2006). However, visual inspection suggests that the number of outliers detected is consistent with the evolution of inflation in this country.

The empirical evidence shows a good performance of the procedure τ_d in selecting the number and location of the additive outliers. Comparison with a very popular and used procedure as the TRAMO-SEATS indicates that τ_d performs better. There are some evident additive outliers in the inflation series of some countries and the procedure TRAMO-SEATS cannot find them as additive outlier.

5 Conclusions

Perron and Rodríguez (2003) claimed that their procedure to detect for additive outliers (τ_d) is powerful even when we have departures from the unit root case. In this note, we use simulations to show that τ_d is powerful when we have $ARFIMA(p, d, q)$ errors. Using simulations, we calculate the expected number of additive outliers found in this context and the number of times that the approach τ_d identifies the true location of the additive outliers. The results indicate that the power of the procedure τ_d depends of the size of the additive outliers. When we have a DGP with big sized additive outliers the percentage of time that τ_d detects correctly the location of the additive outliers is 100.0%.

An empirical application using eight Latin-American inflation series is performed. Countries like Argentina, Bolivia, Chile and Peru show a larger number of outliers because are the countries where the problem of high inflation or hyperinflation was presented. Other countries (Colombia, Ecuador, Uruguay and Venezuela) show some additive outliers but size of them appears to be smaller compared to the other four countries. A comparison between τ_d and the procedure TRAMO-SEATS is included and the results shows that the former identifies more additive outliers and their location is consistent with the visual inspection and with the historical evolution of the inflation rates in these countries.

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Table 1. Power of τ_d; ARFIMA(0,d,0) Errors

	Probability to find	$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3,$ $\delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$d = 0.00$	1st outlier	0.047	0.049	0.996	0.999	1.000	1.000
	2nd outlier	0.002	0.001	0.671	0.714	1.000	1.000
	3rd outlier	0.000	0.000	0.226	0.222	1.000	1.000
	4th outlier	0.000	0.000	0.039	0.034	0.998	0.998
$d = 0.12$	1st outlier	0.046	0.048	0.999	1.000	1.000	1.000
	2nd outtlier	0.002	0.002	0.740	0.790	1.000	1.000
	3rd outlier	0.000	0.000	0.289	0.292	1.000	1.000
	4th outlier	0.000	0.000	0.059	0.056	0.999	1.000
$d = 0.24$	1st outlier	0.045	0.048	0.999	1.000	1.000	1.000
	2nd outlier	0.003	0.002	0.802	0.856	1.000	1.000
	3rd outlier	0.000	0.000	0.357	0.374	1.000	1.000
	4th outlier	0.000	0.000	0.088	0.087	1.000	1.000
$d = 0.36$	1st outlier	0.044	0.047	1.000	1.000	1.000	1.000
	2nd outlier	0.003	0.003	0.854	0.904	1.000	1.000
	3rd outlier	0.000	0.000	0.429	0.457	1.000	1.000
	4th outlier	0.000	0.000	0.132	0.134	1.000	1.000
$d = 0.48$	1st outlier	0.040	0.044	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.005	0.892	0.935	1.000	1.000
	3rd outlier	0.002	0.002	0.494	0.544	1.000	1.000
	4th outlier	0.002	0.003	0.182	0.203	1.000	1.000

Table 2a. Power of τ_d; ARFIMA(0,d,1) Errors with d=0.00

	Probability to find	$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3, \delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\theta = -0.80$	1st outlier	0.055	0.053	0.743	0.767	1.000	1.000
	2nd outlier	0.002	0.002	0.177	0.162	0.921	0.964
	3rd outlier	0.000	0.000	0.019	0.016	0.777	0.820
	4th outlier	0.000	0.000	0.002	0.001	0.515	0.479
$\theta = -0.40$	1st outlier	0.053	0.050	0.940	0.965	1.000	1.000
	2nd outlier	0.002	0.001	0.393	0.393	0.997	1.000
	3rd outlier	0.000	0.000	0.075	0.065	0.984	0.993
	4th outlier	0.000	0.000	0.007	0.006	0.912	0.907
$\theta = 0.00$	1st outlier	0.047	0.049	0.996	0.999	1.000	1.000
	2nd outlier	0.002	0.001	0.671	0.714	1.000	1.000
	3rd outlier	0.000	0.000	0.226	0.222	1.000	1.000
	4th outlier	0.000	0.000	0.039	0.034	0.998	0.998
$\theta = 0.40$	1st outlier	0.037	0.042	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.004	0.819	0.876	1.000	1.000
	3rd outlier	0.001	0.000	0.377	0.407	1.000	1.000
	4th outlier	0.000	0.000	0.105	0.108	1.000	1.000
$\theta = 0.80$	1st outlier	0.021	0.028	0.999	1.000	1.000	1.000
	2nd outlier	0.005	0.007	0.747	0.820	1.000	1.000
	3rd outlier	0.001	0.001	0.294	0.343	1.000	1.000
	4th outlier	0.001	0.000	0.086	0.096	0.999	1.000

Table 2b. Power of τ_d; ARFIMA(0,d,1) Errors with d=0.24

	Probability to find	$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3, \delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\theta = -0.80$	1st outlier	0.053	0.051	0.883	0.913	1.000	1.000
	2nd outlier	0.002	0.002	0.299	0.289	0.984	0.997
	3rd outlier	0.000	0.000	0.045	0.037	0.944	0.969
	4th outlier	0.000	0.000	0.003	0.002	0.798	0.782
$\theta = -0.40$	1st outlier	0.052	0.050	0.986	0.994	1.000	1.000
	2nd outlier	0.002	0.002	0.557	0.587	1.000	1.000
	3rd outlier	0.000	0.000	0.148	0.138	0.999	1.000
	4th outlier	0.000	0.000	0.021	0.016	0.989	0.988
$\theta = 0.00$	1st outlier	0.045	0.048	0.999	1.000	1.000	1.000
	2nd outlier	0.003	0.002	0.802	0.856	1.000	1.000
	3rd outlier	0.000	0.000	0.357	0.374	1.000	1.000
	4th outlier	0.000	0.000	0.088	0.087	1.000	1.000
$\theta = 0.40$	1st outlier	0.032	0.036	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.005	0.890	0.941	1.000	1.000
	3rd outlier	0.001	0.001	0.497	0.548	1.000	1.000
	4th outlier	0.001	0.001	0.185	0.207	1.000	1.000
$\theta = 0.80$	1st outlier	0.017	0.025	1.000	1.000	1.000	1.000
	2nd outlier	0.005	0.009	0.805	0.879	1.000	1.000
	3rd outlier	0.002	0.003	0.366	0.437	1.000	1.000
	4th outlier	0.001	0.001	0.145	0.018	1.000	1.000

Table 2c. Power of τ_d; ARFIMA(0,d,1) Errors with d=0.48

	Probability to find	$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3, \delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\theta = -0.80$	1st outlier	0.051	0.049	0.962	0.981	1.000	1.000
	2nd outlier	0.002	0.001	0.448	0.459	0.999	1.000
	3rd outlier	0.000	0.000	0.095	0.086	0.994	0.997
	4th outlier	0.000	0.000	0.010	0.007	0.950	0.948
$\theta = -0.40$	1st outlier	0.047	0.051	0.998	1.000	1.000	1.000
	2nd outlier	0.002	0.002	0.715	0.761	1.000	1.000
	3rd outlier	0.000	0.000	0.265	0.267	1.000	1.000
	4th outlier	0.000	0.000	0.052	0.050	1.000	1.000
$\theta = 0.00$	1st outlier	0.040	0.044	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.005	0.892	0.935	1.000	1.000
	3rd outlier	0.002	0.002	0.494	0.544	1.000	1.000
	4th outlier	0.002	0.002	0.182	0.203	1.000	1.000
$\theta = 0.40$	1st outlier	0.022	0.030	1.000	1.000	1.000	1.000
	2nd outlier	0.006	0.011	0.919	0.967	1.000	1.000
	3rd outlier	0.004	0.006	0.569	0.672	1.000	1.000
	4th outlier	0.003	0.005	0.297	0.385	1.000	1.000
$\theta = 0.80$	1st outlier	0.012	0.021	1.000	1.000	1.000	1.000
	2nd outlier	0.007	0.013	0.821	0.906	1.000	1.000
	3rd outlier	0.004	0.007	0.422	0.550	1.000	1.000
	4th outlier	0.003	0.005	0.255	0.362	1.000	1.000

Table 3a. Power of τ_d; ARFIMA(1,d,0) Errors with d=0.00

	Probability to find	$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3,$ $\delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\rho = -0.80$	1st outlier	0.028	0.028	0.373	0.329	0.964	0.984
	2nd outlier	0.003	0.002	0.043	0.028	0.563	0.570
	3rd outlier	0.001	0.000	0.003	0.002	0.277	0.220
	4th outlier	0.000	0.000	0.000	0.000	0.097	0.054
$\rho = -0.40$	1st outlier	0.054	0.049	0.919	0.948	1.000	1.000
	2nd outlier	0.002	0.002	0.359	0.357	0.992	0.999
	3rd outlier	0.000	0.000	0.066	0.054	0.972	0.986
	4th outlier	0.000	0.000	0.006	0.004	0.879	0.866
$\rho = 0.00$	1st outlier	0.047	0.049	0.996	0.999	1.000	1.000
	2nd outlier	0.002	0.001	0.671	0.714	1.000	1.000
	3rd outlier	0.000	0.000	0.226	0.222	1.000	1.000
	4th outlier	0.000	0.000	0.039	0.034	0.998	0.998
$\rho = 0.40$	1st outlier	0.041	0.044	1.000	1.000	1.000	1.000
	2nd outlier	0.003	0.002	0.855	0.906	1.000	1.000
	3rd outlier	0.000	0.000	0.425	0.458	1.000	1.000
	4th outlier	0.000	0.000	0.129	0.132	1.000	1.000
$\rho = 0.80$	1st outlier	0.029	0.0034	1.000	1.000	1.000	1.0000
	2nd outlier	0.006	0.008	0.934	0.975	1.000	1.0000
	3rd outlier	0.004	0.005	0.605	0.691	1.000	1.0000
	4th outlier	0.004	0.005	0.306	0.359	1.000	1.0000

Table 3b. Power of τ_d; ARFIMA(1,d,0) Errors with d=0.24

	Probability to find	$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3,$ $\delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\rho = -0.80$	1st outlier	0.0300	0.029	0.531	0.511	0.993	0.999
	2nd outlier	0.003	0.002	0.088	0.066	0.743	0.783
	3rd outlier	0.000	0.000	0.008	0.004	0.503	0.467
	4th outlier	0.000	0.000	0.001	0.000	0.250	0.181
$\rho = -0.40$	1st outlier	0.053	0.050	0.978	0.991	1.000	1.000
	2nd outlier	0.002	0.002	0.533	0.547	1.000	1.000
	3rd outlier	0.000	0.000	0.138	0.122	0.999	0.999
	4th outlier	0.000	0.000	0.018	0.013	0.980	0.978
$\rho = 0.00$	1st outlier	0.045	0.048	0.999	1.000	1.000	1.000
	2nd outlier	0.003	0.002	0.802	0.856	1.000	1.000
	3rd outlier	0.000	0.000	0.357	0.374	1.000	1.000
	4th outlier	0.000	0.000	0.088	0.087	1.000	1.000
$\rho = 0.40$	1st outlier	0.034	0.040	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.005	0.915	0.957	1.000	1.000
	3rd outlier	0.002	0.002	0.544	0.605	1.000	1.000
	4th outlier	0.001	0.002	0.221	0.246	1.000	1.000
$\rho = 0.80$	1st outlier	0.016	0.026	1.000	1.000	1.000	1.000
	2nd outlier	0.008	0.014	0.926	0.981	1.000	1.000
	3rd outlier	0.006	0.011	0.647	0.780	1.000	1.000
	4th outlier	0.005	0.009	0.472	0.589	1.000	1.000

Table 3c. Power of τ_d; ARFIMA(1,d,0) Errors with d=0.48

	Probability to find	$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 5, \delta_2 = 3,$ $\delta_3 = 2, \delta_4 = 2$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200	T=100	T=200
$\rho = -0.80$	1st outlier	0.031	0.029	0.709	0.710	0.999	1.000
	2nd outlier	0.002	0.002	0.176	0.143	0.895	0.930
	3rd outlier	0.000	0.000	0.021	0.012	0.738	0.752
	4th outlier	0.000	0.000	0.002	0.001	0.499	0.424
$\rho = -0.40$	1st outlier	0.051	0.049	0.997	1.000	1.000	1.000
	2nd outlier	0.003	0.002	0.698	0.739	1.000	1.000
	3rd outlier	0.000	0.000	0.248	0.245	1.000	1.000
	4th outlier	0.000	0.000	0.048	0.042	0.998	0.999
$\rho = 0.00$	1st outlier	0.040	0.044	1.000	1.000	1.000	1.000
	2nd outlier	0.004	0.005	0.892	0.935	1.000	1.000
	3rd outlier	0.002	0.002	0.494	0.544	1.000	1.000
	4th outlier	0.002	0.002	0.182	0.203	1.000	1.000
$\rho = 0.40$	1st outlier	0.024	0.031	1.000	1.000	1.000	1.000
	2nd outlier	0.007	0.012	0.932	0.979	1.000	1.000
	3rd outlier	0.005	0.009	0.615	0.728	1.000	1.000
	4th outlier	0.005	0.008	0.363	0.466	1.000	1.000
$\rho = 0.80$	1st outlier	0.005	0.012	0.999	1.000	1.000	1.000
	2nd outlier	0.004	0.010	0.834	0.946	0.998	1.000
	3rd outlier	0.004	0.009	0.657	0.826	0.995	1.000
	4th outlier	0.003	0.009	0.613	0.787	0.993	1.000

Table 4. Matches of τ_d; ARFIMA(0,d,0) Errors

		$\delta_1= 0, \delta_2= 0, \delta_3= 0, \delta_4= 0$		$\delta_1= 10, \delta_2= 5, \delta_3= 5, \delta_4= 5$	
		T=100	T=200	T=100	T=200
$d = -0.48$	Average found outliers	0.052	0.052	3.842	3.854
	Matches	0.000	0.000	0.870	0.859
$d = -0.36$	Average found outliers	0.053	0.051	3.932	3.932
	Matches	0.000	0.000	0.937	0.929
$d = -0.24$	Average found outliers	0.052	0.052	3.976	3.974
	Matches	0.000	0.000	0.975	0.971
$d = -0.12$	Average found outliers	0.050	0.052	3.994	3.994
	Matches	0.000	0.000	0.991	0.992
$d = 0.00$	Average found outliers	0.049	0.051	3.998	3.998
	Matches	0.000	0.000	0.996	0.997
$d = 0.12$	Average found outliers	0.049	0.050	3.999	4.000
	Matches	0.000	0.000	0.999	0.999
$d = 0.24$	Average found outliers	0.048	0.051	4.000	4.000
	Matches	0.000	0.000	1.000	0.999
$d = 0.36$	Average found outliers	0.048	0.050	4.000	4.000
	Matches	0.000	0.000	1.000	1.000
$d = 0.48$	Average found outliers	0.047	0.052	4.000	4.000
	Matches	0.000	0.000	1.000	1.000

Table 5a. Matches of τ_d Test; ARFIMA(0,d,1) Errors with d=0.00

		$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\theta = -0.80$	Average found outliers	0.057	0.055	3.212	3.263
	Matches	0.000	0.000	0.499	0.460
$\theta = -0.40$	Average found outliers	0.055	0.052	3.893	3.899
	Matches	0.000	0.000	0.907	0.900
$\theta = 0.00$	Average found outliers	0.049	0.051	3.998	3.998
	Matches	0.000	0.000	0.996	0.997
$\theta = 0.40$	Average found outliers	0.041	0.046	4.000	4.000
	Matches	0.000	0.000	0.998	0.998
$\theta = 0.80$	Average found outliers	0.028	0.036	3.999	4.000
	Matches	0.000	0.000	0.986	0.986

Tables 5b. Matches of τ_d Test; ARFIMA(0,d,1) Errors with d=0.24

		$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\theta = -0.80$	Average found outliers	0.054	0.053	3.725	3.747
	Matches	0.000	0.000	0.789	0.770
$\theta = -0.40$	Average found outliers	0.054	0.052	3.988	3.987
	Matches	0.000	0.000	0.987	0.986
$\theta = 0.00$	Average found outliers	0.048	0.051	4.000	4.000
	Matches	0.000	0.000	1.000	0.999
$\theta = 0.40$	Average found outliers	0.038	0.043	4.000	4.000
	Matches	0.000	0.000	0.998	0.998
$\theta = 0.80$	Average found outliers	0.025	0.038	4.000	4.000
	Matches	0.000	0.000	0.990	0.989

Table 5c. Matches of τ_d Test; ARFIMA(0,d,1) Errors with d=0.48

		$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\theta = -0.80$	Average found outliers	0.053	0.050	3.943	3.945
	Matches	0.000	0.000	0.947	0.943
$\theta = -0.40$	Average found outliers	0.050	0.054	3.999	4.000
	Matches	0.000	0.000	0.998	0.999
$\theta = 0.00$	Average found outliers	0.047	0.052	4.000	4.000
	Matches	0.000	0.000	1.000	1.000
$\theta = 0.40$	Average found outliers	0.034	0.051	4.000	4.000
	Matches	0.000	0.000	0.998	0.997
$\theta = 0.80$	Average found outliers	0.025	0.046	4.000	4.000
	Matches	0.000	0.000	0.978	0.971

Table 6a. Matches of τ_d Test; ARFIMA(1,d,0) Errors with d=0.00

		$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\rho = -0.80$	Average found outliers	0.031	0.030	1.902	1.828
	Matches	0.000	0.000	0.093	0.049
$\rho = -0.40$	Average found outliers	0.056	0.051	3.843	3.851
	Matches	0.000	0.000	0.875	0.859
$\rho = 0.00$	Average found outliers	0.049	0.051	3.998	3.998
	Matches	0.000	0.000	0.996	0.997
$\rho = 0.40$	Average found outliers	0.045	0.046	4.000	4.000
	Matches	0.000	0.000	0.999	0.999
$\rho = 0.80$	Average found outliers	0.043	0.053	4.000	4.000
	Matches	0.000	0.000	0.999	0.999

Table 6b. Matches of τ_d Test; ARFIMA(1,d,0) Errors with d=0.24

		$\delta_1 = 0, \delta_2 = 0, \delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5, \delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\rho = -0.80$	Average found outliers	0.033	0.031	2.495	2.429
	Matches	0.000	0.000	0.245	0.172
$\rho = -0.40$	Average found outliers	0.056	0.052	3.979	3.977
	Matches	0.000	0.000	0.979	0.976
$\rho = 0.00$	Average found outliers	0.048	0.051	4.000	4.000
	Matches	0.000	0.000	1.000	0.999
$\rho = 0.40$	Average found outliers	0.042	0.049	4.000	4.000
	Matches	0.000	0.000	0.999	0.999
$\rho = 0.80$	Average found outliers	0.035	0.059	4.000	4.000
	Matches	0.000	0.000	0.983	0.983

Table 6c. Matches of τ_d Test; ARFIMA(1,d,0) Errors with d=0.48

		$\delta_1 = 0, \delta_2 = 0,$ $\delta_3 = 0, \delta_4 = 0$		$\delta_1 = 10, \delta_2 = 5,$ $\delta_3 = 5, \delta_4 = 5$	
		T=100	T=200	T=100	T=200
$\rho = -0.80$	Average found outliers	0.033	0.031	3.131	3.105
	Matches	0.000	0.000	0.495	0.415
$\rho = -0.40$	Average found outliers	0.054	0.053	3.998	3.999
	Matches	0.000	0.000	0.998	0.999
$\rho = 0.00$	Average found outliers	0.047	0.052	4.000	4.000
	Matches	0.000	0.000	1.000	1.000
$\rho = 0.40$	Average found outliers	0.041	0.059	4.000	4.000
	Matches	0.000	0.000	0.998	0.995
$\rho = 0.80$	Average found outliers	0.016	0.040	3.986	4.000
	Matches	0.000	0.000	0.691	0.622

Table 7. Empirical Application: Detection of Additive Outliers in Quarterly Latin-American Inflation Series

		TRAMO-SEATS	τ_d	
		(Additive Outliers)	Additive Outliers	Same location
Argentina	Number of Additive Outliers	10 (6)	3	2
Bolivia	Number of Additive Outliers	17 (7)	16	4
Chile	Number of Additive Outliers	23 (8)	14	4
Colombia	Number of Additive Outliers	10 (4)	3	3
Ecuador	Number of Additive Outliers	9 (2)	2	0
Peru	Number of Additive Outliers	7 (3)	19	1
Uruguay	Number of Additive Outliers	5 (2)	3	2
Venezuela	Number of Additive Outliers	3 (3)	5	1

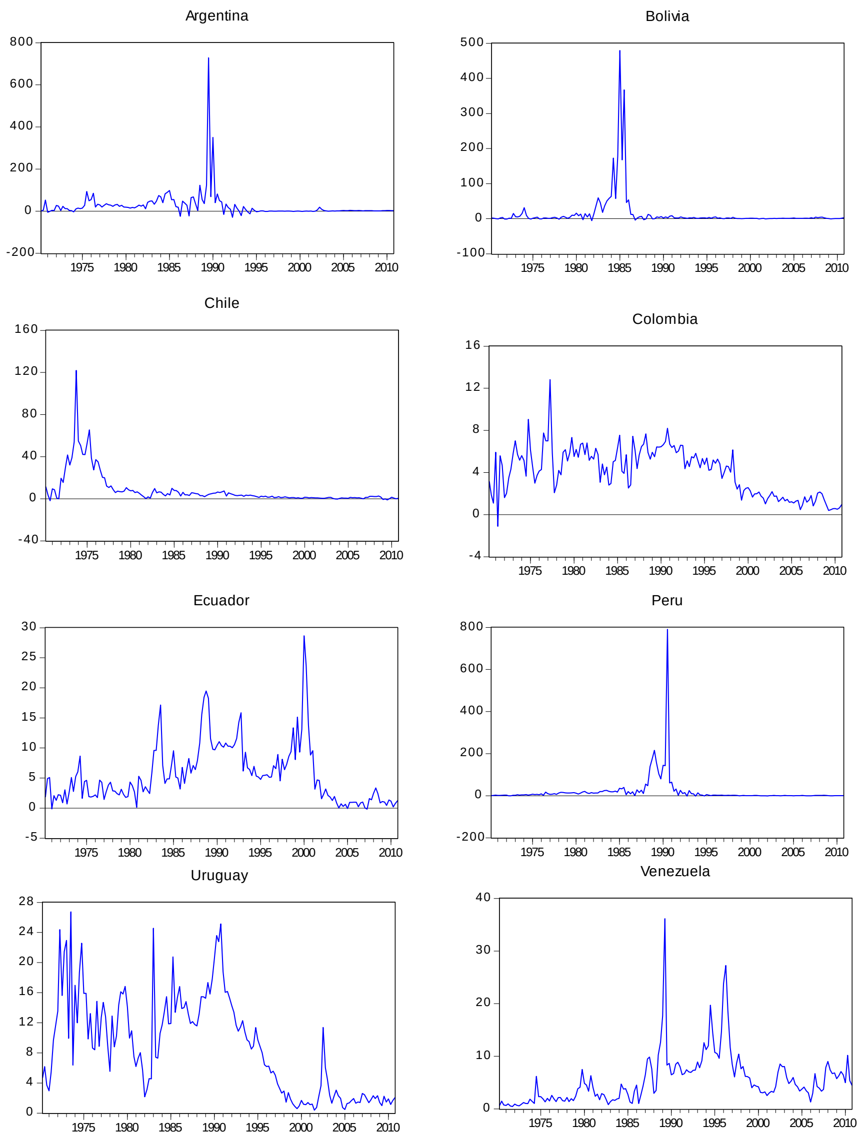

Figure 1. Quarterly Latin-American Inflation Series

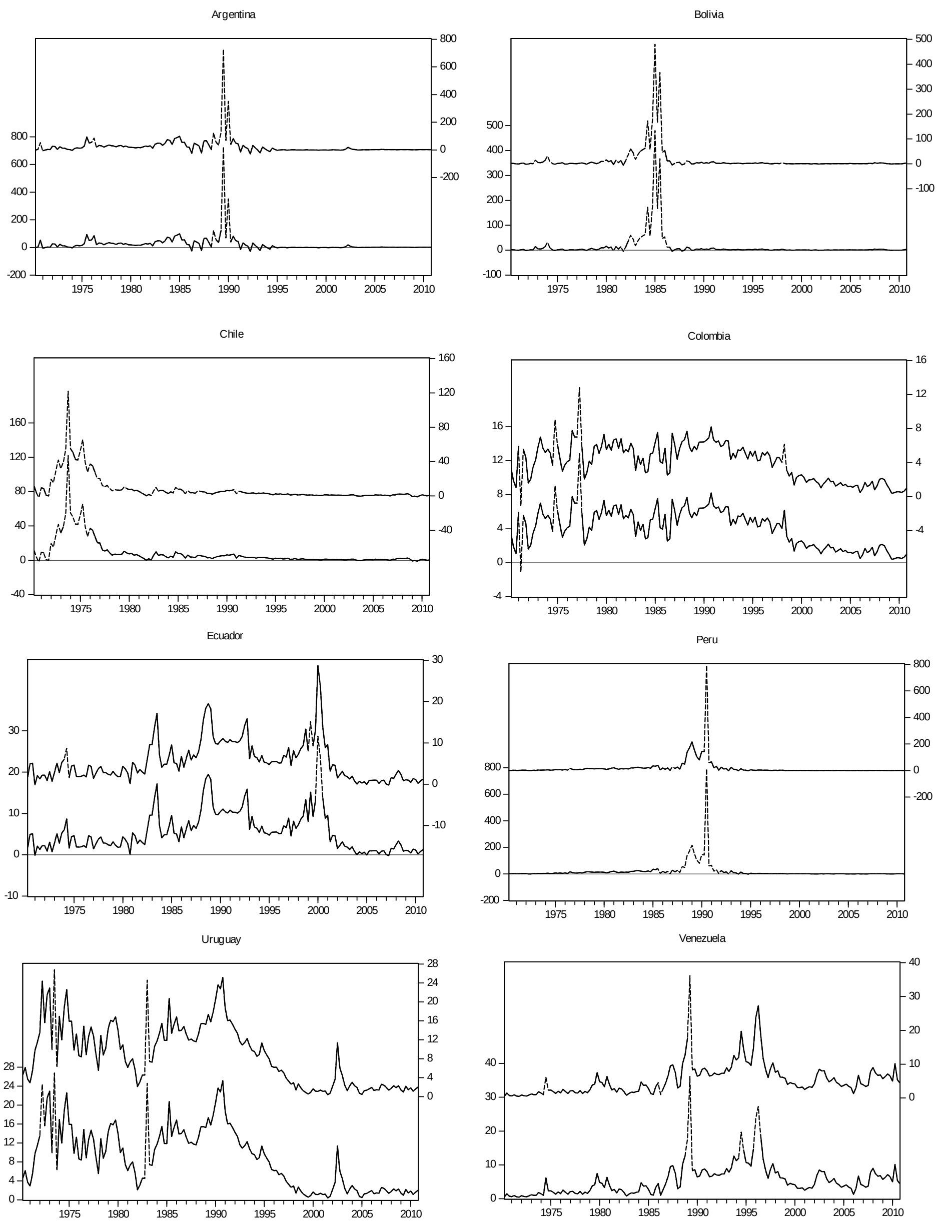

Figure 2. Quarterly Latin-American Inflation Series. Left Axis: Inflation (Original: dotted line) and Inflation using PR (2003): solid line; Rigth Axis: Inflation (Original: dotted line) Inflation Series using TRAMO-SEATS: solid line)

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