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APPROXIMATE BAYESIAN ESTIMATION OF STOCHASTIC VOLATILITY IN MEAN MODELS USING HIDDEN MARKOV MODELS: EMPIRICAL EVIDENCE FROM STOCK LATIN AMERICAN MARKETS

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Approximate Bayesian Estimation of Stochastic Volatility in Mean Models using Hidden Markov Models: Empirical Evidence from Stock Latin American Markets

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September 27, 2021

Abstract

The stochastic volatility in mean (SVM) model proposed by Koopman and Uspensky (2002) is revisited. This paper has two goals. The first is to offer a methodology that requires less computational time in simulations and estimates compared with others proposed in the literature as in Abanto-Valle et al. (2021) and others. To achieve the first goal, we propose to approximate the likelihood function of the SVM model applying Hidden Markov Models (HMM) machinery to make possible Bayesian inference in real-time. We sample from the posterior distribution of parameters with a multivariate Normal distribution with mean and variance given by the posterior mode and the inverse of the Hessian matrix evaluated at this posterior mode using importance sampling (IS). The frequentist properties of estimators is anlyzed conducting a simulation study. The second goal is to provide empirical evidence estimating the SVM model using daily data for five Latin American stock markets. The results indicate that volatility negatively impacts returns, suggesting that the volatility feedback effect is stronger than the effect related to the expected volatility. This result is exact and opposite to the finding of Koopman and Uspensky (2002). We compare our methodology with the Hamiltonian Monte Carlo (HMC) and Riemannian HMC methods based on Abanto-Valle et al. (2021).

Keywords: Stock Latin American Markets, Stochastic Volatility in Mean, Feed-Back Effect, Hamiltonian Monte Carlo, Hidden Markov Models, Riemannian Manifold Hamiltonian Monte Carlo, Non Linear State Space Models.

JEL Classification: C11, C15, C22, C51, C52, C58, G12.

Estimación Bayesiana Aproximada de Modelos de Volatilidad Estocástica en la Media usando Modelos Hidden Markov: Evidencia Empírica para Mercados Bursátiles de América Latina

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Resumen

El modelo de volatilidad estocástica en la media (SVM) propuesto por Koopman and Uspensky (2002) es revisado. Este documento tiene dos objetivos. El primero es ofrecer una metodología que requiere menos tiempo computacional en simulaciones y estimaciones comparadas con otras propuestas en la literatura como en Abanto-Valle et al. (2021) y otros. Para lograr el primer objetivo, proponemos aproximamos la función de verosimilitud del modelo SVM aplicando *Hidden Markov Models (HMM)* para hacer posible la inferencia bayesiana en tiempo real. Tomamos muestras de la distribución posterior de los parámetros usando una distribución Normal multivariada con media y varianza dadas por la moda posterior y la inversa de la matriz Hessiana evaluada en esta moda posterior usando *importance sampling* (IS). Usando simulaciones, las propiedades frecuentistas de los estimadores son analizadas. El segundo objetivo es proporcionar evidencia empírica estimando el modelo SVM utilizando datos diarios para cinco mercados bursátiles de América Latina. Los resultados indican que la volatilidad impacta negativamente en los rendimientos sugiriendo que el efecto de retroalimentación de la volatilidad es más fuerte que el efecto relacionado con la volatilidad esperada. Este resultado es exacto y opuesto al hallazgo de Koopman and Uspensky (2002). Nosotros comparamos nuestra metodología con los algoritmos de *Hamiltoniano Montecarlo (HMC)* y *Riemannian HMC* basados en Abanto-Valle et al. (2021).

Palabras Claves: Mercado Bursátiles de América Latina, Volatilidad Estocástica en Media, Efecto Feed-Back, *Hamiltonian Monte Carlo*, *Hidden Markov Models*, *Riemannian Manifold Hamiltonian Monte Carlo*, Modelos Espacio Estado No Lineales.

Clasificación JEL: C11, C15, C22, C51, C52, C58, G12.

1 Introduction

Stochastic volatility (SV) models initially proposed by Taylor (1982, 2008), compose a well-known class of models to estimate volatility. These models have gained attention in the financial econometrics literature because of their flexibility to capture the nonlinear behavior observed in financial time series returns[1]. According to Melino and Turnbull (1990) and Carnero et al. (2004), an appealing aspect of the SV model is its close association to financial economic theories and its ability to capture the stylized facts often observed in daily series of financial returns in a more appropriate way.

The daily asymmetrical relation between equity market returns and volatility has received a lot of attention in the financial literature; see for instance, Black (1976), Campbell and Entchel (1992), and Bekaert and Wu (2000). Asymmetric equity market volatility is important for al least three reasons. First, it is an important characteristic of the market volatility dynamics, has asset pricing implications and is a feature of priced risk factors. Second, it plays an important role in risk prediction, hedging and option pricing. Finally, asymmetric volatility implies negatively skewed returns distributions, i.e. it may help explain some of the market's chances of losing.

On the other hand, the relation between expected returns and expected volatility have been extensively examined in recent years. Overall, there appears to be stronger evidence of a negative relationship between unexpected returns and innovations to the volatility process, which French et al. (1987) interpreted as indirect evidence of a positive correlation between the expected risk premium and *ex ante* volatility. If expected volatility and expected returns are positively related and future cash flows are unaffected, the current stock index price should fall. Conversely, small shocks to the return process lead to an increase in contemporaneous stock index prices. This theory, known as the volatility feedback theory hinges on two assumptions: first, the existence of a positive relation between the expected components of the return and volatility process and second, volatility persistence. An alternative explanation for asymmetric volatility where causality runs in the opposite direction is the leverage effect put forward by Black (1976), who asserted that a negative (positive) return shock leads to an increase (decrease) in the firm's financial leverage ratio, which has an upward (downward) effect on the volatility of its stock returns. However, French et al. (1987) and Schwert (1989) argued that leverage alone cannot account for the magnitude of the negative relationship. For example, Campbell and Entchel (1992) found evidence of both volatility feedback and leverage effects, whereas Bekaert and Wu (2000) presented results suggesting that the volatility feedback effect dominates the leverage effect empirically.

[1]The other important branch of the literature is GARCH models where time-varying variance is modeled as a deterministic function of past squared perturbations and lagged conditional variances. Details and explanations of the extensive GARCH literature may be found in Bollerslev et al. (1992, 1994) and Diebold and Lopes (1995). On the other hand, SV models are reviewed in Taylor (1994), Ghysels et al. (1994), and Shephard (1996); among others.

Frequently, the volatility of daily stock returns has been estimated with SV models, but the results have relied on an extensive pre-modeling of these series to avoid the problem of simultaneous estimation of the mean and variance. Koopman and Uspensky (2002) introduced the SV in mean (SVM) model to deal with this problem. In the SVM setup, the unobserved volatility is incorporated as an explanatory variable in the mean equation of the returns under the normality assumption of the innovations. Moreover, they derived an exact maximum likelihood estimation based on Monte Carlo simulation methods. Abanto-Valle et al. (2012) extended the SVM model to the class of scale mixture of Normal distributions and developed an Markov Chain Monte Carlo (MCMC) algorithm to sample parameters and the log-volatilities from a Bayesian perspective. Recently, Abanto-Valle et al. (2021) apply Hamiltonian Monte Carlo (HMC) and Riemann Manifold HMC (RMHMC) methods within the MCMC algorithm to update the log-volatilities and parameters of the SVM model, respectively. However, the resulting MCMC algorithms has some undesirable features. In particular, the procedure is quite involved, requiring a large amount of computer-intensive simulations. In addition, the computational cost increases rapidly with the sample size.

This paper has two objectives. The first is to offer an algorithm that requires less computational time in simulations and estimates even when the sample increases as compared with MCMC algorithms as proposed in Abanto-Valle et al. (2021). For this, this article applies an alternative approximate Bayesian estimation method to the SVM model considered by Abanto-Valle et al. (2012) and Abanto-Valle et al. (2021). First, we approximate the likelihood function by integrating out the log-volatilities as suggested by Langrock (2011), Langrock et al. (2012) and Abanto-Valle et al. (2017). Second, we get the maximum a posteriori by using a numerical optimization routine, and third, we use importance sampling to sample from the posterior distribution of the parameters using a multivariate normal distribution where the mean and variance are given by the maximum a posteriori and the inverse of the Hessian matrix evaluated at the maximum a posteriori, respectively.

The second objective is to provide empirical evidence estimating the SVM model using daily data for five Latin American stock markets. Time-varying volatility for developed economies' financial variables have been studied extensively; see for instance, Liesenfeld and Jung (2000), Jacquier et al. (2004) and Abanto-Valle et al. (2010). However, empirical studies of the volatility characteristics of the financial markets in Latin America are very scarce and are far from being thoroughly analyzed despite their growth in recent years, see Abanto-Valle et al. (2011), Rodríguez (2016), Rodríguez (2017), Rodríguez (2017), Lengua Lafosse and Rodríguez (2018), Alanya and Rodríguez (2019). Moreover, Abanto-Valle et al. (2021) use HMC amd RMHMC methods to analyse the SVM model using Latin American markets. For this reason, we perform a detailed empirical study of five Latin American indexes: MERVAL (Argentina), IBOVESPA (Brazil), IPSA (Chile), MEXBOL (Mexico) and IGBVL (Peru) in the context of the SVM model using the HMM approach. We also include the S&P 500 returns in order to perform some comparisons. All the results are in line with Abanto-Valle et al. (2021).

The remainder of this paper is organized as follows. Section 2 describes the SVM model, the approximated likelihood of the SVM model based on hiden Markov models (HMM) techniques and the Bayesian inference procedure. In section 3, we conduct a simulation study to verify the frequentist properties of estimators compared to the methods used in Abanto-Valle et al. (2021) including computational time intensity. Section 4 is devoted to the application of the proposed methodology to five indexes of Latin American countries and the S&P 500. Finally, some concluding remarks and suggestions for future developments are given in Section 5.

2 The Stochastic Volatility in Mean (SVM) Model

The SVM model is defined by

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 e^{h_t} + e^{\frac{h_t}{2}} \epsilon_t, \tag{1a}$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \sigma\eta_t, \tag{1b}$$

where y_t and h_t are, respectively, the compounded return and the log-volatility at time t, for $t = 1, \ldots, T$. We assume that $|\beta_1| < 1$, $|\phi| < 1$, i.e., that the returns and the log-volatility process are stationary and that the initial value $h_1 \sim \mathcal{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$. Furthermore, the innovations ϵ_t and η_t are assumed to be mutually independent and normally distributed with mean zero and unit variance. The SVM model incorporates the unobserved volatility as an explanatory variable in the mean equation. Therefore, the aim of the SVM model is to estimate the ex-ante relation between returns and volatility and the volatility feedback effect (parameter β_2).

2.1 Likelihood evaluation by iterated numerical integration and fast evaluation of the approximate likelihood using HMM techniques

To formulate the likelihood, we require the conditional *pdfs* of the random variables y_t, given h_t and y_{t-1} ($t = 1, \ldots, T$), and of the random variables h_t, given h_{t-1} ($t = 2, \ldots, T$). We denote these by $p(y_t \mid y_{t-1}, h_t)$ and $p(h_t \mid h_{t-1})$, respectively. The likelihood of the SVM model defined by equations (1a) and (1b) can then be derived as

$$\begin{aligned} \mathcal{L} &= \int \ldots \int p(y_1, \ldots, y_T, h_1, \ldots, h_T \mid y_0) dh_T \ldots dh_1 \\ &= \int \ldots \int p(y_1, \ldots, y_T \mid y_0, h_1, \ldots, h_T) p(h_1, \ldots, h_T) dh_T \ldots dh_1 \\ &= \int \ldots \int p(h_1) p(y_1 \mid y_0, h_1) \prod_{t=2}^{T} p(y_t \mid y_{t-1}, h_t) p(h_t \mid h_{t-1}) dh_T \ldots dh_1. \end{aligned}$$

Hence, the likelihood is a high-order multiple integral that cannot be evaluated analytically. Through numerical integration, using a simple rectangular rule based on m equidistant intervals, $B_i = (b_{i-1}, b_i)$, $i = 1, \ldots, m$, with midpoints b_i^* and length b, the likelihood can be approximated as follows:

$$\begin{aligned}\mathcal{L} &\approx b^T \sum_{i_1=1}^{m} \ldots \sum_{i_T=1}^{m} p(h_1 = b_{i_1}^*) p(y_1 \mid y_0, h_1 = b_{i_1}^*) \\ &\quad \times \prod_{t=2}^{T} p(y_t \mid y_{t-1}, h_t = b_{i_t}^*) p(h_t = b_{i_t}^* \mid h_{t-1} = b_{i_{t-1}}^*) = \mathcal{L}_{\text{approx}} \,. \end{aligned} \tag{2}$$

This approximation can be made arbitrarily accurate by increasing m, provided that the interval (b_0, b_m) covers the essential range of the log-volatility process. We note that this simple midpoint quadrature is by no means the only way in which the integral can be approximated (cf. Langrock et al., 2012). The numerical evaluation of approximate likelihood given in (2) will usually be computationally intractable since it involves m^T summands. However, it can be evaluated numerically using the tools well-established for HMMs, which are the models that have exactly the same dependence structure as the stochastic volatility in mean models, but with a finite and hence discrete state space (cf. Langrock, 2011; Langrock et al., 2012). In the given scenario, the discrete states correspond to the intervals B_i, $i = 1, \ldots, m$, in which the state space has been partitioned. A fundamental property of HMM, which we exploit here, is that the likelihood can be evaluated efficiently using the so-called forward algorithm, a recursive scheme which iteratively traverses forward along the time series, updating the likelihood and the state probabilities in each step (Zucchini et al., 2016). Under the HMM, to apply the forward algorithm results in a convenient closed-form matrix product expression for the likelihood. For the SVM model is given by:

$$\mathcal{L}_{\text{approx}} = \boldsymbol{\delta}\mathbf{P}(y_1)\boldsymbol{\Gamma}\mathbf{P}(y_2)\boldsymbol{\Gamma}\mathbf{P}(y_3)\cdots\boldsymbol{\Gamma}\mathbf{P}(y_{T-1})\boldsymbol{\Gamma}\mathbf{P}(y_T)\mathbf{1}' \,. \tag{3}$$

Note that, in equation (3), the $m \times m$-matrix $\boldsymbol{\Gamma} = (\gamma_{ij})$ plays the role of the transition probability matrix in case of an HMM, defined by $\gamma_{ij} = p(h_t = b_j^* \mid h_{t-1} = b_i^*) \cdot b$, which is an approximation of the probability of the log-volatility process changing from some value in the interval B_i to some value in the interval B_j; this conditional probability is determined by (1b). The vector $\boldsymbol{\delta}$ is the analogue to the Markov chain initial distribution in case of an HMM. It is defined such that δ_i is the density of the $\mathcal{N}(\mu, \frac{\sigma_\eta^2}{1-\phi^2})$-distribution –the stationary distribution of the log-volatility process– multiplied by b. Furthermore, $\mathbf{P}(y_t)$ is an $m \times m$ diagonal matrix with the ith diagonal entry $p(y_t \mid y_{t-1}, h_t = b_i^*)$ determined by (1a). Finally, $\mathbf{1}'$ is a column vector of ones.

Using the matrix product expression given in (3), the computational effort required to evaluate the approximate likelihood is linear in the number of observations, say T, and quadratic in the number of intervals used in the discretization, say m. In other words, the likelihood can be calculated in a fraction of seconds, even for high values of T and m. Furthermore, the approximation can be‘

Although we are using the HMM forward algorithm to evaluate the (approximate) likelihood, the specifications of $\boldsymbol{\delta}$, $\boldsymbol{\Gamma}$ and $\mathbf{P}(x_t)$ given above do not define precisely an HMM. In general, the row sums of $\boldsymbol{\Gamma}$ will be only approximately equal to one, and the vector components $\boldsymbol{\delta}$ will only approximately sum to one. If desired, this can be remedied by scaling each row of $\boldsymbol{\Gamma}$ and the vector $\boldsymbol{\delta}$ to total 1.

2.2 Bayesian Inference for the SVM model

Because we have some constraints in the original parametric space of the SVM model ($|\beta_1| < 1, |\phi| < 1, \sigma_\eta > 0$), we consider the transformations for the following parameters : $\gamma = \log\left(\frac{1+\beta_1}{1-\beta_1}\right)$, $\psi = \log\left(\frac{1+\phi}{1-\phi}\right)$, and $\omega = \log(\sigma)$. Let $\boldsymbol{\theta} = (\beta_0, \gamma, \beta_2, \mu, \psi, \omega)'$ and $p(\boldsymbol{\theta})$ be the prior distribution of $\boldsymbol{\theta}$. As the likelihood function is invariant to 1:1 transformations, from equation (3), we obtain the posterior distribution up to a normalization constant, namely:

$$p(\boldsymbol{\theta} \mid y_0, \mathbf{y}_T) \propto p(\boldsymbol{\theta})\mathcal{L}_{\text{approx}}(\boldsymbol{\theta}), \tag{4}$$

where $\mathbf{y}_T = (y_1, \ldots, y_T)'$. Suposse we wish to calculate an expectation $E_{p(\boldsymbol{\theta}|y_0,\mathbf{y}_T)}[h(\boldsymbol{\theta})]$, which can be calculated by using the importance density $q(\boldsymbol{\theta})$ as follows:

$$\begin{aligned}
E_{p(\boldsymbol{\theta}|y_0,\mathbf{y}_T)}[h(\boldsymbol{\theta})] &= \frac{\int h(\boldsymbol{\theta})p(\boldsymbol{\theta} \mid y_0, \mathbf{y}_T)d\boldsymbol{\theta}}{\int p(\boldsymbol{\theta} \mid y_0, \mathbf{y}_T)d\boldsymbol{\theta}} \\
&= \frac{\int \frac{h(\boldsymbol{\theta})p(\boldsymbol{\theta}|y_0,\mathbf{y}_T)}{q(\boldsymbol{\theta})}q(\boldsymbol{\theta})d\boldsymbol{\theta}}{\int \frac{p(\boldsymbol{\theta}|y_0,\mathbf{y}_T)}{q(\boldsymbol{\theta})}q(\boldsymbol{\theta})d\boldsymbol{\theta}} = \frac{E_{q(\boldsymbol{\theta})}\Big[h(\boldsymbol{\theta})\boldsymbol{\omega}(\boldsymbol{\theta})\Big]}{E_{q(\boldsymbol{\theta})}\Big[\boldsymbol{\omega}(\boldsymbol{\theta})\Big]},
\end{aligned} \tag{5}$$

where $\boldsymbol{\omega}(\boldsymbol{\theta}) = \frac{p(\boldsymbol{\theta}|y_0,\mathbf{y}_T)}{q(\boldsymbol{\theta})}$ and $E_q[.]$ denotes an expected value with respect to the importance density $q(\boldsymbol{\theta})$. Therefore a sample of independent draws $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_m$ from $q(\boldsymbol{\theta})$ can be used to estimate $E_{p(\theta|y_0,\mathbf{y}_T)}[h(\boldsymbol{\theta})]$ by

$$\bar{h} = \frac{\sum_{i=1}^m h(\boldsymbol{\theta}_i)\boldsymbol{\omega}(\theta_i)}{\sum_{i=1}^m \boldsymbol{\omega}(\theta_i)}. \tag{6}$$

It is shown that using one sample $\boldsymbol{\theta}_i's$ in estimating the ratio in (5) is more efficient than using two samples (one for the numerator and another for denominator); see Chen et al. (2008). It follows from the strong law of large numbers that $\bar{h} \to E_{p(\theta|y_0,\mathbf{y}_T)}[h(\boldsymbol{\theta})]$ as $m \to \infty$ almost surely; see Geweke (1989). In the same way, a variance of $\bar{h}(\boldsymbol{\theta})$ can be consistenly estimated by $\sum_{i=1}^m \boldsymbol{\omega}(\boldsymbol{\theta}_i)^2[h(\boldsymbol{\theta}_i) - \bar{h}]^2/[\sum_{i=1}^m \boldsymbol{\omega}(\boldsymbol{\theta}_i)]^2$.

3 Simulation Study

To assess the performance of the methodology described in the previous Section, we conducted some simulation experiments. All the calculations were performed using stand-alone code developed by the authors using the Rcpp interface inside the R package. First, we simulated a data set comprising $T = 6000$ observations from the SVM model, specifying $\boldsymbol{\beta} = (0.14, 0.03, -0.10)'$, $\mu = 0.3$, $\phi = 0.98$, $\sigma_\eta = 0.2$ and $y_0 = 0.2$, which correspond to typical values found in daily series of returns; see for example Leão et al. (2017) and Abanto-Valle et al. (2017). The resulting transformed true parameter vector $\boldsymbol{\theta} = (\beta_0, \gamma, \beta_2, \mu, \psi, \omega)' =$

$(0.14, 0.06, -0.10, 0.30, 4.5951, -1.6094)'$. Figure 1 shows the resulting artificial data set. We set the priors distributions as follows: $(\beta_0, \gamma, \beta_2) \sim \mathcal{N}_3(\mathbf{0_3}, 100\mathbf{I}_3)$, $\mu \sim \mathcal{N}(0, 100)$, $\psi \sim \mathcal{N}(4.5, 100)$ and $\omega \sim \mathcal{N}(-1.5, 100)$, where $\mathcal{N}_r(.,.)$ and $\mathcal{N}(.,.)$ denote the $r-$variate and univariate normal distributions and $\mathbf{0_r}$ and $\mathbf{I}_r$ are the $r \times 1$ vector of zeros and the $r \times r$ the identity matrix, respectively.

In order to investigate the influence of the choice of m on the accuracy of the likelihood approximation in the posterior distribution, and of the sample size T on the computing time, we fitted the SVM model using $m = 50, 100, 150, 200$ (i.e., different levels of accuracy), $b_m = -b_0 = 4$, to subsamples of length $T = 1500, 3000, 6000$ of the original simulated series. Table 1 reports the results of the maximum to posteriori (MAP). In general, we observe that all MAP estimates approach their true values as we go from $T = 1500$ to $T = 6000$. This convergence is faster and clearer for ψ and ω. In the case of μ, the MAP estimates are different for $T = 1500$ observations but they converge to their true value rapidly when sample size increases. In particular, the MAP for β_2 approaches its true value when T=6000. The log likelihood value is fairly stable for any value of m and sample size. Most of MAP estimates obtained by numerical maximization become stable for values of m around 100, for all the sample sizes considered here. However, when $T = 6000$, we observe that stabilization is reached for $m = 150$ or more. Therefore, we recommend to use $m = 150, 200$ in empirical applications.

Next, the multivariate normal distribution with mean and covariace matrix being the MAP and the inverse of the hessian matrix evaluated at the MAP is used as an importance density. We draw a sample of size 1000 using sampling importance resampling. Based on it, we calculate the expected value (posterior mean) and the standar deviation in the original escale using equation (6). The results are reported in Table 2. In all the cases the posterior credibility intervals of 95% contain the true value of the parameters. Considering a time serie of size 6000, the proposal methodology spend 174.18 seconds to simulate and report the results, making useful our proposal in real time applications.

We also investigated the influence of the choice of b_0 and b_m (the results are not presented here for space reasons). Overall, we observe that the estimator performance is not affected much. However, when these values are chosen either too small (not covering the support of the log-volatility process, e.g., $b_m = -b_0 = 2$) or too large (leading to a partition of the support into unnecessarily wide intervals and poor approximation of the likelihood, e.g. $m = 50$ and $b_m = -b_0 = 15$), the estimator performance could be affected. In practice, it can easily be checked *post-hoc* if the chosen range, specified by b_0 and b_m, is adequate, by investigating the stationary distribution of the fitted log-volatility process.

The second simulation experiment studies the properties of the estimators of the SVM model parameters. We generated 300 datasets from the SVM model, specifying $\boldsymbol{\beta} = (0.14, 0.03, -0.10)'$, $\phi = 0.98$, $\sigma = 0.2$, $\mu = 0.30$. For each generated data set, we fitted the SVM model using $m = 50, 100, 150, 200$ and $b_m = -b_0 = 4$, for $T = 1500$, $T = 3000$ and $T = 6000$, respectively. Tables 3, 4 and 5 report the sample mean, the mean relative bias (MRB), the mean relative absolute deviation (MRAD) and the mean squared error (MSE) of the parameter estimates for $T = 1500, 3000, 6000$, respectively.

For all the sample sizes, i.e., $T = 1500$, $T = 3000$, and $T = 6000$, higher values of MRAD are found for the estimator of β_1 and μ, while none of the other estimators exhibited a notable bias. The bias found for μ does not substantially affect the resulting model and its performance in forecasting, since it merely indicates a minor shift of the volatility process. It is important to stress that the MSEs are smaller for the larger sample size, as presumed. The obtained results for $m = 50$ are similar to those using higher values of m. Consequently, a more acceptable approximation of the likelihood is achieved.

Overall, it can be concluded that the use of the HMM machinery to maximize the approximate posterior distributions of SVM models numerically leads to a good estimator performance, considering a modest computational effort.

4 Empirical Application

We consider the daily closing prices of five Latin American stock markets: MERVAL (Argentina), IBOVESPA (Brazil), IPSA (Chile), MEXBOL (Mexico) and IGBVL (Peru). We use the S&P 500 in order to compare the results with Latin American stock markets because the U.S. stock market could be considered as a good benchmark. The data sets were obtained from the Yahoo finance web site available to download at http://finance.yahoo.com. The period of analysis is from January 6, 1998, until December 30, 2016. Stock returns are computed as $y_t = 100 \times (\log P_t - \log P_{t-1})$, where P_t is the (adjusted) closing price on day t.

Table 6 shows the number of observations and summary descriptive statistics. The sample size differs between countries due to holidays and stock market non-trading days. According to Table 6, the IGBVL and S&P 500 returns are negatively skewed whereas the rest are positively skewed. The IGBVL returns are the most negatively skewed with -0.3915 and the IBOVESPA returns the most positively skewed with 0.5313. Regarding the kurtosis, all the daily returns of the five Latin American returns and the S&P 500 are leptokurtic (all kurtosis coefficients are higher than 3). Brazil, Peru, and Chile are the markets with the highest degree of kurtosis with the USA near Chile's value. Although there are high differences between the minimum and maximum values, the most outstanding values correspond to Argentina and Brazil.

We further observe that the IGBVL and IPSA returns show the highest level of first-order autocorrelation. These values decrease fast for the other orders of autocorrelation. In the case of returns, high first-order autocorrelation reflects the effects of non-synchronous or thin trading. The squared returns show high level of autocorrelation of order one, which can be seen as an indication of volatility clustering. We further observe that high-order autocorrelations for squared returns are still high and decrease slowly[2]. The $Q(12)$ test statistic,

[2]This behavior has suggested that the literature considers that there is long memory in the volatility of returns, as well as the possibility that infrequent level shifts cause such behavior. For a discussion on this, see Diebold and Inoue

which is a joint test for the hypothesis that the first twelve autocorrelation coefficients are equal to zero, indicates that this hypothesis has to be rejected at the 5% significance level for all return and squared return series.

We set the priors distributions as follows: $(\beta_0, \gamma, \beta_2) \sim \mathcal{N}_3(\mathbf{0_3}, 100\mathbf{I}_3)$, $\mu \sim \mathcal{N}(0, 100)$, $\psi \sim \mathcal{N}(4.5, 100)$ and $\omega \sim \mathcal{N}(-1.5, 100)$, where $\mathcal{N}_r(.,.)$ and $\mathcal{N}(.,.)$ denote the $r-$variate and univariate normal distributions and $\mathbf{0_r}$ and $\mathbf{I}_r$ are the $r \times 1$ vector of zeros and the $r \times r$ the identity matrix, respectively. We use the procedure described in Section 2 by using $b_m = -b_0 = 4$ and $m = 200$, to ensure numerical stability in the results. We use the multivariate normal distribution with mean and variance being the MAP and the inverse of the hessian matrix evaluated at the MAP as importance density. To compare our methodology, we use the MCMC procedure based on HMC and RMHMC algorithms as described in Abanto-Valle et al. (2021) based on 30000 iterations and discarding the first 10000, only every 10-th values of the chain are stored.

Table 7 summarizes these results for the five Latin American stock market and the S&P 500 returns using our proposal and MCMC methods based on Abanto-Valle et al. (2021). It is important to note tha all the estimates are similar for all markets considered here. The value of ϕ is very similar among all markets, suggesting similar degrees of persistence (ranging from 0.9523 for Argentina to 0.9851 for Mexico) with the HMM aproach. The MEXBOL, IBOVESPA and S&P 500 are more persistent than the other markets. The main difference between the results in in the case of Brazil, where the posterior mean of ϕ is 0.9841 with the HMM approach and 0.9730 with MCMC.

The posterior mean estimates of σ show that all returns have similar estimates in the range from 0.1330 to 0.2757. The higest value is 0.2757 for the IGBVL jointly with the estimate of ϕ indicates that IGBVL is the most volatile stock market index in the region. Regarding the posterior mean of μ, we found that the estimates are statistically significant for the MERVAL, IBOVESPA and IPSA markets. For the MEXBOL, IGBVL and S&P 500 markets, the parameter μ is not significant because the credibility interval contains the null value.

We observe that the posterior mean parameter β_0 is always positive and statistically significant for all series. The value of β_1 that measures the correlation of returns is as expected, small and very similar to the first-order autocorrelation coefficients reported in Table 6. The estimates of β_1 are statistically significant for all series with the exception of Brazil and, although in the cases of Chile and Peru these values are 0.1876 and 0.1873, respectively, these values indicate a weak persistence with a rapid mean reversion.

Regarding the parameter of interest (β_2), this is more negative in the cases of USA, Brazil and Chile. Intermediate values are observed in Argentina and Mexico, while Peru presents the smallest value in absolute terms. Moreover, while all countries have a credibility interval that excludes the zero value, this does not happen in the case of Peru, so it is difficult to argue for an uncertainty effect in this market. It is important to note that the right side of the credibility interval is very close to zero in all markets except the U.S.. Therefore, the

(2001) and Perron and Qu (2010), among others. For applications to different financial markets in Latin America, see Rodríguez (2017) and the references mentioned therein.

posterior mean of β_2 parameter, which measures both the ex ante relationship between returns and volatility and the volatility feedback effect, is negative for all series and statistically significant for all the series with the exception of Peru. These findings are very similar and consistent with those found by Abanto-Valle et al. (2021).

Following Koopman and Uspensky (2002), the volatility feedback effect (negative) dominates the positive effect which links the returns with the expected volatility. Our estimates are more negative compared to those of Koopman and Uspensky (2002) where the hypothesis that $\beta_2 = 0$ can never be rejected at the conventional 5% siginificance level. Therefore, the volatility feedback effect is clearly dominant in our results (except for Peru) in comparison to those of Koopman and Uspensky (2002). These results confirm the hypothesis that investors require higher expected returns when unanticipated increases in future volatility are highly persistent.

It is important to stress that these findings are consistent with higher values of ϕ combined with larger negative values for the in-mean parameter. We have indirect evidence of a positive intertemporal relation between expected excess market returns and its volatility as this is one of the assumptions underlying the volatility feedback hypothesis.

Figure 2 shows the estimates of $e^{\frac{h_t}{2}}$ using our proposal and the smoothed mean of $e^{\frac{h_t}{2}}$ obtained via the MCMC output for all the series considered here. The dotted red line is the value obtained using our approach and the full black line via MCMC. We observe a great similarity between the estimating results for both methods. From a practitioners' viewpoint, implementing the MCMC procedure developed in Abanto-Valle et al. (2021) requires to write about 800 lines of C++ code. In stark contrast, the approach discussed in the present paper is easily implemented using Rcpp inside R, writing less than 200 lines of code. In all the applications considered here the MCMC procedure takes about 1 hour and our HMM procedure takes about 20 minutes, for all the countries. Therefore, from the practical viewpoint, there is substantial merit in considering the HMM approach as an alternative to MCMC schemes.

5 Discussion

This paper has two objectives. The first is to show gains in computational time using HMM methods versus MCMC methods developed and used, for example, in Abanto-Valle et al. (2021). The second objective is to empirically estimate the effect of the volatility in the mean (SVM model) using five Latin American stock markets comparing with the results obtained by Koopman and Uspensky (2002) and Abanto-Valle et al. (2021). Regarding, the first goal, this article introduces an approximate Bayesian inference via importance sampling, of the SVM model proposed by Koopman and Uspensky (2002). The likelihood function of the SVM model is approximated using HMMs machinery. The empirical application reveals similar results between our proposal methodology and the MCMC methods used by Abanto-Valle et al. (2021). However, our proposal is less timeconsuming, which is very important in real time applications.

The SVM model allows us to investigate the dynamic relationship between returns and their time-varying volatility. Therefore, concerning the second goal, we illustrated our methods through an empirical application of five Latin American return series and the S&P 500 return. The β_2 estimate, which measures both the ex-ante relationship between returns and volatility and the volatility feedback effect, was negative and significant for all the indexes considered here except for the IGBVL. The results are in line with those of French et al. (1987), who found a similar relationship between unexpected volatility dynamics and returns and confirm the hypothesis that investors require higher expected returns when unanticipated increases in future volatility are highly persistent. This fact is consistent with our findings of higher values of ϕ combined with larger negative values for the in-mean parameter.

Future research considers extending the model and algorithm to include time-varying parameters, including the in-mean parameter. This fact would allow us a comparison with other algorithms, such as the one proposed in Chan (2017). Another extension is to incorporate heavy-tails as in Abanto-Valle et al. (2012) or skewness and heavy-tails simultaneously, as in Leão et al. (2017).

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Table 1: SVM Model, simulated data set: maximum a posteriori (MAP) of the parameters and computing times in seconds for the HMM method ($b_m = -b_0 = 4$). True values of the parameters: $\boldsymbol{\theta} = (\beta_0, \gamma, \beta_2, \mu, \psi, \omega)' = (0.1400, 0.0600, -0.1000, 0.3000, 4.5951, -1.6094)'$

size	m	$\log p(\boldsymbol{\theta} \mid \mathbf{y}_T)$	β_0	γ	β_2	μ	ψ	ω	time
1500	50	-2339.82	0.1037	0.0322	-0.0248	-0.0135	4.4144	-1.4568	3.75
	100	-2339.82	0.1038	0.0325	-0.0249	-0.0139	4.4141	-1.4562	8.76
	150	-2339.82	0.1038	0.0325	-0.0249	-0.0140	4.4141	-1.4562	15.33
	200	-2339.82	0.1038	0.0325	-0.0249	-0.0140	4.4141	-1.44562	23.02
3000	50	-5197.48	0.1196	0.0485	-0.0710	0.4390	4.4783	-1.4421	8.01
	100	-5197.49	0.1210	0.0544	-0.0718	0.4201	4.4496	-1.4439	18.40
	150	-5197.49	0.1210	0.0544	-0.0718	0.4201	4.4496	-1.4439	35.65
	200	-5197.49	0.1210	0.0544	-0.0718	0.4201	4.4496	-1.4439	65.06
6000	50	-9934.81	0.1231	0.0592	-0.0800	0.3116	4.6130	-1.5051	14.75
	100	-9934.80	0.1232	0.0589	-0.0802	0.3133	4.6103	-1.5050	31.43
	150	-9934.80	0.1230	0.0596	-0.0800	0.3082	4.6034	-1.5028	62.41
	200	-9934.80	0.1230	0.0596	-0.0800	0.3082	4.6034	-1.5028	89.45

Table 2: SVM Model estimation results using HMM approach, simulated data set. First line: posterior mean and the standar deviation and the computing time in seconds. Second line: 95% posterior credibility interval. True values of the parameters: $\boldsymbol{\beta} = (0.14, 0.03, -0.10)'$, $\mu = 0.3$, $\phi = 0.98$ and $\sigma_\eta = 0.2$

size	m	β_0	β_1	β_2	μ	ϕ	σ_η	time
1500	50	0.1053 (0.0291)	0.0169 (0.0267)	-0.0248 (0.0253)	-0.0229 (0.2552)	0.9789 (0.0072)	0.2321 (0.0274)	13.41
		(0.0505,0.1567)	(-0.0406,0.0683)	(-0.1059,0.0228)	(-0.5081,0.4594)	(0.9556,0.9876)	(0.1816,0.2871)	
	100	0.1035 (0.0288)	0.0162 (0.0266)	-0.0247 (0.0262)	-0.0683 (0.3247)	0.9789 (0.0074)	0.2291 (0.0279)	15.29
		(0.0506,0.1565)	(-0.0380,0.0718)	(-0.1037,0.0222)	(-0.4863,0.4206)	(0.9573,0.9878)	(0.1850,0.2937)	
	150	0.1039 (0.0283)	0.0172 (0.0260)	-0.0244 (0.0253)	-0.0006 (0.2589)	0.9780 (0.0072)	0.2330 (0.0289)	28.39
		(0.0539,0.1603)	(-0.0369,0.0694)	(-0.1030,0.0243)	(-0.5030,0.5158)	(0.9544,0.9869)	(0.1850,0.3005)	
	200	0.1037 (0.0278)	0.0149 (0.0279)	-0.0236 (0.0248)	-0.0229 (0.2590)	0.9792 (0.0071)	0.2306 (0.0279)	43.22
		(0.0495,0.1598)	(-0.0378,0.0697)	(-0.1033,0.0254)	(-0.5113,0.5124)	(0.9558,0.9873)	(0.1861,0.2922)	
3000	50	0.1181 (0.0230)	0.0245 (0.0189)	-0.0705 (0.0144)	0.4325 (0.1974)	0.9792 (0.0047)	0.2305 (0.0199)	13.27
		(0.0774,0.1635)	(-0.0115,0.0620)	(-0.1188,-0.0438)	(0.0586,0.7896)	(0.9651,0.9857)	(0.1986,0.2811)	
	100	0.1173 (0.0226)	0.0240 (0.0196)	-0.0709 (0.0141)	0.4209 (0.1932)	0.9787	0.2312 (0.0194)	30.69
		(0.0746,0.1678)	(-0.0130,0.0609)	(-0.1077,-0.0446)	(0.0522,0.7749)	(0.9662,0.9850)	(0.1995,0.2758)	
	150	0.1179 (0.0234)	0.0266 (0.0190)	-0.0715 (0.0140)	0.4447 (0.1899)	0.9788 (0.0048)	0.2314 (0.0179)	55.68
		(0.0780,0.1663)	(-0.0070,0.0663)	(-0.1099,-0.0444)	(0.0859,0.7933)	(0.9651,0.9848)	(0.1992,0.2751)	
	200	0.1171 (0.0230)	0.0239 (0.0193)	-0.0709 (0.0145)	0.4377 (0.1966)	0.9791 (0.0049)	0.2319 (0.0189)	85.8
		(0.0761,0.1656)	(-0.0081,0.0645)	(-0.1119,-0.0434)	(0.0665,0.7899)	(0.9658,0.9850)	(0.1991,0.2799)	
6000	50	0.1237 (0.0152)	0.0297 (0.0134)	-0.0800 (0.0103)	0.313 (0.1513)	0.9809 (0.0032)	0.2213 (0.0128)	28.21
		(0.0946,0.1537)	(0.0029,0.0567)	(-0.1112,-0.0609)	(0.0423,0.5759)	(0.9731,0.9850)	(0.1988,0.2485)	
	100	0.1233 (0.0153)	0.0297 (0.0132)	-0.0796 (0.0108)	0.3101 (0.1539)	0.9811 (0.0033)	0.2213 (0.0129)	59.90
		(0.0949,0.1531)	(0.0044,0.0567)	(-0.1107,-0.0588)	(0.0219,0.6027)	(0.9727,0.9859)	(0.1957,0.2478)	
	150	0.1230 (0.0153)	0.0298 (0.0137)	-0.0801 (0.0103)	0.3123 (0.1421)	0.9808 (0.0032)	0.2220 (0.0131)	124.07
		(0.0974,0.1002)	(0.0321,0.1255)	(-0.1106,-0.0589)	(0.0471,0.3372)	(0.0197, 0.1177)	(0.1977,0.2497)	
	200	0.1229 (0.0155)	0.0293 (0.0302)	-0.0800 (0.0107)	0.3092 (0.1495)	0.9805 (0.0031)	0.2220 (0.0130)	174.18
		(0.0950,0.1509)	(0.0048,0.1509)	(-0.1099,-0.0589)	(0.0200,0.5939)	(0.9728, 0.9854)	(0.1988,0.2500)	

Table 3: SVM Model: Simulaton study results based on 300 replicates using the HMM method ($b_{max} = -b_{min} = 4$ and $T = 1500$)

Parameter	True value	mean	MRB	MARD	MSE
$m = 50$					
β_0	0.14	0.1340	-0.0428	0.2208	0.0015
β_1	0.03	0.0288	-0.0387	0.7322	0.0008
β_2	-0.10	-0.0983	-0.0172	0.1976	0.0007
μ	0.30	0.3079	0.0263	0.6619	0.0647
ϕ	0.98	0.9732	-0.0069	0.0083	0.0001
σ	0.20	0.2395	0.1974	0.2018	0.0022
$m = 100$					
β_0	0.14	0.1344	-0.0397	0.2193	0.0015
β_1	0.03	0.0285	-0.0492	0.7219	0.0008
β_2	-0.10	-0.0984	-0.0158	0.1974	0.0007
μ	0.30	0.3094	0.0312	0.6582	0.0632
ϕ	0.98	0.9732	-0.0069	0.0084	0.0001
σ	0.20	0.2391	0.1958	0.2013	0.0022
$m = 150$					
β_0	0.14	0.1348	-0.0401	0.2210	0.0015
β_1	0.03	0.0286	-0.0409	0.7316	0.0008
β_2	-0.10	-0.0988	-0.0182	0.1975	0.0007
μ	0.30	0.3205	0.0305	0.6703	0.0662
ϕ	0.98	0.9735	-0.0068	0.0083	0.0001
σ	0.20	0.2389	0.1956	0.2006	0.0022
$m = 200$					
β_0	0.14	0.1340	-0.0372	0.1436	0.0015
β_1	0.03	0.0288	-0.0443	0.7313	0.0008
β_2	-0.10	-0.0983	-0.0124	0.1965	0.0007
μ	0.30	0.3079	0.0685	0.6948	0.0695
ϕ	0.98	0.9732	-0.0067	0.0083	0.0001
σ	0.20	0.2395	0.1943	0.1987	0.0022

Table 4: SVM Model: Simulaton study results based on 300 replicates using the HMM method ($b_{max} = -b_{min} = 4$ and $T = 3000$)

Parameter	True value	mean	MRB	MARD	MSE
$m = 50$					
β_0	0.14	0.1368	-0.0225	0.1436	0.0006
β_1	0.03	0.0296	-0.0134	0.5113	0.0003
β_2	-0.10	-0.0990	-0.0104	0.1349	0.0003
μ	0.30	0.3097	0.0326	0.4465	0.0293
ϕ	0.98	0.9772	-0.0028	0.0046	0.00004
σ	0.20	0.2201	0.1006	0.1077	0.0007
$m = 100$					
β_0	0.14	0.1369	-0.0221	0.1446	0.0006
β_1	0.03	0.0293	-0.0232	0.5047	0.0003
β_2	-0.10	-0.0990	-0.0103	0.1349	0.0003
μ	0.30	0.3097	0.0324	0.4463	0.0281
ϕ	0.98	0.9772	-0.0028	0.0047	0.00004
σ	0.20	0.2200	0.0998	0.1074	0.0007
$m = 150$					
β_0	0.14	0.1371	-0.0206	0.1439	0.0006
β_1	0.03	0.0294	-0.0216	0.5014	0.0003
β_2	-0.10	-0.0990	-0.0094	0.1348	0.0003
μ	0.30	0.3205	0.0333	0.4520	0.0297
ϕ	0.98	0.9772	-0.0028	0.0047	0.00004
σ	0.20	0.2199	0.0997	0.1069	0.0007
$m = 200$					
β_0	0.14	0.1369	-0.0218	0.1436	0.0006
β_1	0.03	0.0294	-0.0185	0.5074	0.0003
β_2	-0.10	-0.0990	-0.0095	0.1350	0.0003
μ	0.30	0.3053	0.0179	0.4571	0.0303
ϕ	0.98	0.9772	-0.0028	0.0046	0.00004
σ	0.20	0.2199	0.0994	0.1067	0.0007

Table 5: SVM Model: Simulaton study results based on 300 replicates using the HMM method ($b_{max} = -b_{min} = 4$ and $T = 6000$)

Parameter	True value	mean	MRB	MARD	MSE
$m = 50$					
β_0	0.14	0.1396	-0.0038	0.1432	0.0003
β_1	0.03	0.0293	-0.0211	0.3412	0.0002
β_2	-0.10	-0.0994	-0.0063	0.0959	0.0001
μ	0.30	0.3101	0.0336	0.3382	0.0156
ϕ	0.98	0.9787	-0.0013	0.0027	0.00001
σ	0.20	0.2098	0.0488	0.0637	0.0002
$m = 100$					
β_0	0.14	0.1396	-0.0025	0.0940	0.0003
β_1	0.03	0.0294	-0.0181	0.3407	0.0002
β_2	-0.10	-0.0993	-0.0062	0.0970	0.0001
μ	0.30	0.3110	0.0366	0.3431	0.0160
ϕ	0.98	0.9786	-0.0013	0.0027	0.00001
σ	0.20	0.2097	0.0485	0.0637	0.0002
$m = 150$					
β_0	0.14	0.1396	-0.0031	0.0935	0.0003
β_1	0.03	0.0295	-0.0166	0.3398	0.0002
β_2	-0.10	-0.0993	-0.0067	0.0958	0.0001
μ	0.30	0.3113	0.0378	0.3401	0.0159
ϕ	0.98	0.9787	-0.0013	0.0027	0.00001
σ	0.20	0.2097	0.0484	0.06346	0.0002
$m = 200$					
β_0	0.14	0.1396	-0.0027	0.0930	0.0003
β_1	0.03	0.0294	-0.0196	0.3400	0.0002
β_2	-0.10	-0.0993	-0.0066	0.0960	0.0001
μ	0.30	0.3107	0.0358	0.3391	0.0156
ϕ	0.98	0.9782	-0.0012	0.0027	0.00001
σ	0.20	0.2097	0.0485	0.0637	0.0002

Table 6: Summary Statistics for daily stock returns data

INDEX	MERVAL	IBOVESPA	IPSA	MEXBOL	IGBVL	S&P 500
Size	4651	4698	4737	4759	4597	4777
Mean	0.0701	0.0376	0.0296	0.0464	0.0478	0.0177
S. D.	2.2125	2.0262	1.0695	1.4276	1.4111	1.2418
Minimum	-14.2896	-17.2082	-7.6381	-10.3410	-13.2908	-9.4695
Maximum	16.1165	28.8325	11.8034	12.1536	12.8156	10.9572
Skewness	2.2091	0.5313	0.1372	0.1458	-0.3915	-0.2086
Kurtosis	7.3418	16.8094	11.6866	8.7449	13.5715	10.6576
Returns						
$\hat{\rho}_1$	0.0550	0.0130	0.1840	0.0910	0.1890	-0.0700
$\hat{\rho}_2$	0.0020	-0.0180	0.0220	-0.0300	0.0080	-0.0450
$\hat{\rho}_3$	0.0240	-0.0390	-0.0190	-0.0301	0.0680	0.0100
$\hat{\rho}_4$	0.0070	-0.0320	0.0250	-0.0030	0.0640	-0.0080
$\hat{\rho}_5$	-0.0090	-0.0170	0.0270	-0.0150	0.0250	-0.0460
$Q(12)$	33.37	44.51	190.52	54.57	240.53	66.95
Squared Returns						
$\hat{\rho}_1$	0.2580	0.1990	0.2320	0.1430	0.4210	0.2040
$\hat{\rho}_2$	0.2160	0.1640	0.2130	0.1780	0.3890	0.3720
$\hat{\rho}_3$	0.1780	0.1860	0.1720	0.2540	0.3920	0.1920
$\hat{\rho}_4$	0.1660	0.1170	0.1550	0.1300	0.2840	0.2880
$\hat{\rho}_5$	0.2130	0.0990	0.2910	0.2420	0.2140	0.3220
$Q(12)$	1763.4	1069.3	2086.0	2147.1	3960.2	4643.7

Table 7: Estimation of the SVM Model using HMM machinery and Importance Sampling

	HMM		MCMC			HMM		MCMC	
Parameter	Mean	95% interval	Mean	95 % interval	Parameter	Mean	95% interval	Mean	95 % interval
MERVAL (Argentina)					IBOVESPA (Brazil)				
μ	1.1726	(1.0166,1.3341)	1.1647	(0.9931,1.3291)	μ	1.0332	(0.8228,1.2679)	1.2898	(0.9807,1.6022)
ϕ	0.9523	(0.9329,0.9658)	0.9501	(0.9353,0.9629)	ϕ	0.9841	(0.9738,0.9890)	0.9730	(0.9565,0.9863)
σ	0.2661	(0.2257,0.3148)	0.2688	(0.2399,0.3044)	σ	0.1330	(0.1122,0.1596)	0.1649	(0.1325,0.1968)
β_0	0.2080	(0.1329,0.2794)	0.2052	(0.1291,0.2815)	β_0	0.1303	(0.0535,0.2027)	0.2575	(0.1101,0.4091)
β_1	0.0047	(0.0180,0.0776)	0.0478	(0.0157,0.0783)	β_1	0.0082	(-0.0213,0.0382)	0.0309	(-0.0161,0.0743)
β_2	-0.0295	(-0.0501,-0.0091)	-0.0287	(-0.0510,-0.0073)	β_2	-0.0343	(-0.0497,0.0000)	-0.0400	(-0.0777,-0.0046)
IPSA (Chile)					MEXBOL (Mexico)				
μ	-0.3459	(-0.5675,-0.1377)	-0.3596	(-0.5706,-0.1555)	μ	0.2462	(-0.0427,0.5264)	0.2316	(-0.0634,0.5200)
ϕ	0.9736	(0.9615,0.9812)	0.9697	(0.9599,0.9791)	ϕ	0.9851	(0.9756,0.9895)	0.9803	(0.9725,0.9870)
σ	0.1970	(0.1708,0.2325)	0.2111	(0.1880,0.2385)	σ	0.1624	(0.1389,0.1937)	0.1859	(0.1655,0.2102)
β_0	0.0708	(0.0400,0.1041)	0.0710	(0.0385,0.1057)	β_0	0.1018	(0.0601,0.1414)	0.1039	(0.0655,0.2102)
β_1	0.1876	(0.1578,0.2167)	0.1885	(0.1578,0.2184)	β_1	0.0724	(0.0424,0.1009)	0.0742	(0.0438,0.1051)
β_2	-0.0433	(-0.0843,-0.0035)	-0.0442	(-0.0581,-0.0021)	β_2	-0.0296	(-0.0577,-0.0015)	-0.0301	(-0.0581,-0.0020)
IGBVL (Peru)					S & P 500 (USA)				
μ	-0.0042	(-0.2219,0.1921)	-0.0095	(-0.2342,0.2044)	μ	-0.1355	(-0.3691,0.1058)	-0.1332	(-0.4096,0.1375)
ϕ	0.9619	(0.9464,0.9714)	0.9618	(0.9490,0.9732)	ϕ	0.9814	(0.9679,0.9838)	0.9791	(0.9705,0.9864)
σ	0.2757	(0.2400,0.3203)	0.2725	(0.2351,0.3102)	σ	0.1833	(0.1635,0.2219)	0.1968	(0.1686,0.2311)
β_0	0.0601	(0.0252,0.0941)	0.0596	(0.0232,0.0960)	β_0	0.1091	(0.0751,0.1373)	0.1085	(0.0754,0.1404)
β_1	0.1873	(0.1560,0.2196)	0.1875	(0.1563,0.2199)	β_1	-0.0600	(-0.0919,-0.0361)	-0.0504	(-0.0853,-0.0232)
β_2	-0.0116	(-0.0398,0.0172)	-0.0114	(-0.0414,0.0179)	β_2	-0.0603	(-0.0889,-0.0259)	-0.0595	(-0.0924,-0.0272)

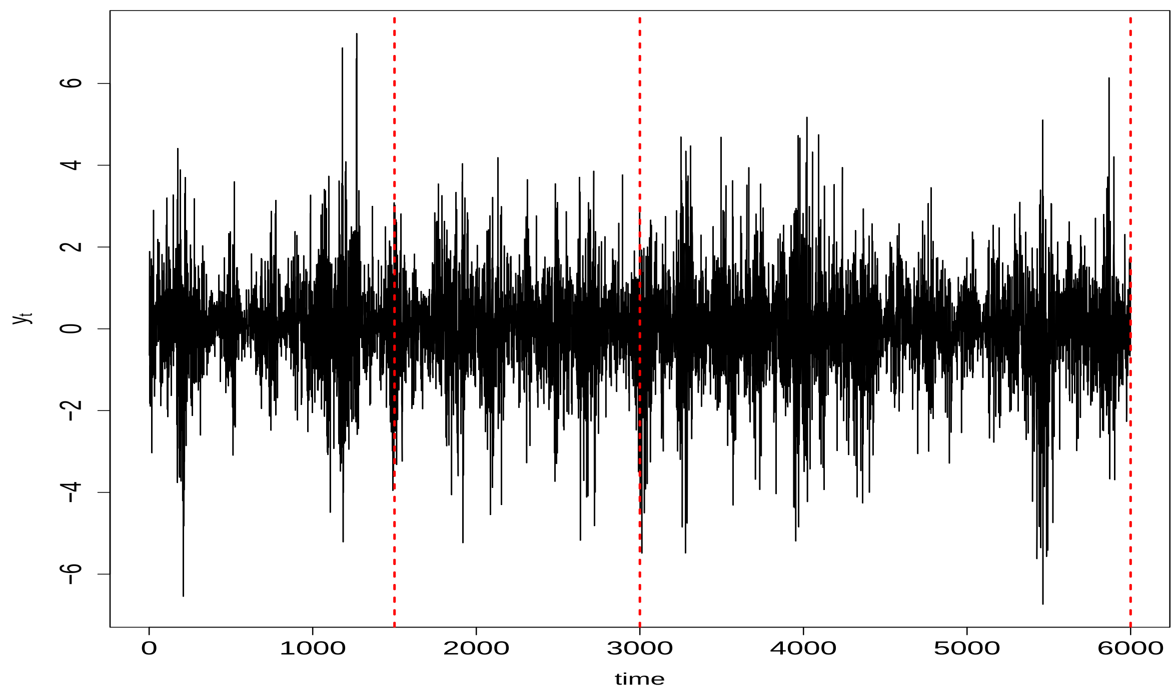

Figure 1: Simulated data set from the SVM model with $\boldsymbol{\beta} = (0.14, 0.03, -0.10)'$, $\mu = 0.3$, $\phi = 0.98$ and $\sigma = 0.2$. The vertical dotted lines (red) indicate the sample size $T = 1500, 3000$ and 6000, respectively.

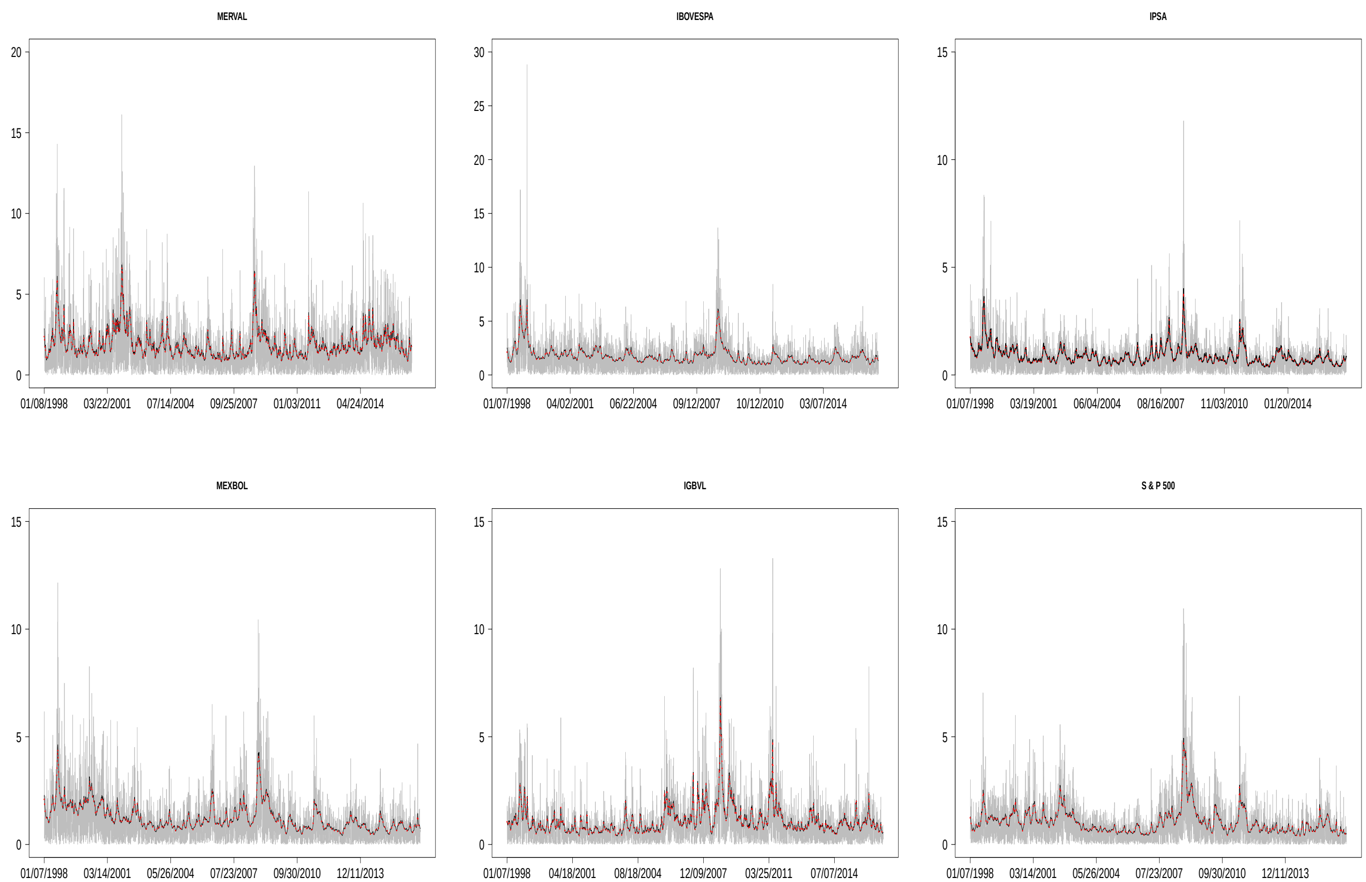

Figure 2: MERVAL, IBOVESPA, IPSA, MEXBOL, IGBVL and S&P 500 returns data sets: absolute returns (full gray line), $e^{\frac{h_t}{2}}$ estimator usign HMM machinery (dotted red line) and posterior smoothed mean of $e^{\frac{h_t}{2}}$ of the SVM model using MCMC (black line).

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