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PALABRAS CLAVE: Cointegration, Residual-Based Unit Root Test, ECR Test, OLS and GLS Detrented Data, Hypothesis Testing.

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Residual Based Tests for Cointegration with GLS Detrended Data

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Abstract

We analyze different residual-based tests for the null of no cointegration using GLS detrended data. We find and simulate the limiting distributions of these statistics when GLS demeaned and GLS detrended data are used. The distributions depend of the number of right-hand side variables, the type of deterministic components used in the cointegration equation, and a nuisance parameter R^2 which measures the long-run correlation between x_t and y_t. We present an extensive number of Figures which show the asymptotic power functions of the different statistics analyzed in this paper. The results show that GLS allows to obtain more asymptotic power in comparison with OLS detrending. The more simple residual-based tests (as the ADF) shows power gains for small values of R^2 and for only one right-hand side variable. This evidence is valid for R^2 less than 0.4. Figures shows that when R^2 is larger, the ECR statistics are better for any value of the right-hand side variables. In particular, evidence shows that the ECR statistic which assumes a known cointegration vector is the most powerful. A set of simulated asymptotic critical values are also presented. Unlike other references, in the present framework we use different $\overline{c}$ for different number of right-hand side variables (x_t variables) and according to the set of deterministic components. In this selection, we use a $R^2 = 0.4$, which appears to be a sensible choice.

Keywords: Cointegration, Residual-Based Unit Root Tests, ECR Tests, OLS and GLS Detrended Data, Hypothesis Testing.

JEL Classification: C2, C3, C5

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Resumen

Este documento analiza diferentes estadísticos basados en los residuos para la hipótesis nula de no cointegración utilizando MCG para eliminar los componentes determinísticos. Las distribuciones asintóticas son simuladas para los casos donde un intercepto y un intercepto y una tendencia son incluídos en la ecuación de cointegración. Los resultados muestran que las distribuciones asintóticas dependen del número de regresores (variables x_t), el número y clase de componentes determinísticos y un parámtro de nuisamce R^2 que mide la correlación de largo plazo entre los regresores x_t y la variable y_t. Los resultados muestran que MCG permiten obtener más potencia que el uso de MCO. Esto es más claro para valores de R^2 menores que 0.4 y un solo regresor x_t. Para valores mayores de R^2 los denominados estadísticos ECR son mejores para cualquier número de regresores. En particular el estadítico ECR basado en un vector de cointegración conocido es el más potente. Se presenta una Tabla con valores criticos asintóticos simulados utilizando diferente parámetro $\overline{c}$ de acuerdo al número de regresores y al tipo de componentes determinísticos. Se eligió $R^2 = 0.4$ por ser un valor sensible en términos prácticos y posiblemente empíricos.

Palabras Claves: Cointegración, Estadísticos Basados en los Residuos, Estadísticos ECR, MCG, MCO, Tests de Hipótesis.

Classificación JEL: C2, C3, C5.

Residual Based Tests for Cointegration with GLS Detrended Data[1]

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1 Introduction

Even though they are applicable only under some specific conditions, residuals based tests for cointegration, developed by Phillips and Ouliaris (1990), have been quite popular in applied work mostly because of their computational simplicity. The statistics introduced are designed to test the null hypothesis of no cointegration in a single equation setting assuming that the variables introduced as regressors are not cointegrated. These tests also have some appeal because they follow quite intuitively from the basic definition of cointegration as laid out in Engle and Granger (1987). If the system of variables is cointegrated, then there exists a linear combination (given by the cointegrating vector) that is stationary. In this case, the residuals from a simple static regression are stationary and, as shown by Stock (1987), this regression estimated by OLS will provide a consistent estimate of the cointegrating vector. In the absence of cointegration, the residuals from the static regression are nonstationary for any choice of the parameter vector and we have what has been labelled, following Granger and Newbold (1974) and later Phillips (1986), a spurious regression. Hence, an obvious testing strategy is to test the null hypothesis of no cointegration using some unit root test on the estimated residuals from the simple static regression.

Of course, there are many alternative approaches available, some applicable under less restrictive conditions; for example, the system based tests of Johansen (1991) and Stock and Watson (1988). The reader is referred to one of the many available surveys; for example, Watson (1994), Perron and Campbell (1992), Banerjee, Dolado, Galbraith and Hendry (1993) and Lütkepohl (1999).

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[1] This paper is a preliminary version of an ongoing research by the authors and it is a substantial revised version of an earlier document. Perron acknowledges financial support from the National Science Foundation. Rodríguez acknowledges financial support from the Department of Economics of the Pontificia Universidad Católica del Perú. This version was worked while the second author was visiting the Department of Economics of Boston University in August 2010.

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In an important paper, Elliott, Rothenberg and Stock (1996, hereafter ERS), following the work of Dufour and King (1991), show that several unit root tests constructed using GLS or quasi-differenced data [3] have asymptotic power functions close to the Gaussian local asymptotic power envelope. Hence, they enjoy some optimal properties over tests constructed using OLS detrended data and the simulations in ERS showed substantial power gains in finite samples. If such a detrending device is beneficial for unit root tests, it is natural to think that it would also be for cointegration tests.

Our aim, accordingly, is to analyze residual based tests for cointegration when they are constructed using GLS detrended or quasi-differenced data. We consider the standard ADF and the Z_α and Z_t tests analyzed in Phillips and Ouliaris (1990) as well as the class of modified unit root tests analyzed in Stock (1999), Perron and Ng (1996) and Ng and Perron (2001). We derive their asymptotic distribution assuming a number m of right-hand side variables (x_t variables), two types of deterministic components (demeaned and detrended cases, respectively). As in other references in the literature (see Pesavento (2007)) these asymptotic distributions depend of a nuisance parameter denoted by R^2 which measures the long-run correlation between the errors in the right-hand side variables and the error in the dependent (y_t) variable. In other words, this parameter contains long-run information from the covariates or regressors. Asymptotic power functions are derived and they are shown in different Figures. The results reveal that important power gains can indeed be achieved by using GLS detrended data, which has been also suggested by Pesavento (2007). Our work is related to that of Lütkepohl and Saikkonen (1999), Saikkonen and Lütkepohl (1997, 1998) and Xiao and Phillips (1999) who considered the use of GLS or quasi-differenced data when testing for cointegration in a multivariate setting, i.e. extending the tests proposed by Johansen (1991).

Another kind of cointegration tests based on estimates of a single equation are the so called ECM tests; see Banerjee, Dolado and Mestre (1998), Kremers, Ericsson and Dolado (1992), Boswijk (1994) and Zivot (2000) among many others. It is important to mention that the last statistic assumes that the cointegration vector is known. These tests are also easy to

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[3] There is some argument to the fact that the use of the terminology "GLS detrending" is not appropriate given that the procedure does not consider a full GLS transformation (since it only considers the leading root modelled as local to unity). An alternative terminology is that of "quasi-differenced" data. We still believe, for reasons that will become clear later in the text, that the use of "GLS detrending" is meaningful since it is this feature of the procedure that is of importance, even if constructed in a partial fashion. We shall use both terminology interchangeably.

apply empirically although critical values are not available in the same way. For a reference about the critical values and new tabulated critical values, see Ericsson and MacKinnon (2002). Pesavento (2004) proportionates the asymptotic distributions of these statistics when OLS demeaned and OLS detrended data are used. In this paper, we find and simulate the limiting distributions of these statistics when GLS demeaned and GLS detrended data are used. As in the other statistics, the limiting distributions depend of the number of right-hand side variables and the type of deterministic components (demeaned or detrended cases, respectively) used in the cointegration equation. Also, they depend of the nuisance parameter R^2. Of course, the statistic which assumes a known cointegration vector has asymptotic power independent of the number of right-hand side variables.

We present an extensive number of Figures which show the asymptotic power functions of the different statistics analyzed in this paper. The Figures shows dependence of the number of right-hand side variables, the type of the deterministic components and the nuisance parameter (R^2). The Figures shows that GLS allows to obtain more asymptotic power in comparison with OLS detrending. The more simple residual-based tests (as the ADF) shows power gains for small values of R^2 and for only one right-hand side variable. This evidence is valid for R^2 less than 0.4 which is consistent with finding of Pesavento (2007). Figures shows that when R^2 is larger the ECR statistics are better for any value of the right-hand side variables. In particular, evidence shows that the ECR statistic which assumes a known cointegration vector is the most powerful.

This paper is organized as follows. Section 2 presents the data-generating process considered and offers preliminary results concerning the limits of the estimates of the coefficients of the trend function and the estimates from the static regression with GLS detrended data. Section 3 presents the tests considered and derives their asymptotic distribution. Section 4 presents simulated asymptotic critical values and asymptotic power functions of the different residual-based tests for cointegration. Section 5 offers brief concluding remarks and an appendix contains technical derivations.

2 The DGP and Preliminary Results

We consider the following Data Generating Process:

$$\begin{aligned} x_t &= d_{xt} + u_{xt} \\ y_t &= d_{yt} + \beta' x_t + u_t \\ u_{xt} &= u_{xt-1} + v_{1t} \\ u_t &= \rho u_{t-1} + v_{2t} \end{aligned} \tag{1}$$

where $t = 1, 2, ..., T$; x_t is a $m \times 1$ vector, y_t is a scalar, d_{xt} and d_{yt} are the deterministic components: $d_{xt} = \sum_{i=0}^{p} \psi_{ix} t^i = \psi'_{ix} m_t$, $d_{yt} = \sum_{i=0}^{p} \psi_{iy} t^i = \psi'_{iy} m_t$. When $p = 0$, $m_t = \{1\}$, and when $p = 1$, $m_t = \{1, t\}$. The vector $v_t = (v'_{1t}, v_{2t})'$ contains serially correlated errors with $v_t = \Phi(L)\epsilon_t = \sum_{i=0}^{\infty} \Phi_i \epsilon_{t-i}$ with $\sum_{i=0}^{\infty} i \det |\Phi_i| < \infty$ $(\Phi_0 = I_n)$ and $\epsilon_t \sim i.i.d.(0, \Sigma)$. Total dimension of the DGP is $n \times 1$ where $n = m + 1$.

Assumption 1 *$\{\epsilon_t\}$ satisfies a functional central limit theorem, that is, $T^{-1/2} \sum_{t=1}^{[Tr]} \epsilon_t \Rightarrow \Sigma^{1/2} W(r)$ where $W(r)$ is a standard $n \times 1$ Wiener process, $\Rightarrow$ denotes weak convergence and Σ is the variance-covariance matrix of ϵ_t.*

The error term defined in the Assumption 1 is valid under a wide range of different assumptions on the process; see Phillips and Solo (1992), Wooldridge (1994). Furthermore, the Assumption 1 along the assumptions on $\Phi(L)$ means that $T^{-1/2} \sum_{t=1}^{[Tr]} v_t \Rightarrow B(r) = \Omega^{1/2} W(r)$ where $B(r)$ is standard $n \times 1$ Brownian motion a Ω is the spectral density at frequency zero of v_t scaled by 2π, that is, $\Omega = \lim_{T \to \infty} T^{-1} E\{[\sum_{t=1}^{T} v_t][\sum_{t=1}^{T} v'_t]\} = 2\pi f_{vv}(0) = 2\pi \Phi(1) \Sigma \Phi(1)'$.

We define the following partition of the $n \times n$ matrix Ω as

$$\Omega = \left[\begin{array}{cc} \Omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{array} \right],$$

and define $R^2 = \delta' \delta$, where $\delta = \Omega_{11}^{-1/2} \omega_{12} \omega_{22}^{-1/2}$ is a vector containing the bivariate zero frequency correlations of each element of v_{1t} with v_{2t}. The coefficient R^2 lies between zero and one, and represents the contribution of the right-hand side variables in the second equation of (1). The coefficient R^2 is zero when the variables x_t are not correlated in the long run with the errors from the cointegration regression. Under the assumption that x_t are not individually cointegrated, Ω_{11} is non singular.

In this paper, we consider two cases: i) there is a constant in the cointegration equation; and ii) there is a constant and a time trend in the cointegration equation. The null hypothesis is $\rho = 1$ and the alternative hypothesis is $\rho < 1$. Under the alternative hypothesis, the linear combination $y_t - d_{yt} - \beta' x_t$ is stationary, y_t and x_t are cointegrated and the system (1) contains m unit roots. Under the null hypothesis, the two variables are not cointegrated and there are n unit roots in the system. In this paper, unlike other approaches based on OLS detrended data, we demean or detrend the variables (x_t', y_t) using GLS or quasi-differences. This is equivalent to first demeaning or detrending x_t and y_t according to the appropiate deterministic case, and then testing the residuals from the regression:

$$y_t^d = \beta' x_t^d + u_t. \tag{2}$$

Estimation of equation (2) by OLS allows to have a consistent estimator for the true β. Therefore, testing stationarity of the residuals is a good proxy for testing that the linear combination (2) is stationary.

2.1 Estimates of the Trend Function under GLS Detrending

Suppose that $z_t = [x_t', y_t]'$ and $d_t = \sum_{i=0}^{p} \psi_i t^i = \psi' m_t$. When $p = 0$ we have $m_t = \{1\}$ and when $p = 1$ we have $m_t = \{1, t\}$. The vector $n \times 1$ z_t is detrended separately using GLS or quasi-differences. Let $\hat{\psi}$ be the GLS estimates of the coefficients of the trend function obtained using $\overline{\rho} = 1 + \bar{c}/T$. In order to apply the GLS detrending procedure suggested by ERS (1996), we start by defining the transformed data $z_t^{\overline{\rho}}$ and $m_t^{\overline{\rho}}$ as:

$$\begin{aligned} z_t^{\overline{\rho}} &= (1 - \overline{\rho} L) z_t, \\ m_t^{\overline{\rho}} &= (1 - \overline{\rho} L) m_t, \end{aligned}$$

for $t = 2, ..., T$, and $z_1^{\overline{\rho}} = z_1$, $m_1^{\overline{\rho}} = m_1$. Let $Z^{\overline{\rho}} = [z_1^{\overline{\rho}}, ..., z_T^{\overline{\rho}}]'$, $M^{\overline{\rho}} = [m_1^{\overline{\rho}}, ..., m_T^{\overline{\rho}}]'$, then

$$\hat{\psi} = (M^{\overline{\rho}\prime} M^{\overline{\rho}})^{-1} M^{\overline{\rho}\prime} Z^{\overline{\rho}}. \tag{3}$$

This implies that each series is detrended separately by an OLS regression on quasi-transformed data with differencing parameter $\overline{\rho}$. As a preliminary result, we consider the limiting distribution of the estimates $\hat{\psi}$ obtained from local to unity GLS detrending. The following Lemma gives the asymptotic properties of the estimates of the coefficients of the trend function in this case.

Lemma 1 *Suppose that* $z_t = [x_t', y_t]'$ *is generated by (1) with* $d_t = \sum_{i=0}^{p} \psi_i t^i = \psi' m_t$ *and that each variable in the* $n \times 1$ *vector* z_t *is detrended separately. Let* $\hat{\psi}$ *be the GLS estimates of the coefficients of the trend function obtained using* $\overline{\rho} = 1 + \overline{c}/T$*. Then,*

1. *If* $p = 0$*, with* $m_t = 1$ *for all* t*, then:* $\Upsilon_T(\hat{\psi} - \psi)' \Rightarrow \mathbf{0}_n$*, where* $\Upsilon_T = diag(T^{-1/2}, ..., T^{-1/2})$*, a* $n \times n$ *matrix and* $\mathbf{0}_n$ *denotes an* $n \times 1$ *vector of zeros.*

2. *If* $p = 1$*, with* $m_t' = (1, t)$ *for all* t*, then:*

$$\Upsilon_T vec[\hat{\psi} - \psi] \Rightarrow \left[\begin{array}{l} \mathbf{0}_n \\ \lambda B_c(1) + 3(1-\lambda)\int_0^1 s B_c(r) ds \end{array} \right],$$

where $\Upsilon_T = [diag(T^{-1/2}, ..., T^{-1/2}),\ diag(T^{1/2}, ..., T^{1/2})]$*, a* $2n \times 2n$ *matrix,* $\lambda = (1 - \overline{c})/(1 - \overline{c} + \overline{c}^2/3)$*,* $B_c(r)$ *is defined by the stochastic differential equation* $dB_c(r) = cB_c(r)dr + dB(r)$ *with* $B_c(0) = 0$*, and the vec operator stacks the rows of a matrix into a column vector.*

2.2 Limit Distribution of the Cointegration Vector

We now consider residuals-based tests for cointegration in the spirit of Phillips and Ouliaris (1990) but using *GLS* detrended variables defined by $z_t^d = (x_t^{d\prime}, y_t^d)'$. The relevant regression estimated by *OLS* is (2). The following Theorem gives the limiting behavior of the estimate $\widehat{\beta}$ under the null hypothesis of no cointegration.

Theorem 1 *Suppose that* $z_t = [x_t', y_t]'$ *is generated by (1) with* $d_t = \sum_{i=0}^{p} \psi_i t^i = \psi' m_t$*. Let* y_t^d *and* x_t^d *be GLS detrended variables with non-centrality parameter* $\overline{\rho} = 1 + \overline{c}/T$*. Let* $\widehat{\beta}$ *be the OLS estimates, from (2), of the cointegrating vector. We have:*

$$\begin{aligned} (\widehat{\beta} - \beta) &\Rightarrow \omega_{2.1}^{1/2} \Omega_{11}^{-1/2} \left[\int_0^1 \mathbf{W}_1 \mathbf{W}_1^{d\prime} \right]^{-1} \left[\int_0^1 \mathbf{W}_1^d J_{12c}^d \right], \\ &\equiv \omega_{2.1}^{1/2} \Omega_{11}^{-1/2} \boldsymbol{\kappa}_c^d, \end{aligned} \tag{4}$$

where $\mathbf{W} = (\mathbf{W}_1', W_2)'$ *and*

1. $\mathbf{W}_1^d = \mathbf{W}_1$ *and* $J_{12c}^d = J_{12c}$ *when only a constant is included in the cointegration regression (2), that is if* $p = 0$*;*

2. $\mathbf{W}_1^d = \mathbf{W}_1 - \left[\lambda \mathbf{W}_1(1) + 3(1-\lambda)\int_0^1 s\mathbf{W}_1(s)ds\right] r$, $J_{12c}^d = J_{12c} - \left[\lambda J_{12c}(1) + 3(1-\lambda)\int_0^1 sJ_{12c}(s)ds\right] r$ *and* $\lambda = (1-\bar{c})/(1-\bar{c}+\bar{c}^2/3)$, *when a constant and a time trend are included in the cointegration regression (2), that is if* $p = 1$.

Furthermore, $\omega_{2.1} = \omega_{22} - \omega_{21}\Omega_{11}^{-1}\omega_{12}$, $\boldsymbol{\delta} = \Omega_{11}^{-1/2}\omega_{12}\omega_{22}^{-1/2}$ and $R^2 = \boldsymbol{\delta}'\boldsymbol{\delta}$, $W_{12}(r) = \overline{\boldsymbol{\delta}}'\mathbf{W}_1 + W_2$. Note that $\overline{\boldsymbol{\delta}}'$ is a $1 \times m$ vector and consequently W_{12} depends of this dimension of parameters of nuisance. However using Lemma 5.6 of Park and Phillips (1988), W_{12} may be written in such way that it depends of only one parameter of nuisance which is R^2. Because Because $\overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} = \frac{R^2}{1-R^2}$, $W_{12} = \left[\frac{R^2}{1-R^2}\right]^{1/2}\overline{W}_1(r) + W_2(r)$ where $\overline{W}_1(r) = \frac{\sum_{i=1}^m W_i(r)}{\sqrt{m}}$. Furthermore, $J_{12c}(r) = W_{12}(r) + c\int_0^s e^{(s-r)c}W_{12}(r)dr$, where $J_{12c}(r)$ is an Ornstein-Uhlenbeck process.

3 The Tests and their Asymptotic Distributions

3.1 The Tests

All tests considered are based on $\widehat{u}_t$, the residual obtained from the static cointegration regression (2), i.e.,

$$\widehat{u}_t = y_t^d - \widehat{\beta}' x_t^d. \tag{5}$$

Firstly, we consider the MP_T^{GLS} defined in Ng and Perron (2001). In our case these statisitics are defined by

$$MP_{T,\mu}^{GLS} = \frac{\bar{c}^2 T^{-2}\sum_{t=1}^T \widehat{u}_{t-1}^2 - \bar{c}T^{-1}\widehat{u}_T^2}{s^2}, \tag{6}$$

$$MP_{T,\tau}^{GLS} = \frac{\bar{c}^2 T^{-2}\sum_{t=1}^T \widehat{u}_{t-1}^2 + (1-\bar{c})T^{-1}\widehat{u}_T^2}{s^2}, \tag{7}$$

for the $p = 0$ and $p = 1$ cases, respectively.

Other considered tests are the class of Z tests, analyzed by Phillips (1987) and Phillips and Perron (1988) in the context of testing for a unit root. These statistics can be applied to test the null hypothesis of no-cointegration as showed by Phillips and Ouliaris (1990). In the present context, with GLS detrended data, these are defined by

$$Z_{\widehat{\rho}}^{GLS} = T(\widehat{\rho}-1) - \frac{(s^2 - s_u^2)}{(2T^{-2}\sum_{t=1}^{T}\widehat{u}_{t-1}^2)}, \tag{8}$$

$$Z_{t_{\widehat{\rho}}}^{GLS} = \frac{s_u}{s}t_{\widehat{\rho}} - \frac{(s^2 - s_u^2)}{(4s^2T^{-2}\sum_{t=1}^{T}\widehat{u}_{t-1}^2)^{1/2}}, \tag{9}$$

where $\widehat{\rho}$ is the OLS estimate in the following regression:

$$\widehat{u}_t = \widehat{\rho}\widehat{u}_{t-1} + \widehat{k}_t, \tag{10}$$

and $t_{\widehat{\rho}}$ is the corresponding t-statistic for testing $\rho = 1$, $s_u^2 = T^{-1}\sum_{t=1}^{T}\widehat{k}_t^2$ and s^2 is described below.

The M-tests, originally proposed by Stock (1999), and further analyzed by Perron and Ng (1996) and Ng and Perron (2001), exploit the feature that a series converges with different rates of normalization under the null and the alternative hypotheses. These were shown to have far less size distortions than the Z tests in the presence of important negative serial correlation in the first-differences of the data. Constructed using the residuals from the cointegrating regression, they are defined by:

$$MZ_{\widehat{\rho}}^{GLS} = \frac{T^{-1}\widehat{u}_T^2 - s^2}{2T^{-2}\sum_{t=1}^{T}\widehat{u}_t^2}, \tag{11}$$

$$MSB^{GLS} = \left[\frac{T^{-2}\sum_{t=1}^{T}\widehat{u}_t^2}{s^2}\right]^{1/2}, \tag{12}$$

$$MZ_{t_{\widehat{\rho}}}^{GLS} = \frac{T^{-1}\widehat{u}_T^2 - s^2}{\left[4s^2T^{-2}\sum_{t=1}^{T}\widehat{u}_t^2\right]^{1/2}}. \tag{13}$$

In all previous definitions, the term s^2 is an autoregressive estimate of (2π times) the spectral density at frequency zero of v_t, defined as:

$$s^2 = \frac{s_{\eta k}^2}{\left[1 - \hat{b}(1)\right]^2}, \tag{14}$$

where $s_{\eta k}^2 = T^{-1}\sum_{t=k+1}^{T}\hat{\eta}_{tk}^2$, $\hat{b}(1) = \sum_{j=1}^{k}\hat{b}_j$, with $\hat{b}_j$ and $\{\hat{\eta}_{tk}\}$ obtained from the autoregression[4]:

$$\Delta\widehat{u}_t = \rho_0\widehat{u}_{t-1} + \sum_{j=1}^{k} b_j\Delta\widehat{u}_{t-j} + \eta_{tk}. \tag{15}$$

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[4] The advantages of using this autoregressive-based spectral density estimator over the more traditional kernel-based methods are discussed in Perron and Ng (1998).

The first statistic is a modified version of the $Z_{\widehat{\rho}}$ test, the second statistic is a modified version of Bhargava's (1986) R_1 statistic which builds upon the work of Sargan and Bhargava (1983), and the third statistic is a modified version of the $Z_{t_{\widehat{\rho}}}$ test. Another test of interest is the so-called ADF test (Dickey and Fuller, 1979, Said and Dickey, 1984) which is the t-statistic for testing $\rho_0 = 0$ in regression (15). We denote this test by ADF^{GLS}.

3.2 The Asymptotic Distributions of the Tests

Assumption 2. *For the regression (15), we need* $T^{-1/3}k \to 0$ *and* $k \to \infty$ *as* $T \to \infty$. *See Pesavento (2004, 2007).*

In the next lines we present the Theorems of the previous residual-based tests. Firstly, we establish the limiting distribution of the MP_T^{GLS} statistic. Second, we show asymptotic distributions of the statistics proposed by Phillips and Ouliaris (1990) but using GLS detrended data. The limiting distribution of the M-tests are also provided. Finally, we present the limiting distributions of the Error Correction (ECR) statistics which are alternative statistics based on single regressions. The limiting distributions of these tests based on OLS detrended data are given in Pesavento (2004) and consequently we only extend the expressions found in that paper but assuming GLS detrended data.

Theorem 2 *When the model is generated according to (1), assumptions 1 and 2 are valid, and the residuals are estimated using GLS detrended variables with a non-centrality parameter* $\overline{\rho} = 1 + \overline{c}/T$, *then, as* $T \to \infty$:

$$MP_{T,\mu}^{GLS} \Rightarrow \frac{\overline{c}^2\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right] - \overline{c}\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d(\mathbf{1})\boldsymbol{\eta}_c^d\right]}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]}, \tag{16}$$

$$MP_{T,\tau}^{GLS} \Rightarrow \frac{\overline{c}^2\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right] + (1-\overline{c})\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d(\mathbf{1})\boldsymbol{\eta}_c^d\right]}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]}, \tag{17}$$

for the $p = 0$ *and* $p = 1$ *cases, respectively and where* $\omega_{2.1} = \omega_{22} - \omega_{21}\Omega_{11}^{-1}\omega_{12}$, $\boldsymbol{\delta} = \Omega_{11}^{-1/2}\omega_{12}\omega_{22}^{-1/2}$, $R^2 = \boldsymbol{\delta}'\boldsymbol{\delta}$, $\overline{\boldsymbol{\delta}}' = \omega_{2.1}^{-1/2}\omega_{21}\Omega_{11}^{-1/2}$, $\boldsymbol{\eta}_c^{d\prime} = \left[-\left(\int \mathbf{W}_1^d J_{12c}^d\right)\left(\int \mathbf{W}_1^d\mathbf{W}_1^{d\prime}\right)^{-1}, 1\right] = \left[-\boldsymbol{\kappa}_c^{d\prime}, 1\right]$, $\mathbf{W}_c^d = \left[\mathbf{W}_1^{d\prime}, J_{12c}^d\right]$, $\mathbf{A}_c^d = \int_0^1 \mathbf{W}_c^d\mathbf{W}_c^{d\prime}$, $W_{12} = \left[\frac{R^2}{1-R^2}\right]^{1/2}\overline{W}_1 + W_2$, *where* $\overline{W}_1(r) = \frac{\sum_{i=1}^m W_i(r)}{\sqrt{m}}$, $\mathbf{D} = \begin{bmatrix} \mathbf{I} & \overline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}}' & 1 + \overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} \end{bmatrix}$, $J_{12c}(r)$ *is an Ornstein-Uhlenbeck process such that* $J_{12c}(r) = W_{12}(r) + c\int_0^s e^{(s-r)c}W_{12}(r)dr$ *and:*

1. $\mathbf{W}_1^d = \mathbf{W}_1$ *and* $J_{12c}^d = J_{12c}$ *when only a constant is included in the cointegration regression (2), that is if* $p = 0$;
2. $\mathbf{W}_1^d = \mathbf{W}_1 - \left[\lambda \mathbf{W}_1(1) + 3(1-\lambda)\int_0^1 s\mathbf{W}_1(s)ds\right] r$, $J_{12c}^d = J_{12c} - \left[\lambda J_{12c}(1) + 3(1-\lambda)\int_0^1 sJ_{12c}(s)ds\right] r$ *and* $\lambda = (1-\bar{c})/(1-\bar{c}+\bar{c}^2/3)$, *when a constant and a time trend are included in the cointegration regression (2), that is if* $p = 1$.

Next Theorem gives asymptotic distributions of the other statistics.

Theorem 3 *When the model is generated according to (1), assumptions 1 and 2 are valid, and the residuals are estimated using GLS detrended variables with a non-centrality parameter* $\bar{\rho} = 1 + \bar{c}/T$, *then, as* $T \to \infty$*:*

$$Z_{t_{\widehat{\rho}}}^{GLS}, ADF^{GLS} \Rightarrow c\frac{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right]^{1/2}}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]^{1/2}} + \frac{\left[\boldsymbol{\eta}_c^{d\prime}\int \mathbf{W}_c^d\mathbf{d}\widetilde{W}\boldsymbol{\eta}_c^d\right]^{1/2}}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right]^{1/2}\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]^{1/2}}, \tag{18}$$

$$Z_{\widehat{\rho}}^{GLS} \Rightarrow c + \frac{\left[\boldsymbol{\eta}_c^{d\prime}\int \mathbf{W}_c^d\mathbf{d}\widetilde{W}\boldsymbol{\eta}_c^d\right]^{1/2}}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right]^{1/2}}, \tag{19}$$

$$MSB^{GLS} \Rightarrow \frac{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right]^{1/2}}{\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]^{1/2}}, \tag{20}$$

$$MZ_{\widehat{\rho}}^{GLS} \Rightarrow \frac{\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d(\mathbf{1})\boldsymbol{\eta}_c^d - \boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d}{2\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d}, \tag{21}$$

$$MZ_{t_{\widehat{\rho}}}^{GLS} \Rightarrow \frac{\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d(\mathbf{1})\boldsymbol{\eta}_c^d - \boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d}{2\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d\right]^{1/2}\left[\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d\right]^{1/2}}, \tag{22}$$

where $\omega_{2.1} = \omega_{22} - \omega_{21}\Omega_{11}^{-1}\omega_{12}$, $\boldsymbol{\delta} = \Omega_{11}^{-1/2}\omega_{12}\omega_{22}^{-1/2}$, $R^2 = \boldsymbol{\delta}'\boldsymbol{\delta}$, $\overline{\boldsymbol{\delta}}' = \omega_{2.1}^{-1/2}\omega_{21}\Omega_{11}^{-1/2}$, $\boldsymbol{\eta}_c^{d\prime} = \left[-\left(\int \mathbf{W}_1^d J_{12c}^d\right)\left(\int \mathbf{W}_1^d\mathbf{W}_1^{d\prime}\right)^{-1}, 1\right] = \left[-\boldsymbol{\kappa}_c^{d\prime}, 1\right]$, $\mathbf{W}_c^d = \left[\mathbf{W}_1^{d\prime}, J_{12c}^d\right]$, $\mathbf{A}_c^d = \int_0^1 \mathbf{W}_c^d\mathbf{W}_c^{d\prime}$, $\widetilde{\mathbf{W}}' = [\mathbf{W}_1', W_{12}]'$, $W_{12} = \left[\frac{R^2}{1-R^2}\right]^{1/2}\overline{W}_1 + W_2$, *where* $\overline{W}_1(r) = \frac{\sum_{i=1}^m W_i(r)}{\sqrt{m}}$, $\mathbf{D} = \begin{bmatrix} \mathbf{I} & \overline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}}' & 1 + \overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} \end{bmatrix}$, $J_{12c}(r)$ *is an Ornstein-Uhlenbeck process such that* $J_{12c}(r) = W_{12}(r) + c\int_0^s e^{(s-r)c}W_{12}(r)dr$ *and*

1. $\mathbf{W}_1^d = \mathbf{W}_1$ *and* $J_{12c}^d = J_{12c}$ *when only a constant is included in the cointegration regression (2), that is if* $p = 0$;

2. *$\mathbf{W}_1^d = \mathbf{W}_1 - \left[\lambda \mathbf{W}_1(1) + 3(1-\lambda)\int_0^1 s\mathbf{W}_1(s)ds\right] r$, $J_{12c}^d = J_{12c} - \left[\lambda J_{12c}(1) + 3(1-\lambda)\int_0^1 sJ_{12c}(s)ds\right] r$ and $\lambda = (1-\bar{c})/(1-\bar{c}+\bar{c}^2/3)$, when a constant and a time trend are included in the cointegration regression (2), that is if $p = 1$.*

Another kind of cointegration tests based on estimates of a single equation are the so called ECM tests; see Banerjee, Dolado and Mestre (1998), Kremers, Ericsson and Dolado (1992), Boswijk (1994) and Zivot (2000) among many others. These tests are also easy to apply empirically although critical values are not available in the same way. For a reference about the critical values and new tabulated critical values, see Ericsson and MacKinnon (2002). The ECR statistics are based on the following equation

$$\Delta y_t = d_t + \boldsymbol{\pi}'_{0x}\boldsymbol{\Delta}\mathbf{x}_t + \gamma_0 y_{t-1} + \boldsymbol{\gamma}'_1\mathbf{x}_{t-1} + \sum_{i=1}^{k}(\boldsymbol{\pi}'_{px}\boldsymbol{\Delta}\mathbf{x}_{t-p} + \pi_{py}\Delta y_{t-p}) + \epsilon_t. \quad (23)$$

The statistic proposed by Banerjee, Dolado and Mestre (1998) is the t-ratio for $H_0 : \gamma_0 = 0$. This tests is denoted by t_{ECR}^{GLS}. On another hand the F statistic suggested by Bowijk (1994) is the F statistic for $H_0 : (\gamma_0, \boldsymbol{\gamma}'_1)' = \mathbf{0}$, which is denoted by F_{ECR}^{GLS}. Furthermore, there is an error correction test asumming a known cointegrating vector which is denoted by t_{EC}^{GLS}. Pesavento (2004) gives the asymptotic distributions of these statistics using OLS residuals. The next Theorem gives asymptotic distributions of the same statistics but based on GLS detrended variables.

Theorem 4 *When the model is generated according to (1), assumptions 1 and 2 are valid, and the residuals are estimated using GLS detrended variables with a non-centrality parameter $\bar{\rho} = 1 + \bar{c}/T$, then, as $T \rightarrow \infty$:*

$$\begin{aligned}
t_{ECR}^{GLS} &\Rightarrow c\left[\int J_{12c}^{d2} - \int \mathbf{W}_1^d J_{12c}^d \left(\int \mathbf{W}_1^d \mathbf{W}_1^{d\prime}\right)^{-1} \int \mathbf{W}_1^d J_{12c}^d\right]^{1/2} && (24)\\
&\quad + \frac{\int J_{12c}^d dW_2 \int \mathbf{W}_1^d J_{12c}^d (\int \mathbf{W}_1^d \mathbf{W}_1^{d\prime})^{-1} \int \mathbf{W}_1^d dW_2}{\left[\int J_{12c}^{d2} - \int \mathbf{W}_1^d J_{12c}^d (\int \mathbf{W}_1^d \mathbf{W}_1^{d\prime})^{-1} \int \mathbf{W}_1^d J_{12c}^d\right]^{1/2}}, \\
F_{ECR}^{GLS} &\Rightarrow c^2 \int J_{12c}^{d2} + 2c \int J_{12c}^d dW_2 && (25)\\
&\quad + \int \mathbf{W}_c^{d\prime} dW_2 (\mathbf{A}_c^d)^{-1} \int \mathbf{W}_c^d dW_2, \\
t_{EC}^{GLS} &\Rightarrow \frac{c}{(\int J_{12c}^{d2})^{-1/2}} + \left(\int J_{12c}^{d2}\right)^{-1/2}\left(\int J_{12c}^d dW_2\right), && (26)
\end{aligned}$$

where $\omega_{2.1} = \omega_{22} - \omega_{21}\Omega_{11}^{-1}\omega_{12}$, $\boldsymbol{\delta} = \Omega_{11}^{-1/2}\omega_{12}\omega_{22}^{-1/2}$, $R^2 = \boldsymbol{\delta}'\boldsymbol{\delta}$, $\overline{\boldsymbol{\delta}}' = \omega_{2.1}^{-1/2}\omega_{21}\Omega_{11}^{-1/2}$, $\mathbf{W}_c^d = \left[\mathbf{W}_1^{d\prime}, J_{12c}^d\right]$, $\mathbf{A}_c^d = \int_0^1 \mathbf{W}_c^d \mathbf{W}_c^{d\prime}$, $J_{12c}(r)$ *is an Ornstein-Uhlenbeck process such that* $J_{12c}(r) = W_{12}(r) + c\int_0^s e^{(s-r)c}W_{12}(r)dr$ *and*

1. $\mathbf{W}_1^d = \mathbf{W}_1$ *and* $J_{12c}^d = J_{12c}$ *when only a constant is included in the cointegration regression (2), that is if* $p = 0$*;*

2. $\mathbf{W}_1^d = \mathbf{W}_1 - \left[\lambda \mathbf{W}_1(1) + 3(1-\lambda)\int_0^1 s\mathbf{W}_1(s)ds\right] r$, $J_{12c}^d = J_{12c} - \left[\lambda J_{12c}(1) + 3(1-\lambda)\int_0^1 sJ_{12c}(s)ds\right] r$ *and* $\lambda = (1-\overline{c})/(1-\overline{c}+\overline{c}^2/3)$, *when a constant and a time trend are included in the cointegration regression (2), that is if* $p = 1$.

4 Results

This section presents simulated asymptotic critical values and asymptotic power functions.

4.1 Asymptotic Critical Values

This section presents asymptotic critical values of the statistics proposed in the theoretical section. All limiting distributions have been simulated using $T = 1000$ and 10,000 replications. Critical values depend of the nuisance parameter R^2. In order to avoid an excessive number of Tables with critical values, we use $R^2 = 0.4$ which appear to be a sensible value useful in the literature; see Pesavento (2007) for a similar argument. However, Pesavento (2007) prefers to use the same critical values as suggested for Elliott, Rothenberg and Stock (1996). However, such values are valid for an univariate context. Given that we are working in a multivariate framework, we prefer to use different critical values. Therefore, different critical values are simulated according to the number of right-hand side variables (x variables), and the number of deterministic components (demeaned and detrended cases, respectively). In fact, simulations show that the nuisance parameter R^2 changes according to the number of x_t variables and the number of deterministic components. We simulate critical values using $\overline{c}$ values obtained using $R^2 = 0.4$ for which an asymptotic power of 50.0% is achieved as is recommended by the literature.

The following Table summarizes the valued used in the simulation of the critical values. It is clear that more negative $\overline{c}$ values are needed for the detrended case in comparison with the demeaned case. Furthermore, $\overline{c}$ values are more negative when more right-hand side regressors enter in the cointegration equation.

Values of $\overline{c}$ used in the simulation of the critical values ($R^2 = 0.4$ is assumed)

Regressors	Demeaned Case	Detrended Case
$x = 1$	-13.75	-20.50
$x = 2$	-18.25	-23.75
$x = 3$	-22.25	-27.25
$x = 4$	-26.25	-30.75
$x = 5$	-30.00	-33.75

4.2 Asymptotic Power Functions

This section presents asymptotic power functions of the statistics proposed in the previous section. All limiting distributions have been obtained using $T = 1000$ and 10,000 replications. Different pictures have been obtained according to the number of right-hand side variables (x_t variables), and the number of deterministic components (demeaned and detrended cases, respectively). Of course, these asymptotic power functions also depend of the nuisance parameter R^2. In order to see the variations and sensibility of the asymptotic power functions regarding this parameter, and unlike the section of the asymptotic critical values, we present different pictures for different values of R^2.

Figure 1a shows the asymptotic power functions of the MP_T^{OLS} and MP_T^{GLS} for the demeaned case. There is a panel for a particular value of R^2 and each panel presents asymptotic power functions for different number of x_t variables. The Figure shows that GLS based statistics are always more powerful than OLS based versions. It is also possible to observe that power is lower when R^2 increases. This issue is not strange given the fact that this statistic does not exploit the information from the x_t variables. It is the same argument as Pesavento (2007) mentions for the ADF and Phillips and Perron statistics. The Figure also shows that power decreases when more x_t variables enter in the cointegrating regression. Figure 1b presents similar evidence but for the detrended case and the conclusions are the same. It is clear that in the detrended case power is smaller because there is a time trend included in the cointegrating regression compared to the demeaned case.

Figures 2a-2c show asymptotic power function for the MP_T^{GLS}, MSB^{GLS} and ADF^{GLS} statistics for the demeaned case and for $x = 1, 2,$ and 3, respectively. When $x = 1$ (Figure 2a) all three statistics have almost the same asymptotic power functions. When $x = 3$ (Figure 2b) and in particular when $x = 5$ (Figure 2c), the ADF^{GLS} statistic has larger power. It is more clear when $R^2 = 0.4$ or larger. However it is important to say that the distances are not very large. Same conclusions (but with lower power) are obtained for the detrended case (see Figures 2d-2f).

Figures 3a-3d show asymptotic power function for the MP_T^{GLS}, MSB^{GLS} and ADF^{GLS} statistics for the demeaned case and for $R^2 = 0.0$, 0.2, 0.4, and 0.8, respectively. When $R^2 = 0.0$ (Figure 3a), the asymptotic power functions are almost the same for $x = 1$ and $x = 2$. The ADF^{GLS} statistic presents slightly higher power when $x = 4$ and $x = 5$. This behavior is the same for other values of R^2 (see Figures 3b-3d). Furthermore, same conclusions are obtained when the detrended case is analyzed (see Figures 3e-3h).

Figures 4a-4c are very similar to previous Figures but now the ECR^{GLS}, EC^{GLS} and F^{GLS} statistics (for the demeaned case) are also presented in the Figures. We observe that the EC^{GLS} statistic has always higher power. Other statistics depend critically on the value of R^2. When $R^2 = 0.0$ (first panel), the ECR^{GLS} statistic has higher power after the EC^{GLS} test. Other tests have relatively same asymptotic power. When $R^2 = 0.2$ (second panel) same performance is observed but now the F^{GLS} test appears in third position of the ranking of power. When $R^2 = 0.4$ (third panel) the ECR^{GLS}, EC^{GLS} and F^{GLS} have better asymptotic power than the other tests presented in the picture. When $R^2 = 0.4$ (last panel) the evidence is more clear in favor of the error correction tests. It is because single-based tests as the MP_T^{GLS}, MSB^{GLS} and ADF^{GLS} do not exploit information contained in R^2, error correction tests perform better. This is mentioned in Pesavento (2004, 2007) and this issue is more important as R^2 increases. Figures 4b and 4c shows more clear evidence of the above issue mentioned. When number of x_t variables is larger, the difference of asymptotic power between single tests and error correction tests is larger. Same conclusions are observed for the detrended case (see Figures 4d-4f).

Figures 5a-5d show asymptotic power for the six above mentioned statistics for the demeaned case and for different values of R^2. We observe that power changes (reduced) when the number of x_t increases. The EC^{GLS} test performs better than the rest of the tests. Second test in the ranking of power is the ECR^{GLS} test. When $R^2 = 0.2$ or larger the F^{GLS} test has same power as the ECR^{GLS} test. The evidence supports the issue that the other tests do not exploit information from the number of x_t contained in R^2. Similar performance and conclusions are observed for the detrended case (Figures 5e-5h).

Figures 6a-6c show asymptotic power functions of the previous statistics but for different values of R^2. More excatly, each Figure shows the asymptotic power functions for all the statistics and for different values of x_t variables ($x = 1, 2$, and 3). In each panel, we present a statistic where its power functions is varying according to the value of R^2. The corresponding Figures for the detrended case are presented in the Figures 6d-6f. Similar Figures are presented in Figures 7a-7d and Figures 7e-7h for the demeaned and detrended cases, respectively. In this case each Figure shows the behavior of the statistics for different values of x_t. Now, each panel shows the behavior of the statistics for different values of R^2. The basic conclusion is that error corrections tests perform better than other statistics. It is more clear when the number of x variables increases and when long-run information contained in R^2 increases. The other statistics experiment loss of power when R^2 increases or when R^2 is very large or close to unity. Notice that this conclusion is obtained comparing all statistic based on GLS detrended data.

Figures 8a-8c show asymptotic power functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS}statistics for the demeaned case. We found that when R^2 is small single-based statistics based on GLS detrended data perform better than ECR statistics based on OLS detrended data. It is the same result as found in Pesavento (2007) for $x = 1$ (see Figure 5 of Pesavento, 2007). As she says, this issue is important because in a bivariate system ($x = 1$) $R^2 = 0.3$ (for example) correspond to a correlation of about 0.5 which is a reasonable estimate for many empirical applications. However we find that if x_t is larger ($x = 3$ or $x = 5$) single-based statistics using GLS detrended data have smaller power compared to ECR statistics based on OLS detrended data. It implies that ECR statistics based on GLS detrended data should be used if we have more than one right-hand side variable. This observation is the same for the detrended case as we may see in Figures 8d-8f.

5 Conclusions

We analyze different residual-based tests for the null of no cointegration using GLS detrended data. We derive their asymptotic distribution assuming a number m of right-hand side variables (x_t variables), two types of deterministic components (demeaned and detrended cases, respectively). These asymptotic distributions depend of a nuisance parameter denoted by

R^2 which measures the long-run correlation between the errors in the right-hand side variables and the error in the dependent (y_t) variable. Asymptotic power functions are derived and they are shown in different Figures. The results reveal that important power gains can indeed be achieved by using GLS detrended data, which has been also suggested by Pesavento (2007).

Another kind of cointegration tests based on estimates of a single equation are the so called ECM tests; see Banerjee, Dolado and Mestre (1998), Kremers, Ericsson and Dolado (1992), Boswijk (1994) and Zivot (2000) among many others. It is important to mention that the last statistic assumes that the cointegration vector is known. Pesavento (2004) proportionates the asymptotic distributions of these statistics when OLS demeaned and OLS detrended data are used. In this paper, we find and simulate the limiting distributions of these statistics when GLS demeaned and GLS detrended data are used. As in the other statistics, the limiting distributions depend of the number of right-hand side variables and the type of deterministic components (demeaned or detrended cases, respectively) used in the cointegration equation. Also, they depend of the nuisance parameter R^2. Of course, the statistic which assumes a known cointegration vector has asymptotic power independent of the number of right-hand side variables.

We present an extensive number of Figures which show the asymptotic power functions of the different statistics analyzed in this paper. The Figures shows dependence of the number of right-hand side variables, the type of the deterministic components and the nuisance parameter (R^2). The Figures show that GLS allows to obtain more asymptotic power in comparison with OLS detrending. The more simple residual-based tests (as the ADF) shows power gains for small values of R^2 and for only one right-hand side variable. This evidence is valid for R^2 less than 0.4. Figures shows that when R^2 is larger the ECR statistics are better for any value of the right-hand side variables. In particular, evidence shows that the ECR statistic which assumes a known cointegration vector is the most powerful. A set of simulated asymptotic critical values are also presented. Unlike other references, in the present framework we use different $\bar{c}$ for different number of right-hand side variables (x_t variables) and according to the set of deterministic components. In this selection, we use a $R^2 = 0.4$, which appears to be a sensible choice.

6 Appendix

Lemma A.1. When the model is generated according to (1) with $T(\rho-1) = c$, then, as $T \rightarrow \infty$, we have:

1. $T^{-2}\sum x_t^d x_t^{d\prime} \Rightarrow \Omega_{11}^{1/2}\int_1^d \mathbf{W}_1^d \mathbf{W}_1^{d\prime}\Omega_{11}^{1/2}$
2. $T^{-2}\sum x_t^d u_t^d \Rightarrow \omega_{2.1}^{1/2}\Omega_{11}^{1/2}\int_1^d \mathbf{W}_1^d J_{12c}^d$
3. $T^{-2}\sum (u_t^d)^2 \Rightarrow \omega_{2.1}\int (J_{12c}^d)^2$
4. $T^{-2}\sum u_{t-1}^d \epsilon_t' \Rightarrow \Omega_{11}^{1/2}\int J_{12c}^d d\mathbf{W}'\Sigma^{1/2}$
5. $T^{-2}\sum x_{t-1}^d \epsilon_t' \Rightarrow \Omega_{11}^{1/2}\int_1^d \mathbf{W}_1^d d\mathbf{W}'\Sigma^{1/2}$
6. $T^{-1}(u_T^d)^2 \Rightarrow \omega_{2.1} J_{12c}^d(1)^2$
7. $T^{-1}(x_T^d)^2 \Rightarrow \Omega_{111}^{1/2}\mathbf{W}_1^d(1)\mathbf{W}_1^d(1)'\Omega_{11}^{1/2}$

Proof of Lemma A.1. Assumption 1 along with the assumptions of $\Phi(L)$ implies that $T^{-1/2}\sum_{t=1}^{[Tr]} v_t \Rightarrow B(r) = \Omega^{1/2}W(r)$ for $r \in [0,1]$, where $\Omega^{1/2} = \begin{bmatrix} \Omega_{11} & 0 \\ \omega_{21}\Omega_{11}^{1/2} & \omega_{2.1}^{1/2} \end{bmatrix}$, $\omega_{2.1} = \omega_{22} - \omega_{21}\Omega_{11}^{-1}\omega_{12}$, and $\mathbf{W}' = [\mathbf{W}_1', W_2]'$. Using this notation, we have $T^{-1/2}\sum_{t=1}^{[Tr]} v_{1t} \Rightarrow \Omega_{11}^{1/2}\mathbf{W}_1(r)$ and $T^{-1/2}\sum_{t=1}^{[Tr]} v_{2t} \Rightarrow \omega_{21}\Omega_{11}^{-1/2}\mathbf{W}_1 + \omega_{2.1}^{1/2}W_2$. Define $\overline{\boldsymbol{\delta}}' = \omega_{2.1}^{-1/2}\omega_{21}\Omega_{11}^{-1/2}$ so that $\overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} = \frac{R^2}{1-R^2}$. Because $\omega_{21}\Omega_{11}^{-1/2}\mathbf{W}_1 = \omega_{2.1}^{1/2}\overline{\boldsymbol{\delta}}'\mathbf{W}_1$. Note that $\overline{\boldsymbol{\delta}}'$ is a $1 \times m$ vector and consequently $T^{-1/2}\sum_{t=1}^{[Tr]} v_{2t}$ depends of this dimension of parameters of nuisance. However using Lemma 5.6 of Park and Phillips (1988), $T^{-1/2}\sum_{t=1}^{[Tr]} v_{2t}$ may be written in such way that it depends of only one parameter of nuisance which is R^2. Because Because $\overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} = \frac{R^2}{1-R^2}$, $W_{12} = \left[\frac{R^2}{1-R^2}\right]^{1/2}\overline{W}_1(r) + W_2(r)$ where $\overline{W}_1(r) = \frac{\sum_{i=1}^m W_i(r)}{\sqrt{m}}$. then, we have $\omega_{2.1}^{1/2}\overline{\boldsymbol{\delta}}'\mathbf{W}_1 = \omega_{2.1}^{1/2}(\frac{R^2}{1-R^2})\overline{W}_1(r)$. With these results we have $T^{-1/2}\sum_{t=1}^{[Tr]} v_{2t} = T^{-1/2}u_{[Tr]} \Rightarrow \omega_{2.1}^{1/2}(\frac{R^2}{1-R^2})^{1/2}\overline{W}_1 + W_{2c} = J_{12c}$, where $\overline{W}_1(r) = \frac{\sum_{i=1}^m W_i(r)}{\sqrt{m}}$. The results in (1, 2, 3, 6, and 7) follows from the continuous mapping theorem (CMT). The other results follows from Chang and Wei (1988) and Phillips (1987). See also Pesavento (2004, 2007). In the present case where the variables are detrended using quasi-differences the results follow from the CMT; see ERS (1996).

Proof of Lemma 1. In both cases, the proof is a multivariate extension of arguments given in ERS (1996) for the univariate case and is, hence, omitted.

Proof of Theorem 1. The cointegration vector $\widehat{\beta}$ is estimated by regressing y_t^d on x_t^d. In the first case ($p = 0$) both variables are demeaned and for second case ($p = 1$) both variables are demeaned and detrended. In the general case:

$$(\widehat{\beta} - \beta) = \left[T^{-2} \sum_{t=1}^{T} x_t^d x_t^{d\prime} \right]^{-1} \left[T^{-2} \sum_{t=1}^{T} x_t^d u_t^d \right].$$

Using results of Lemma A.1 and the CMT we have that

$$\begin{aligned} (\widehat{\beta} - \beta) \;&\Rightarrow\; \omega_{2.1}^{1/2} \Omega_{11}^{-1/2} \left[\int_0^1 \mathbf{W}_1^d \mathbf{W}_1^{d\prime} \right]^{-1} \left[\int_0^1 \mathbf{W}_1^d J_{12c}^d \right] \\ &\equiv\; \omega_{2.1}^{1/2} \Omega_{11}^{-1/2} \boldsymbol{\kappa}_c^d. \end{aligned}$$

where all terms have been defined in the text. Before proving Theorem 2, we introduce some auxiliary results.

Lemma A.2. When the model is generated according to (1), assumptions 1 and 2 are valid, and the residuals are estimated using GLS detrended variables with a non-centrality parameter $\overline{\rho} = 1 + \bar{c}/T$, then, as $T \to \infty$:

1. $T^{-2} \sum_{t=1}^{T} (\widehat{u}_t^d)^2 \Rightarrow \omega_{2.1} \boldsymbol{\eta}_c^{d\prime} \mathbf{A}_c^d \boldsymbol{\eta}_c^d$
2. $T^{-1} (\widehat{u}_T^d)^2 \Rightarrow \omega_{2.1} \boldsymbol{\eta}_c^{d\prime} \mathbf{A}_c^d (\mathbf{1}) \boldsymbol{\eta}_c^d$
3. $T^{-1} \sum_{t=1}^{T} \widehat{u}_t^d \Delta \widehat{u}_t^d \Rightarrow \omega_{2.1} \left[c \boldsymbol{\eta}_c^{d\prime} \mathbf{A}_c^d \boldsymbol{\eta}_c^d + \boldsymbol{\eta}_c^{d\prime} \int \mathbf{W}_c^d d\widetilde{\mathbf{W}}^{\prime} \boldsymbol{\eta}_c^d \right]$
4. $s^2 \Rightarrow \omega_{2.1} \boldsymbol{\eta}_c^{d\prime} D \boldsymbol{\eta}_c^d$

where $\widehat{u}_t^d$ are the residuals from the cointegration regression (5) and s^2 is the estimated of the long-run variance. Furthermore we have $\boldsymbol{\eta}_c^{d\prime} = \left[\begin{array}{cc} -\left(\int \mathbf{W}_1^d J_{12c}^d\right) \left(\int \mathbf{W}_1^d \mathbf{W}_{1c}^{d\prime}\right)^{-1} & 1 \end{array} \right] = \left[\begin{array}{cc} -\boldsymbol{\kappa}_c^{d\prime} & 1 \end{array} \right]$, $\mathbf{W}_c^d = \left[\mathbf{W}_1^{d\prime}, J_{12c}^d \right]$,

$$\mathbf{A}_c^d = \int \mathbf{W}_c^d \mathbf{W}_c^{d\prime} = \left[\begin{array}{cc} \int \mathbf{W}_1^d \mathbf{W}_1^{d\prime} & \int \mathbf{W}_1^d J_{12c}^{d\prime} \\ \int \mathbf{W}_1^d J_{12c}^{d\prime} & \int (J_{12c}^d)^2 \end{array} \right], \quad \mathbf{D} = \left[\begin{array}{cc} \mathbf{I} & \overline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}}^{\prime} & 1 + \overline{\boldsymbol{\delta}}^{\prime} \overline{\boldsymbol{\delta}} \end{array} \right],$$

$\widetilde{\mathbf{W}}' = [\mathbf{W}_1', W_{12}]'$, $W_{12} = \left[\frac{R^2}{1-R^2}\right]^{1/2} \overline{W}_1(r) + W_2$, $J_{12c}(r)$ is an Ornstein-Uhlenbeck process such that $J_{12c}(r) = W_{12}(r) + c\int_0^s e^{(s-r)c} W_{12}(r)dr$.

Proof of Lemma A.2. Following Pesavento (2004, 2007) and by OLS projection we have that $\widehat{u}_t = \widehat{u}_t^d = u_t^d - (\widehat{\beta} - \beta)' x_t^d$.

1. In order to prove $T^{-2}\sum_{t=1}^{T}(\widehat{u}_t^d)^2$ we have:

$$\begin{aligned}
T^{-2}\sum_{t=1}^{T}(\widehat{u}_t^d)^2 &= \begin{bmatrix} -(\widehat{\beta}-\beta)' & 1 \end{bmatrix} \times \begin{bmatrix} T^{-2}\sum x_t^d x_t^{d\prime} & T^{-2}\sum x_t^d u_t^d \\ T^{-2}\sum u_t^d x_t^{d\prime} & T^{-2}\sum u_t^d u_t^{d\prime} \end{bmatrix} \\
&\quad \times \begin{bmatrix} -(\widehat{\beta}-\beta) \\ 1 \end{bmatrix} \\
&\Rightarrow \begin{bmatrix} -\omega_{2.1}^{1/2}\boldsymbol{\kappa}_c^{d\prime}\Omega_{11}^{-1/2} & 1 \end{bmatrix} \\
&\quad \times \begin{bmatrix} \Omega_{11}^{1/2}\int \mathbf{W}_1^d \mathbf{W}_1^{d\prime}\Omega_{11}^{1/2} & \omega_{2.1}^{1/2}\Omega_{11}^{1/2}\int \mathbf{W}_1^d J_{12c}^{d\prime} \\ \omega_{2.1}^{1/2}\Omega_{11}^{1/2}\int J_{12c}^d \mathbf{W}_1^{d\prime} & \omega_{2.1}\int (J_{12c}^d)^2 \end{bmatrix} \\
&\quad \times \begin{bmatrix} -\omega_{2.1}^{1/2}\Omega_{11}^{-1/2}\boldsymbol{\kappa}_c^d \\ 1 \end{bmatrix} \\
&\equiv \omega_{2.1}\boldsymbol{\eta}_c^{d\prime}\mathbf{A}_c^d\boldsymbol{\eta}_c^d
\end{aligned}$$

   where $\boldsymbol{\eta}_c^{d\prime} = \begin{bmatrix} -\boldsymbol{\kappa}_c^{d\prime} & 1 \end{bmatrix}$, and the other terms have been defined in the text and in Lemma A.2.

2. Proof of $T^{-1}(\widehat{u}_T^d)^2$ follows in same way as before.

3. Proof of $T^{-1}\sum_{t=1}^{T}\widehat{u}_t^d\Delta\widehat{u}_t^d$ is in Pesavento (2004)

4. For a proof of s^2 we follow the argument of Phillips and Ouliaris (1990) and Berk (1974). s^2 is a zero frequency spectral density estimate based on $\Delta\widehat{u}_t$. In other terms s^2 is the long-run variance of $\Delta\widehat{u}_t$. Therefore:

$$\begin{aligned}
Var(\Delta\widehat{u}_t) &= Var\left[\Delta u_t^d - (\widehat{\beta}-\beta)'\Delta x_t^d\right] \\
&\Rightarrow \begin{bmatrix} -\omega_{2.1}^{1/2}\boldsymbol{\kappa}_c^{d\prime}\Omega_{11}^{-1/2} & 1 \end{bmatrix} \begin{bmatrix} \Omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} -\omega_{2.1}^{1/2}\Omega_{11}^{-1/2}\boldsymbol{\kappa}_c^d \\ 1 \end{bmatrix} \\
&\equiv \omega_{2.1}\begin{bmatrix} \boldsymbol{\kappa}_c^{d\prime} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_m & \overline{\boldsymbol{\delta}} \\ \overline{\boldsymbol{\delta}}' & 1 + \overline{\boldsymbol{\delta}}'\overline{\boldsymbol{\delta}} \end{bmatrix} \begin{bmatrix} -\boldsymbol{\kappa}_c^d \\ 1 \end{bmatrix} \\
&\equiv \omega_{2.1}\boldsymbol{\eta}_c^{d\prime}\mathbf{D}\boldsymbol{\eta}_c^d.
\end{aligned}$$

Proof of Theorem 2. Proofs follow from results of Lemma A.2.
Proof of Theorem 3. Proofs for all statistics follow from results of Lemma A.2. See also Pesavento (2007) where more elements for the proof may be found.
Proof of Theorem 4. Proofs are extensions of the OLS-based versions found by Pesavento (2004). The proofs follow from results of Lemma A.2. See also Pesavento (2007) where more elements for the proofs may be found.

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Tabla 1. Critical Values for M^{GLS}, ADF^{GLS} and Error Correction Tests

(Demeaned Case)

	MP_T					$MZ_{\widehat{\rho}}$				
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
1.0%	4.275	5.712	6.896	7.905	9.372	-23.633	-30.602	-37.266	-44.944	-49.568
2.5%	5.193	6.667	7.980	9.032	10.473	-19.143	-26.010	-32.252	-39.392	-44.141
5.0%	6.230	7.825	9.086	10.361	11.559	-15.984	-22.064	-28.164	-34.392	-40.040
7.5%	7.025	8.591	9.916	11.256	12.369	-14.169	-20.075	-25.798	-31.695	-37.197
10.0%	7.757	9.315	10.618	11.979	13.117	-12.708	-18.491	-24.113	-29.586	-35.224
15.0%	9.071	10.555	11.798	13.151	14.369	-10.857	-16.282	-21.632	-26.931	-32.047
20.0%	10.294	11.670	12.836	14.174	15.437	-9.466	-14.672	-19.889	-24.907	-29.850
	MSB					MZ_t				
1.0%	0.144	0.126	0.115	0.105	0.100	-3.389	-3.870	-4.260	-4.709	-4.932
2.5%	0.159	0.137	0.123	0.112	0.105	-3.047	-3.563	-3.969	-4.397	-4.658
5.0%	0.172	0.148	0.131	0.119	0.111	-2.768	-3.282	-3.709	-4.108	-4.434
7.5%	0.182	0.155	0.137	0.124	0.115	-2.600	-3.119	-3.545	-3.928	-4.277
10.0%	0.191	0.160	0.141	0.128	0.118	-2.462	-2.992	-3.424	-3.800	-4.156
15.0%	0.206	0.171	0.149	0.134	0.123	-2.259	-2.797	-3.253	-3.622	-3.963
20.0%	0.219	0.179	0.155	0.139	0.128	-2.098	-2.654	-3.098	-3.482	-3.818

Tabla 1 (continued). Critical Values for M^{GLS}, ADF^{GLS} and Error Correction Tests

(Demeaned Case)

	ADF					ECR				
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
1.0%	-3.353	-3.849	-4.258	-4.641	-4.913	-3.163	-3.568	-3.974	-4.236	-4.443
2.5%	-3.028	-3.531	-3.936	-4.345	-4.615	-2.880	-3.263	-3.598	-3.888	-4.105
5.0%	-2.764	-3.279	-3.687	-4.055	-4.384	-2.603	-2.997	-3.320	-3.599	-3.829
7.5%	-2.588	-3.104	-3.520	-3.898	-4.238	-2.414	-2.820	-3.109	-3.406	-3.632
10.0%	-2.452	-2.975	-3.400	-3.783	-4.098	-2.264	-2.686	-2.966	-3.250	-3.476
15.0%	-2.256	-2.780	-3.22	-3.598	-3.917	-2.051	-2.451	-2.759	-3.015	-3.251
20.0%	-2.096	-2.630	-3.080	-3.453	-3.766	-1.867	-2.264	-2.580	-2.841	-3.069
	EC					F				
1.0%	-2.575	-2.575	-2.575	-2.575	-2.575	11.450	15.111	19.187	22.099	25.068
2.5%	-2.248	-2.248	-2.248	-2.248	-2.248	9.710	13.013	16.248	19.290	22.073
5.0%	-1.946	-1.946	-1.946	-1.946	-1.946	8.233	11.344	14.296	17.133	19.729
7.5%	-1.769	-1.769	-1.769	-1.769	-1.769	7.205	10.215	13.084	15.945	18.341
10.0%	-1.615	-1.615	1.615	1.615	1.615	6.552	9.448	12.168	14.900	17.301
15.0%	-1.403	-1.403	-1.403	-1.403	-1.403	5.569	8.327	10.917	13.531	15.796
20.0%	-1.226	-1.226	-1.226	-1.226	-1.226	4.817	7.522	9.946	12.371	14.726

Tabla 2. Critical Values for M^{GLS}, ADF^{GLS} and Error Correction Tests

(Detrended Case)

	MP_T					$MZ_{\widehat{\rho}}$				
	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
1.0%	7.014	7.638	8.778	9.588	10.592	-31.041	-38.102	-43.493	-50.662	-54.794
2.5%	8.166	8.824	9.890	10.906	11.759	-26.416	-33.099	-38.416	-44.482	-49.406
5.0%	9.242	10.121	11.160	12.156	12.944	-23.256	-28.474	-34.073	-39.851	-44.954
7.5%	10.243	11.075	12.083	13.079	13.868	-21.078	-26.111	-31.371	-36.811	-42.207
10.0%	11.093	11.940	12.905	13.861	14.523	-19.449	-24.336	-29.498	-34.822	-40.054
15.0%	12.660	13.204	14.175	15.180	15.723	-17.041	-21.863	-26.814	-31.724	-36.871
20.0%	13.929	14.372	15.370	16.134	16.833	-15.398	-20.065	-24.667	-29.506	-34.496
	MSB					MZ_t				
1.0%	0.126	0.114	0.107	0.099	0.095	-3.911	-4.345	-4.638	-5.011	-5.216
2.5%	0.135	0.122	0.113	0.105	0.100	-3.597	-4.023	-4.351	-4.683	-4.940
5.0%	0.145	0.131	0.120	0.111	0.105	-3.387	-3.745	-4.095	-4.430	-4.711
7.5%	0.152	0.136	0.125	0.115	0.108	-3.207	-3.579	-3.930	-4.266	-4.553
10.0%	0.158	0.141	0.129	0.119	0.111	-3.076	-3.443	-3.803	-4.138	-4.444
15.0%	0.168	0.149	0.135	0.124	0.115	-2.871	-3.265	-3.620	-3.952	-4.262
20.0%	0.177	0.155	0.140	0.129	0.119	-2.724	-3.122	-3.470	-3.806	-4.115

Tabla 2 (continued). Critical Values for M^{GLS}, ADF^{GLS} and Error Correction Tests
(Detrended Case)

	ADF					ECR				
	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
1.0%	-3.913	-4.294	-4.627	-4.923	-5.179	-3.804	-4.094	-4.359	-4.640	-4.754
2.5%	-3.635	-4.007	-4.340	-4.677	-4.910	-3.516	-3.787	-4.087	-4.306	-4.446
5.0%	-3.401	-3.746	-4.064	-4.401	-4.668	-3.259	-3.511	-3.791	-3.981	-4.176
7.5%	-3.229	-3.581	-3.907	-4.219	-4.525	-3.074	-3.333	-3.595	-3.802	-4.010
10.0%	-3.085	-3.454	-3.787	-4.102	-4.402	-2.935	-3.212	-3.455	-3.648	-3.866
15.0%	-2.879	-3.254	-3.606	-3.919	-4.222	-2.733	-2.999	-3.244	-3.443	-3.640
20.0%	-2.721	-3.111	-3.455	-3.778	-4.069	-2.578	-2.836	-3.072	-3.275	-3.468
	EC					F				
1.0%	-3.481	-3.479	-3.477	-3.476	-3.475	15.556	18.754	21.892	25.678	27.565
2.5%	-3.170	-3.173	-3.170	-3.170	-3.168	13.527	16.228	19.465	22.473	24.731
5.0%	-2.895	-2.890	-2.866	-2.886	-2.883	11.746	14.407	17.425	20.022	22.415
7.5%	-2.723	-2.720	-2.721	-2.721	-2.720	10.534	13.272	16.016	18.525	20.995
10.0%	-2.589	-2.586	-2.585	-2.583	-2.582	9.681	12.434	15.025	17.517	20.001
15.0%	-2.396	-2.394	-2.392	-2.389	-2.388	8.588	11.101	13.576	16.019	18.360
20.0%	-2.238	-2.236	-2.236	-2.233	-2.232	7.715	10.171	12.519	14.842	17.149

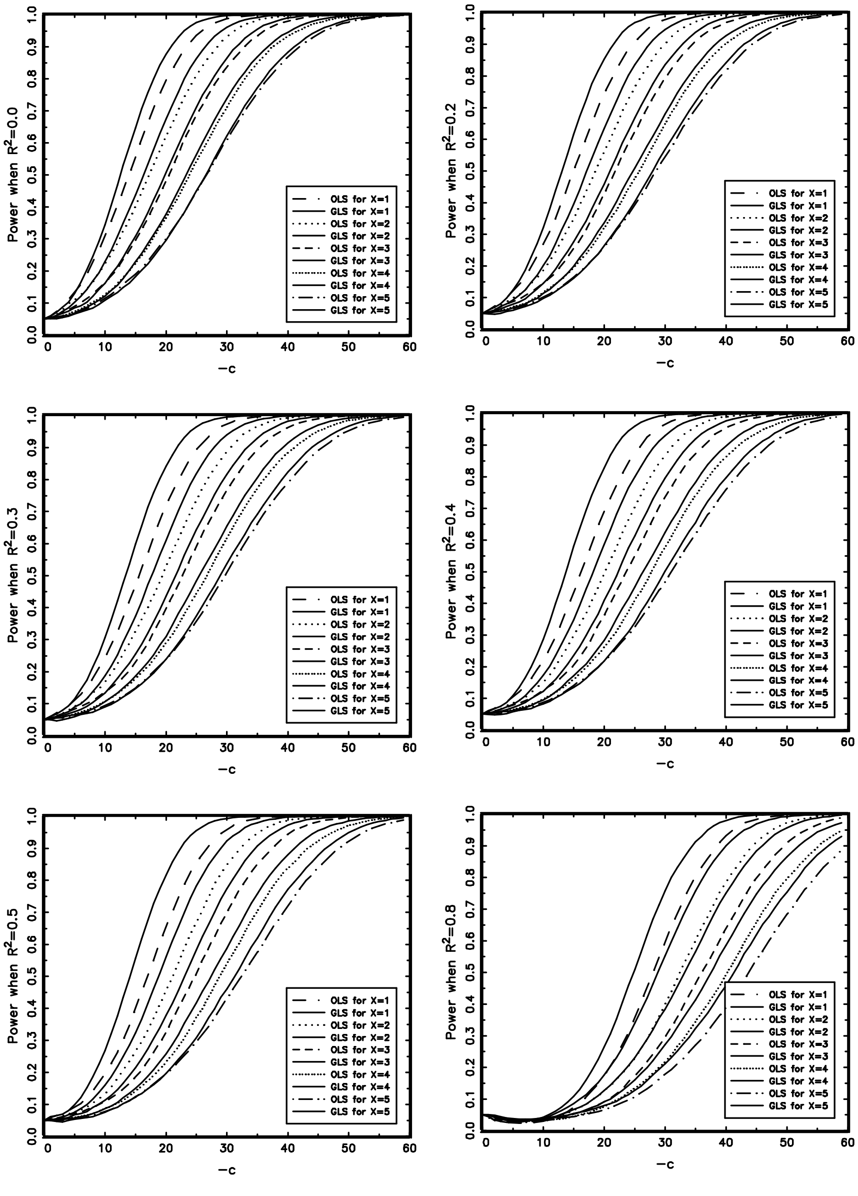

Figure 1a. Asymptotic Power Functions of MP_T^{GLS} for $x = 1, 2, 3, 4, 5$ and different values of R^2. Demeaned Case.

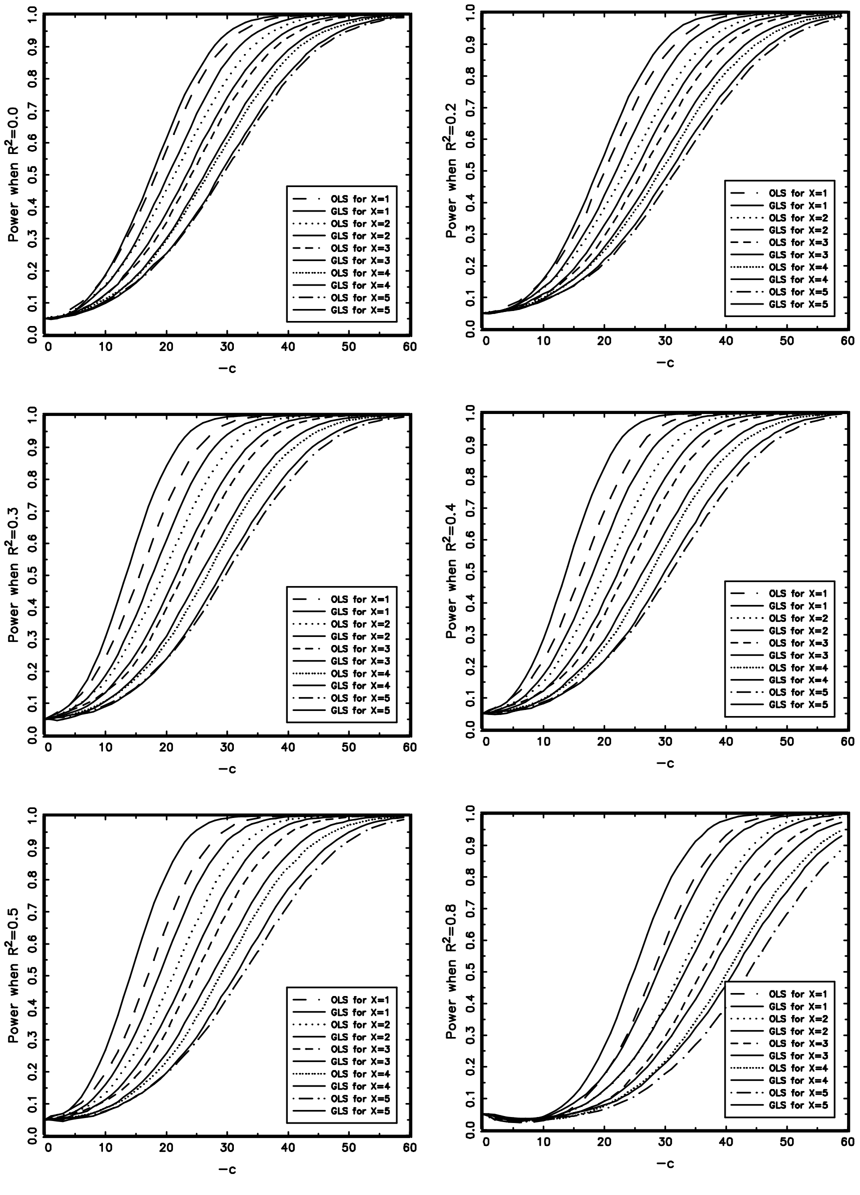

Figure 1b. Asymptotic Power Functions of MP_T^{GLS} for $x = 1, 2, 3, 4, 5$ and different values of R^2. Detrended Case.

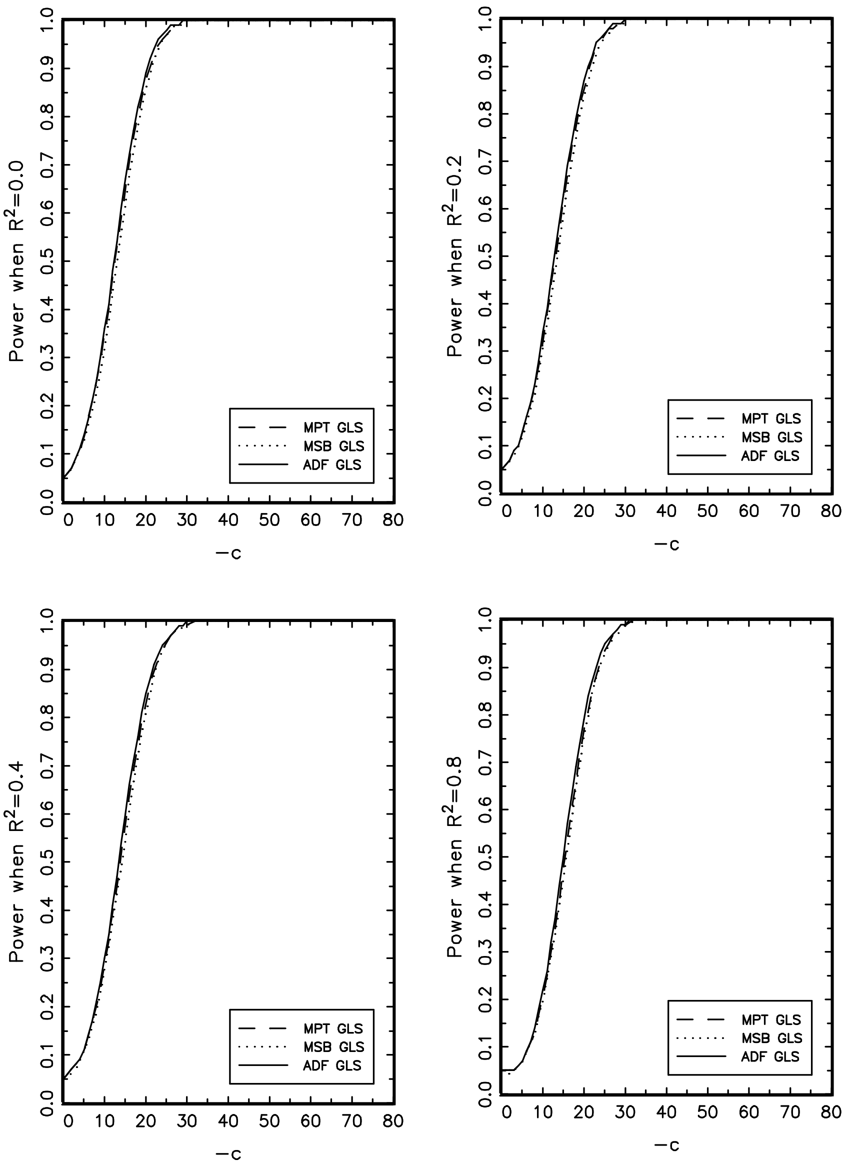

Figure 2a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 1$ and different values of R^2. Demeaned Case.

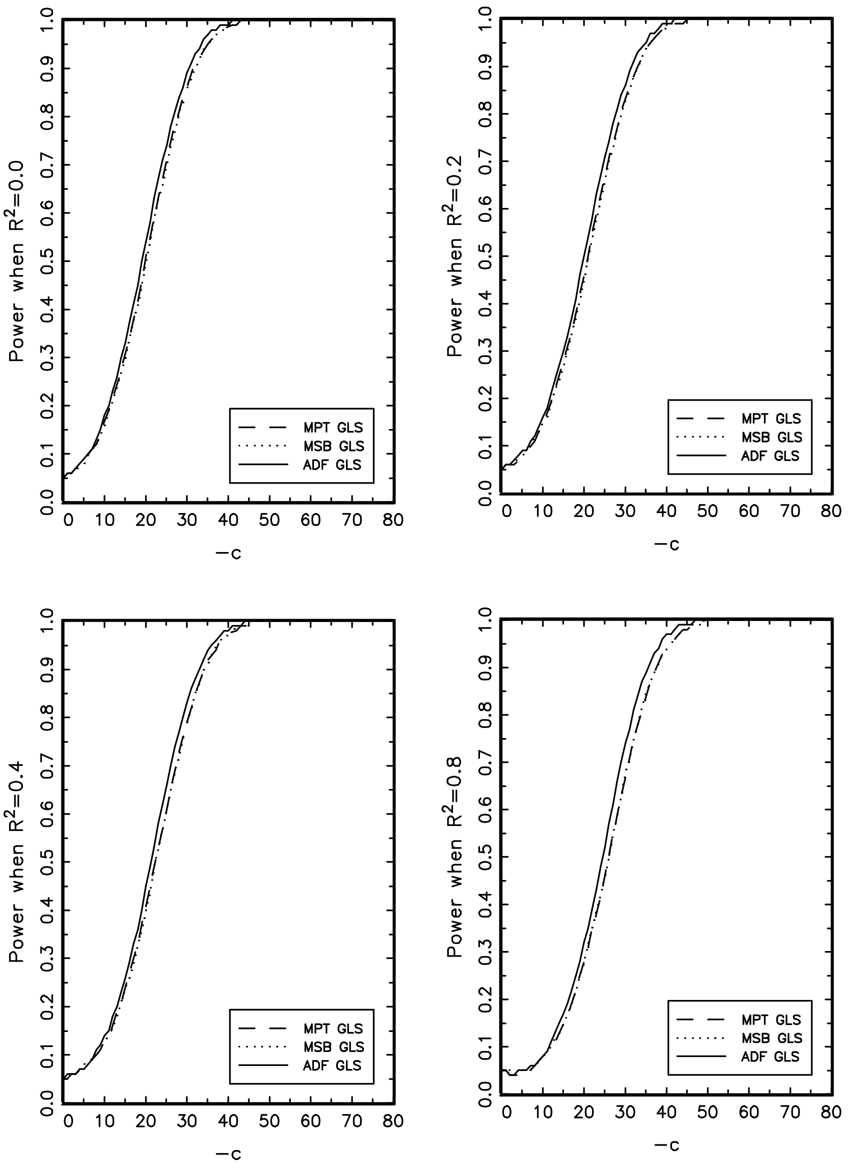

Figure 2b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 3$ and different values of R^2. Demeaned Case.

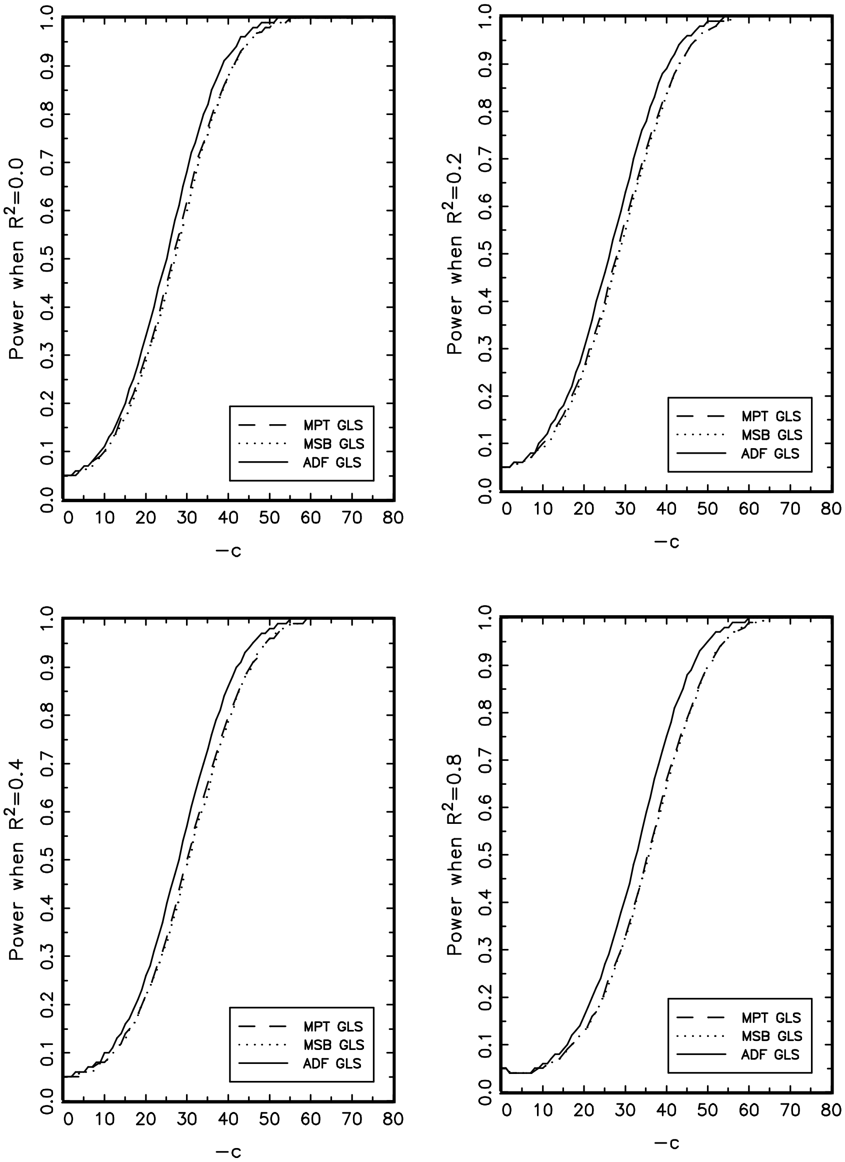

Figure 2c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 5$ and different values of R^2. Demeaned Case.

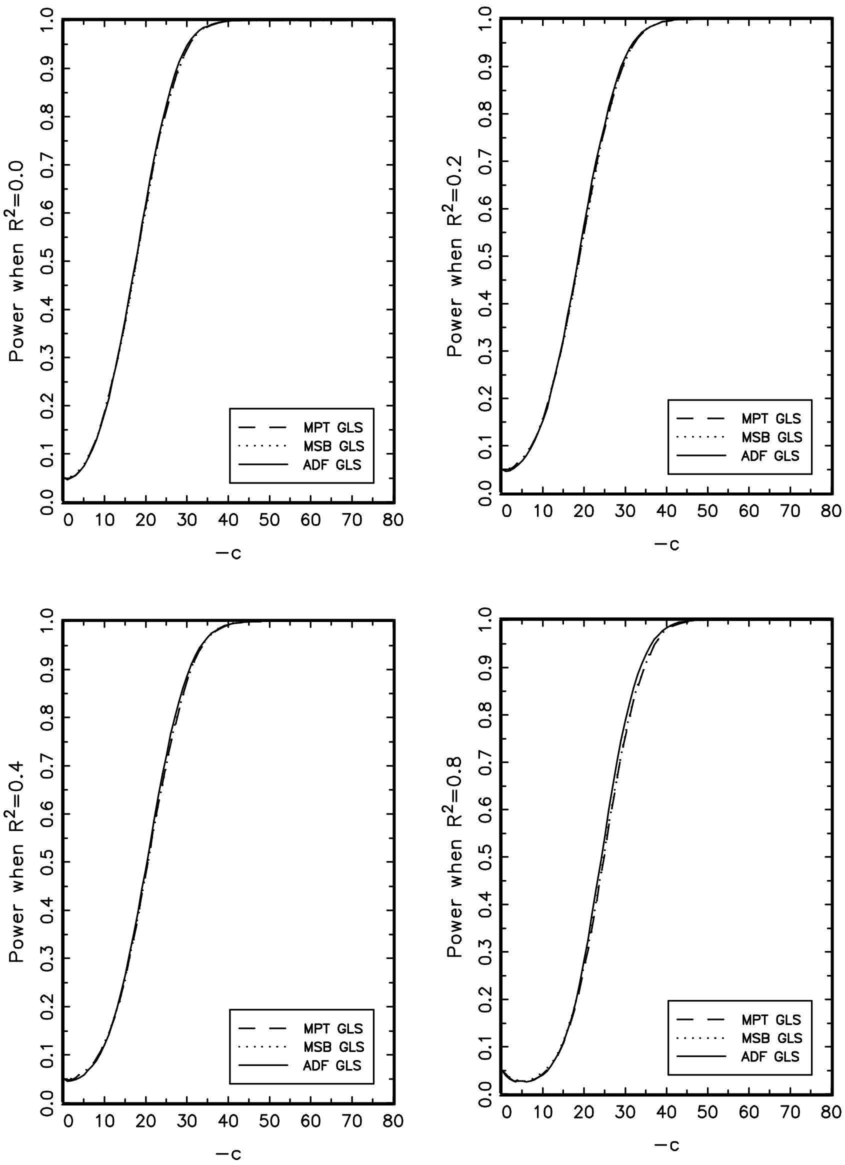

Figure 2d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 1$ and different values of R^2. Detrended Case.

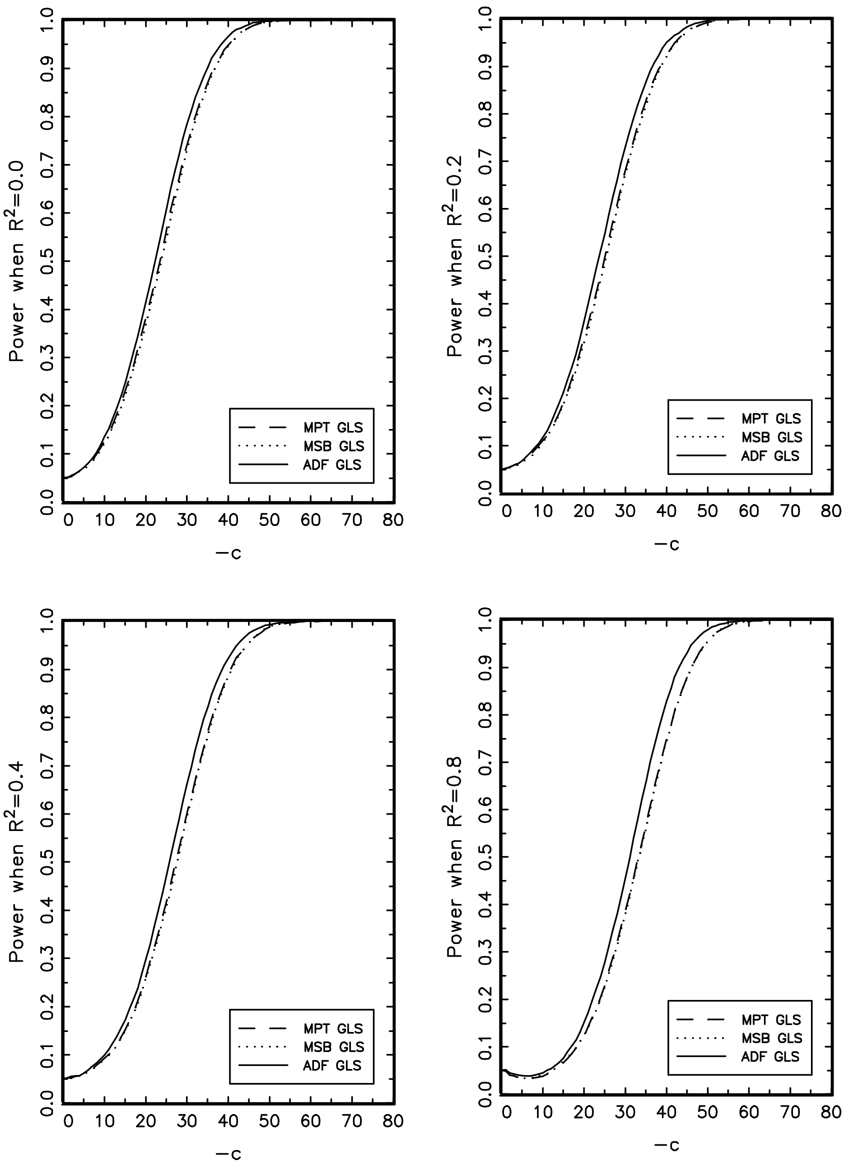

Figure 2e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 3$ and different values of R^2. Detrended Case.

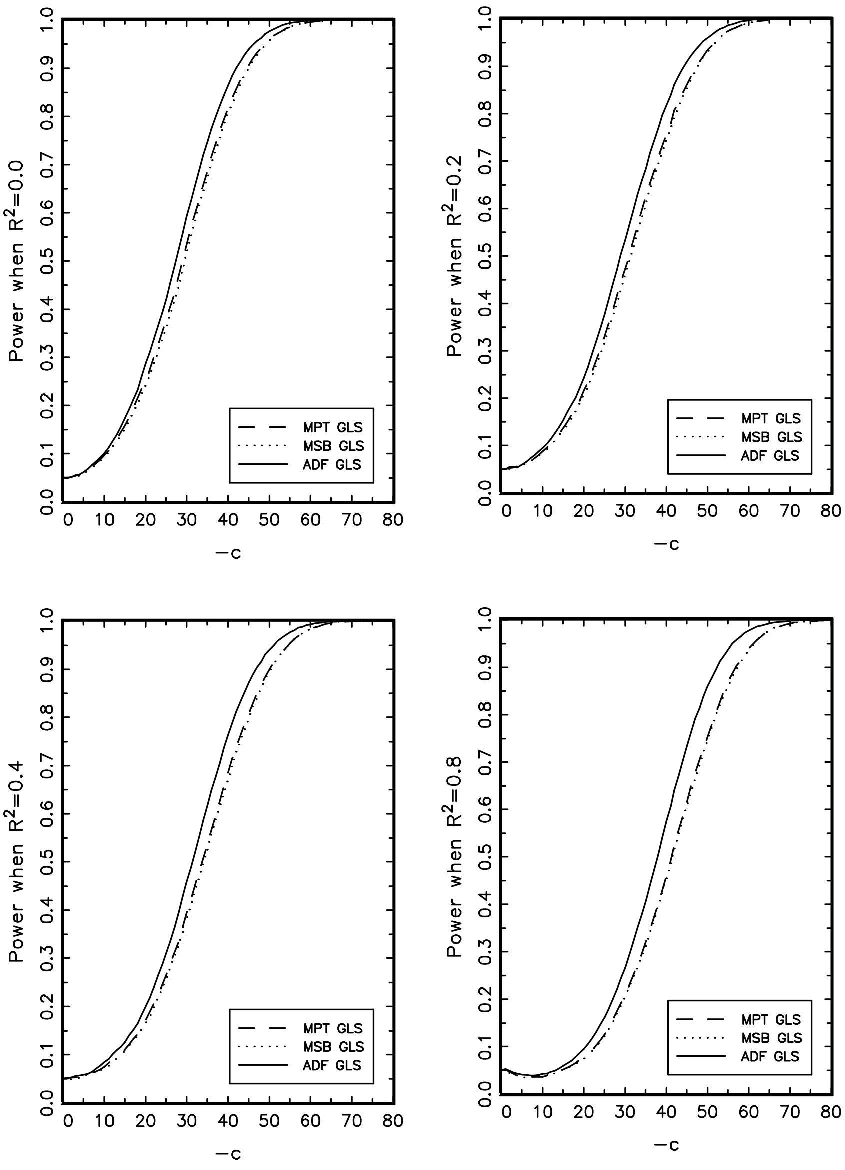

Figure 2f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $x = 5$ and different values of R^2. Detrended Case.

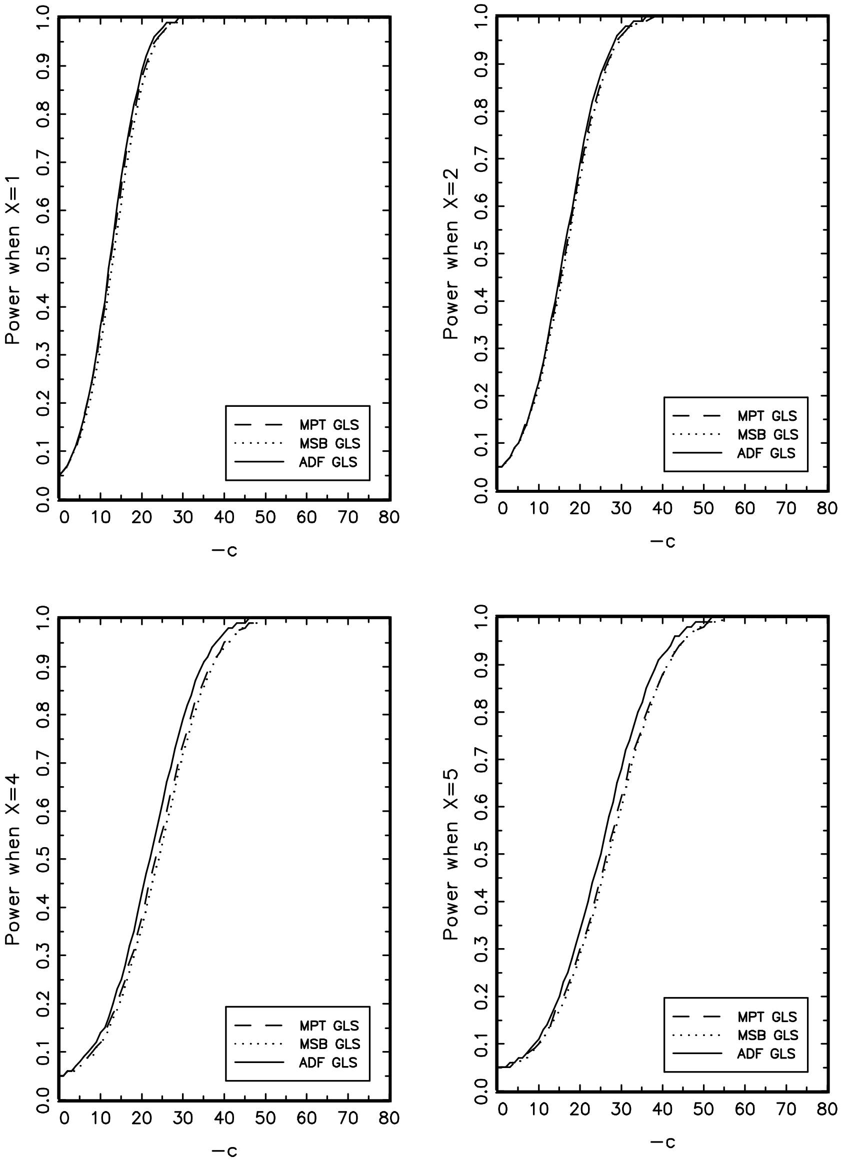

Figure 3a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.0$ and different values of x. Demeaned Case.

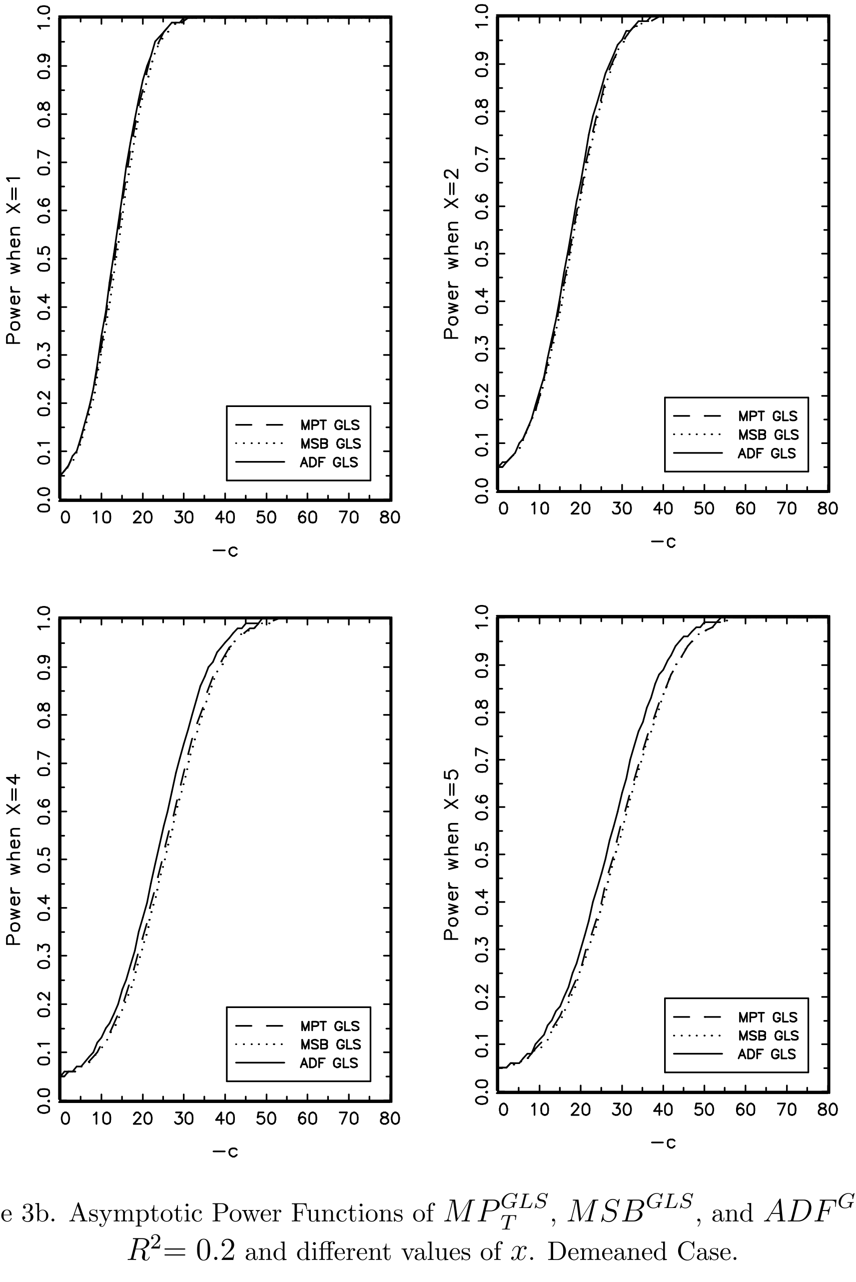

Figure 3b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.2$ and different values of x. Demeaned Case.

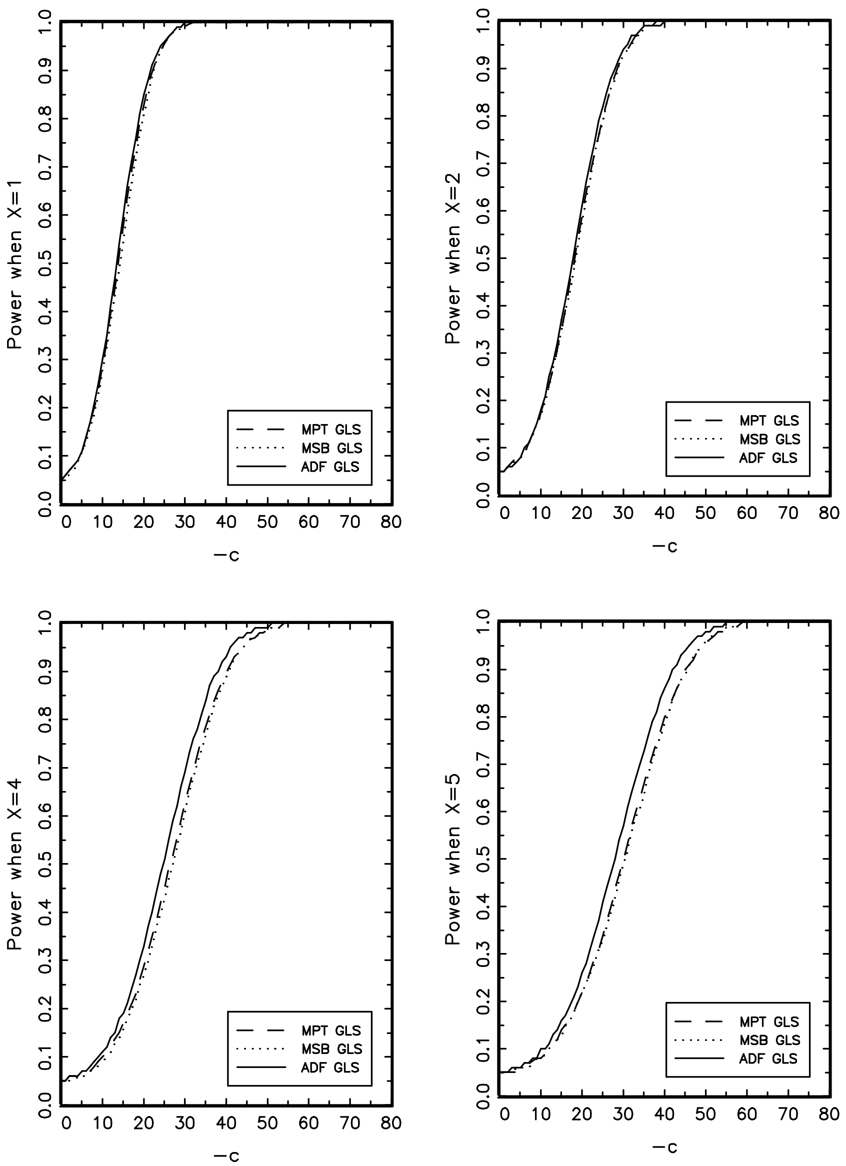

Figure 3c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.4$ and different values of x. Demeaned Case.

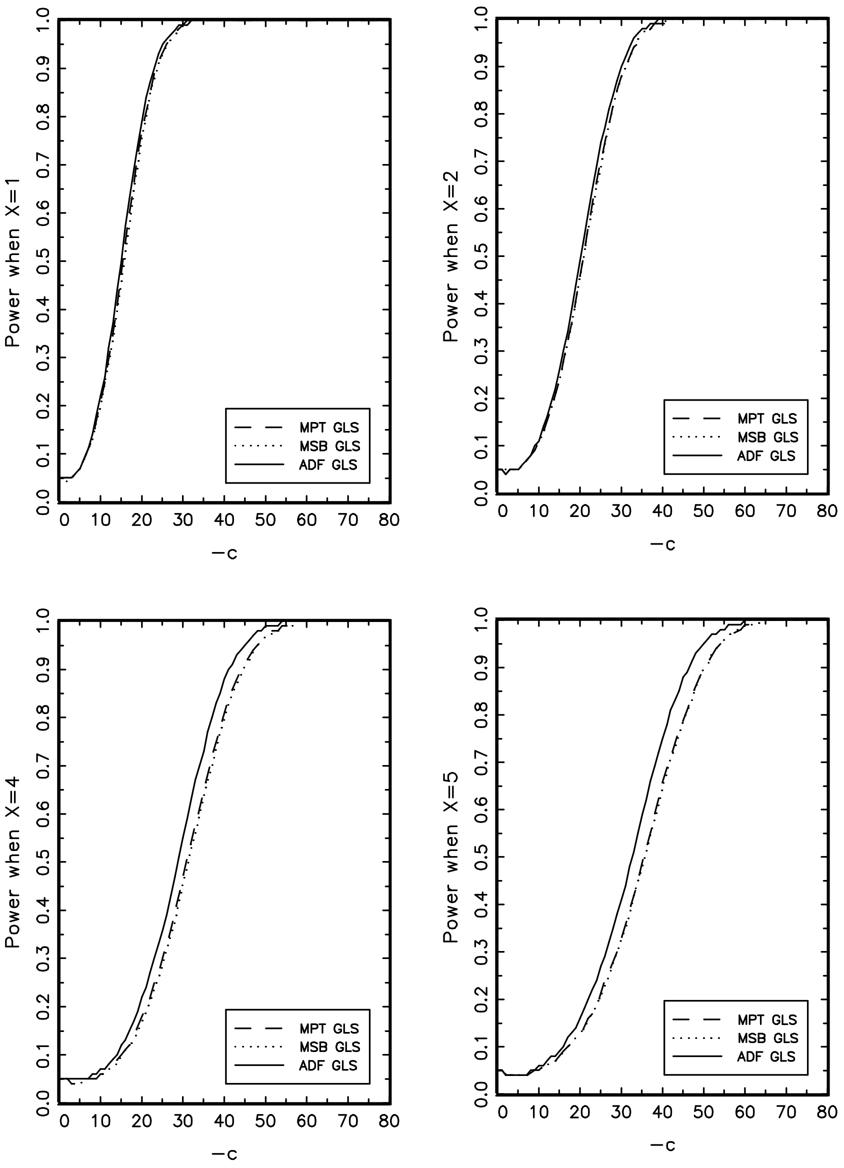

Figure 3d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.8$ and different values of x. Demeaned Case.

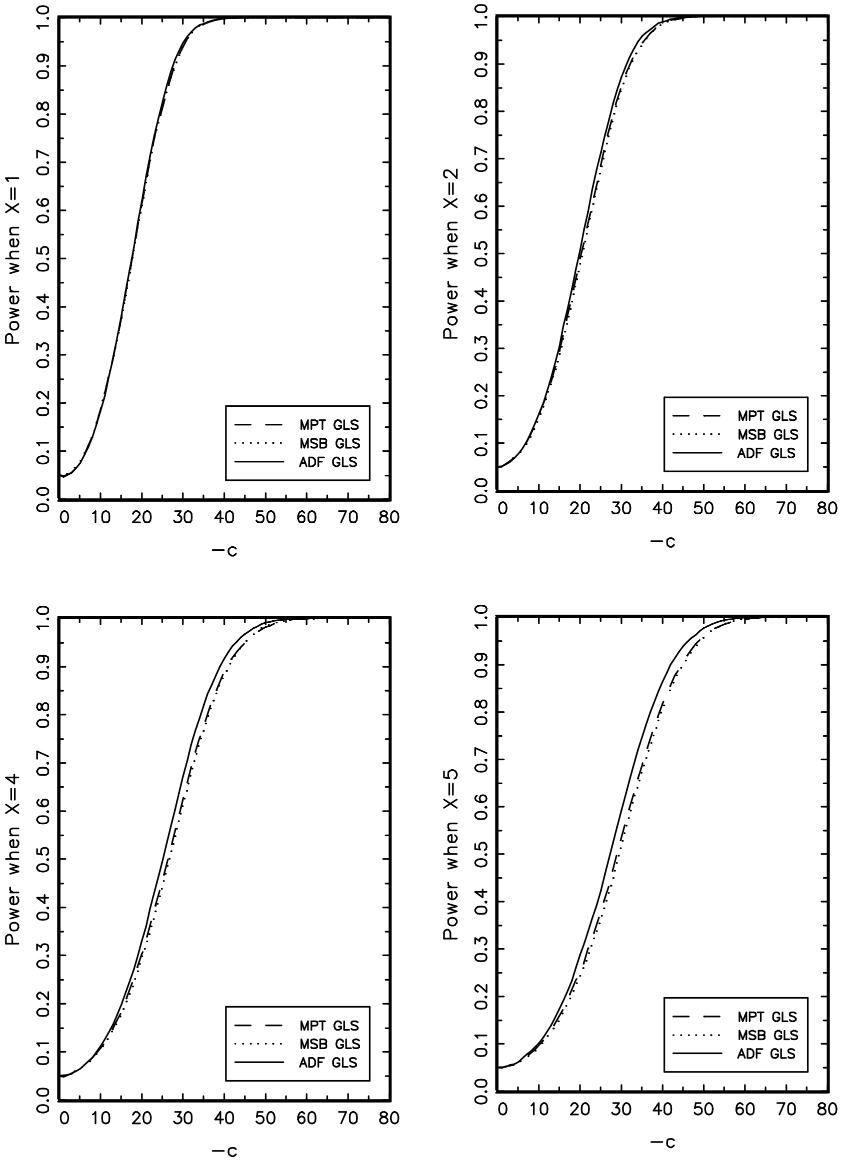

Figure 3e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.0$ and different values of x. Detrended Case.

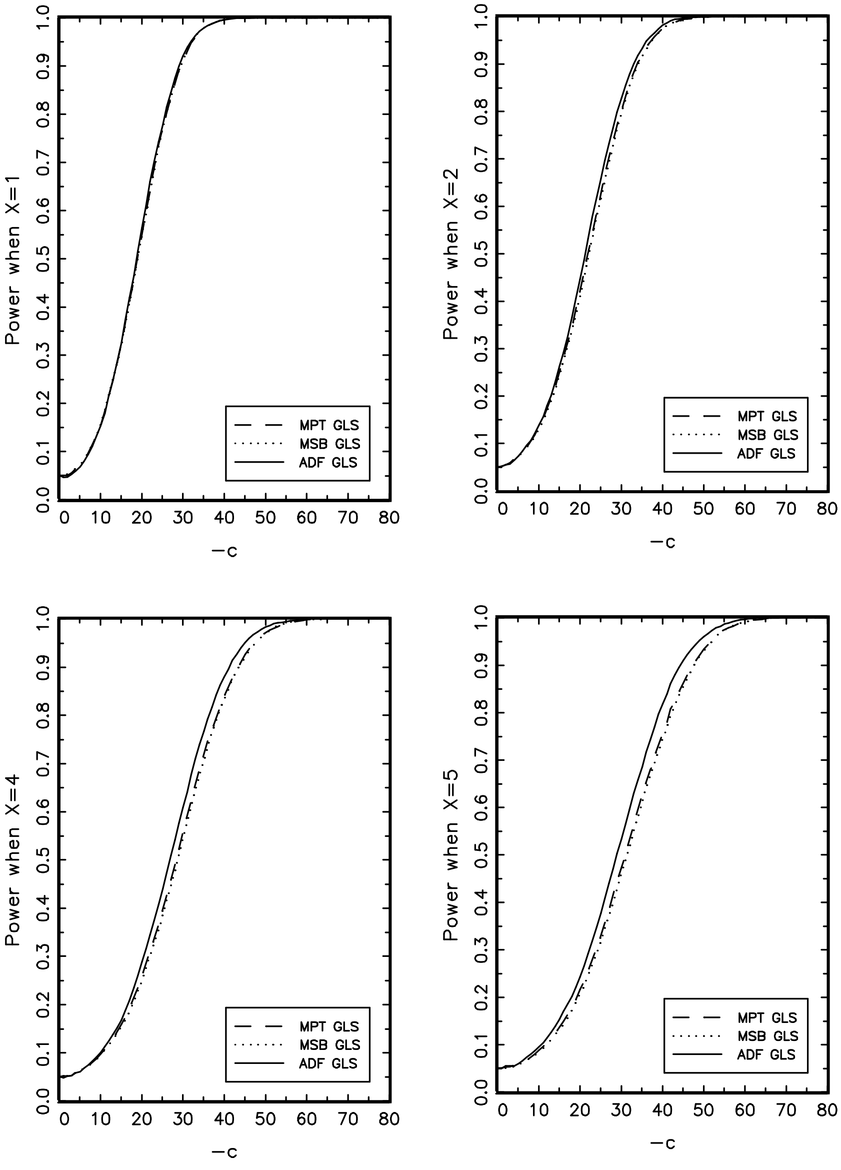

Figure 3f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.2$ and different values of x. Detrended Case.

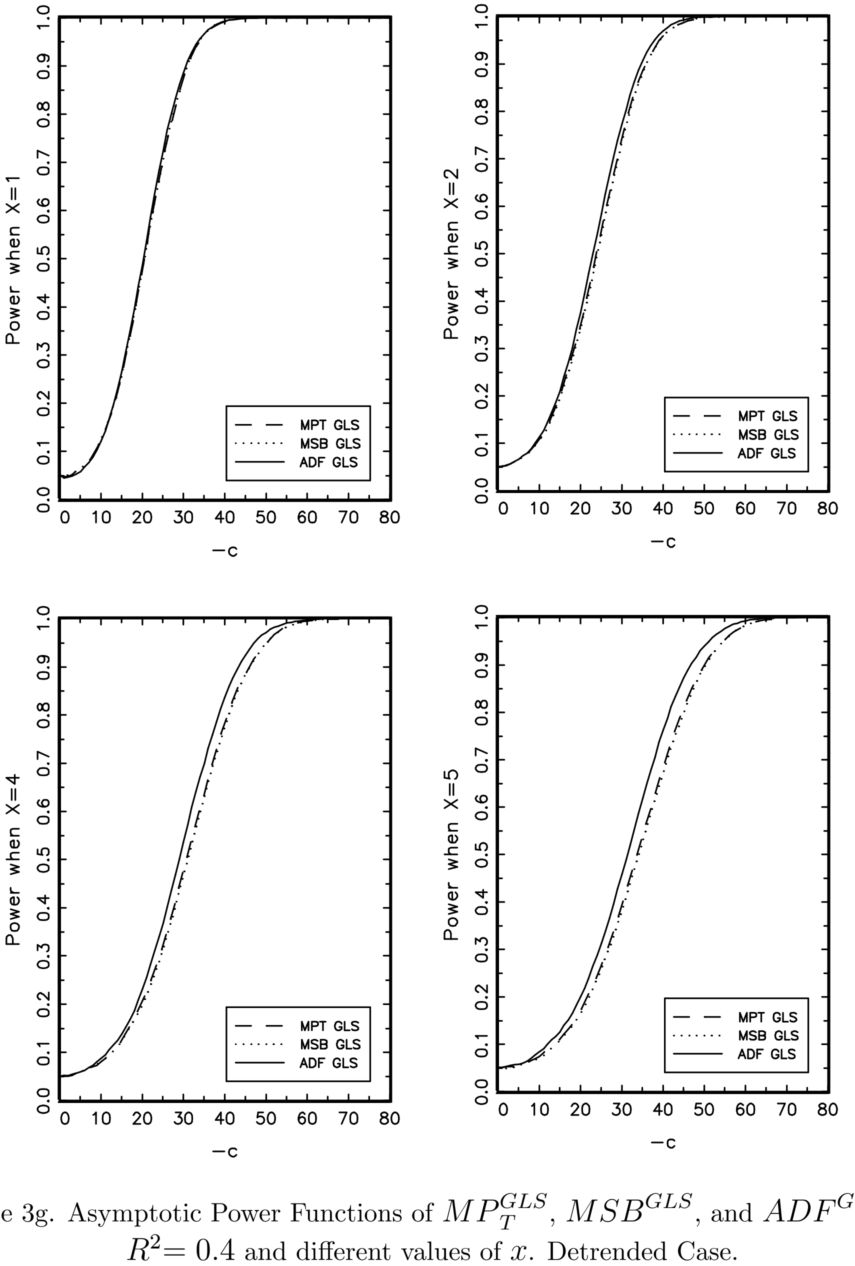

Figure 3g. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for $R^2 = 0.4$ and different values of x. Detrended Case.

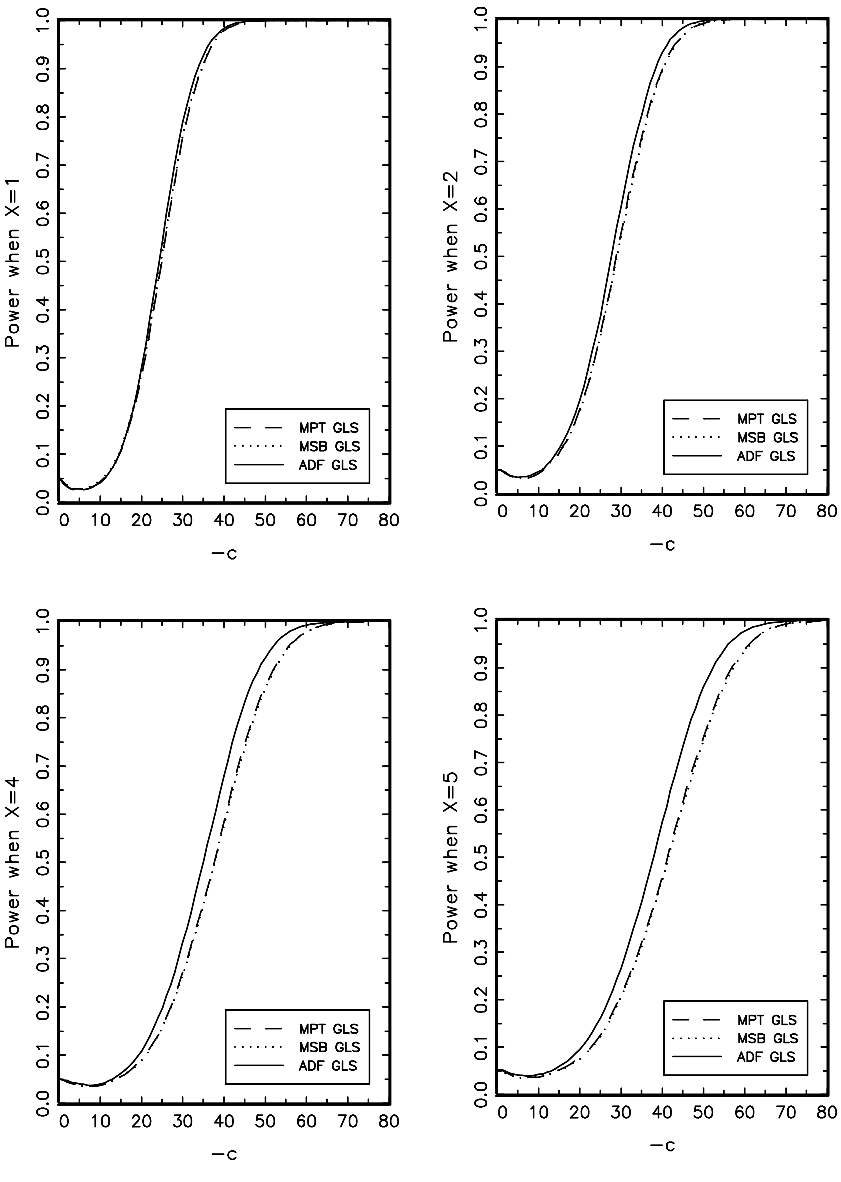

Figure 3h. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, and ADF^{GLS} for R^2= 0.8 and different values of x. Detrended Case.

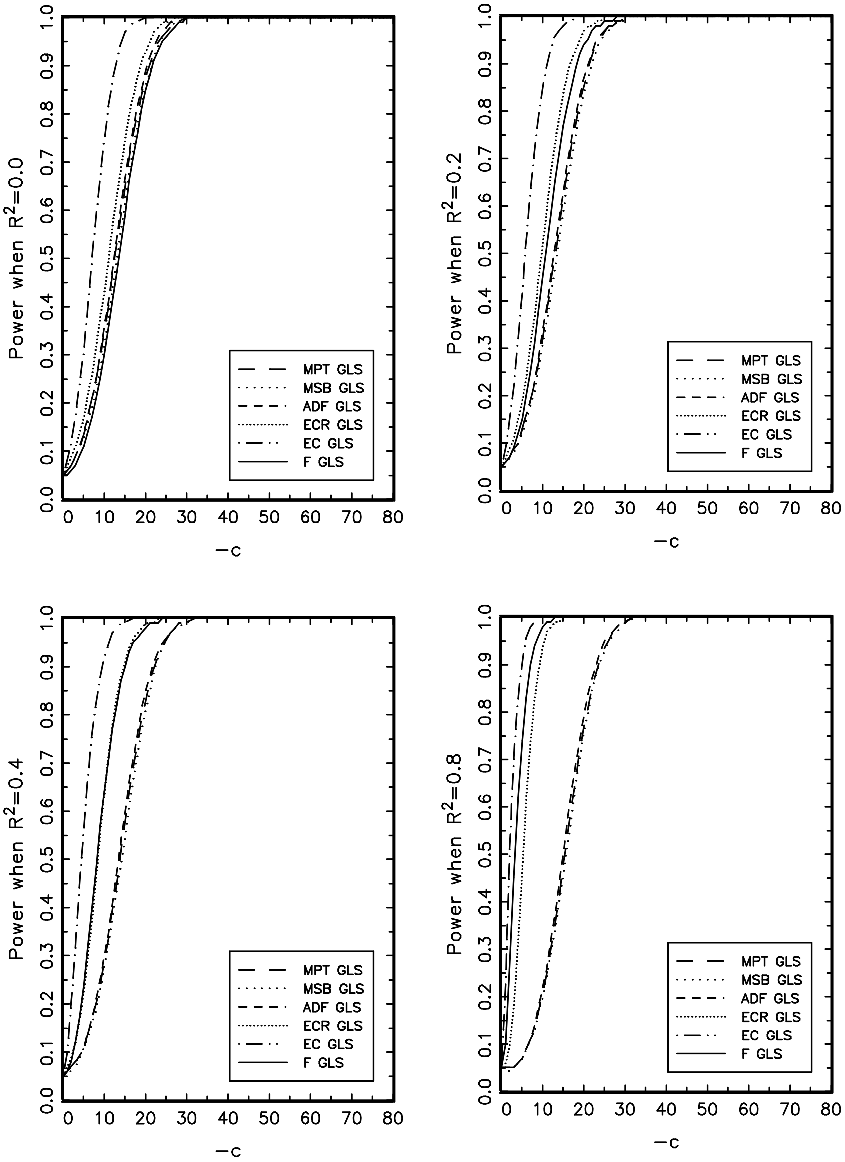

Figure 4a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 1$ and different values of R^2. Demeaned Case.

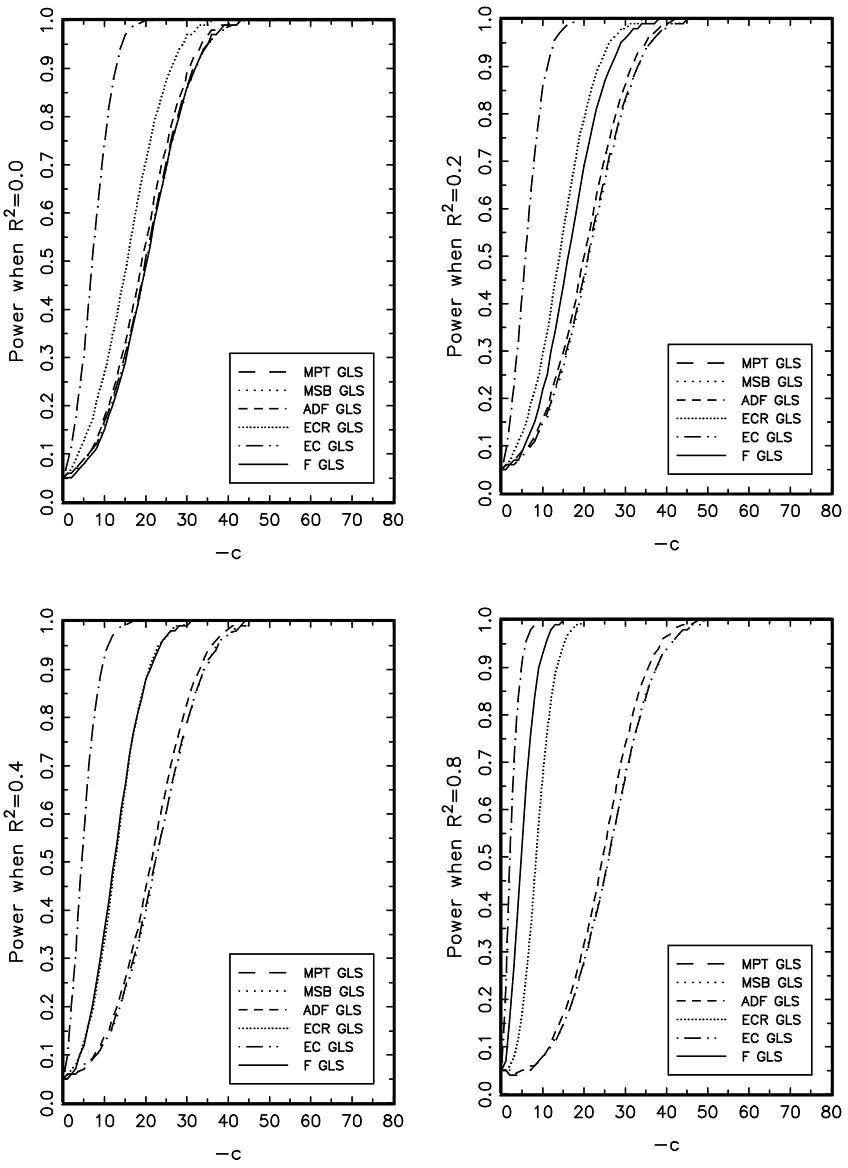

Figure 4b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 3$ and different values of R^2. Demeaned Case.

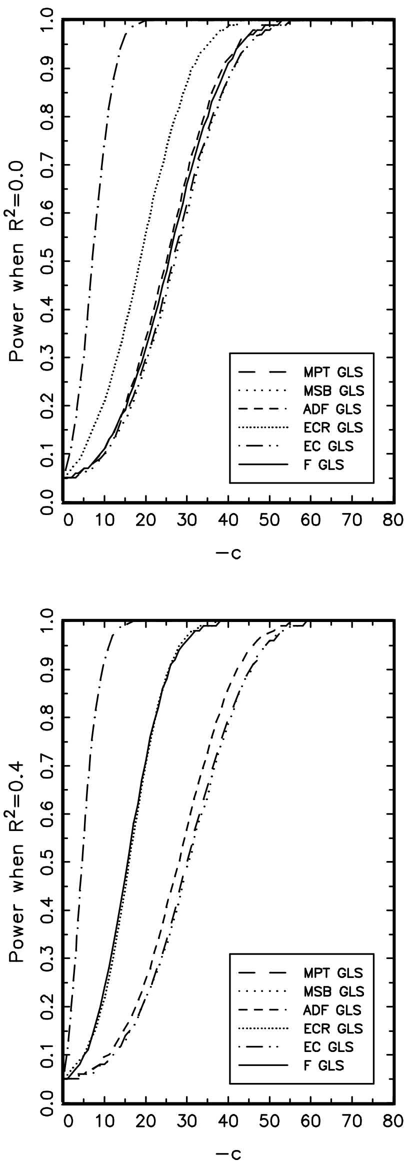

Figure 4c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 5$ and different values of R^2. Demeaned Case.

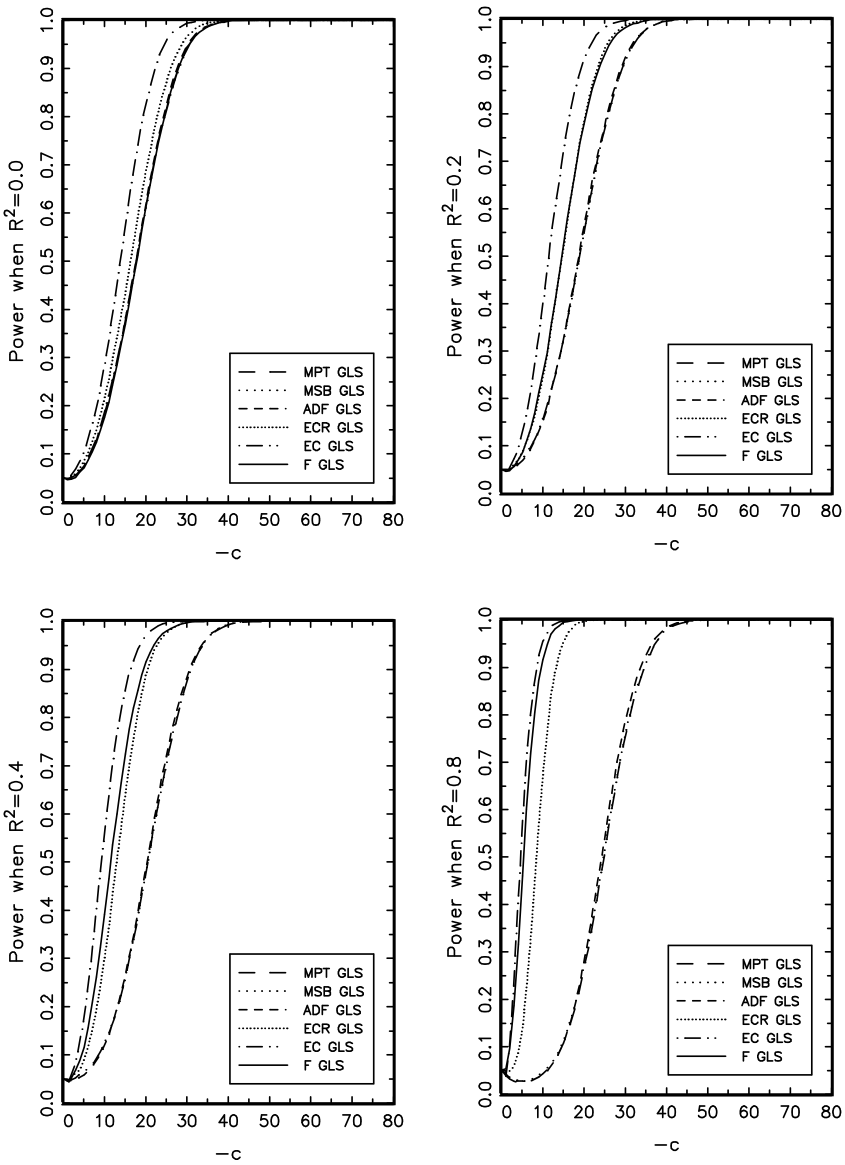

Figure 4d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 1$ and different values of R^2. Detrended Case.

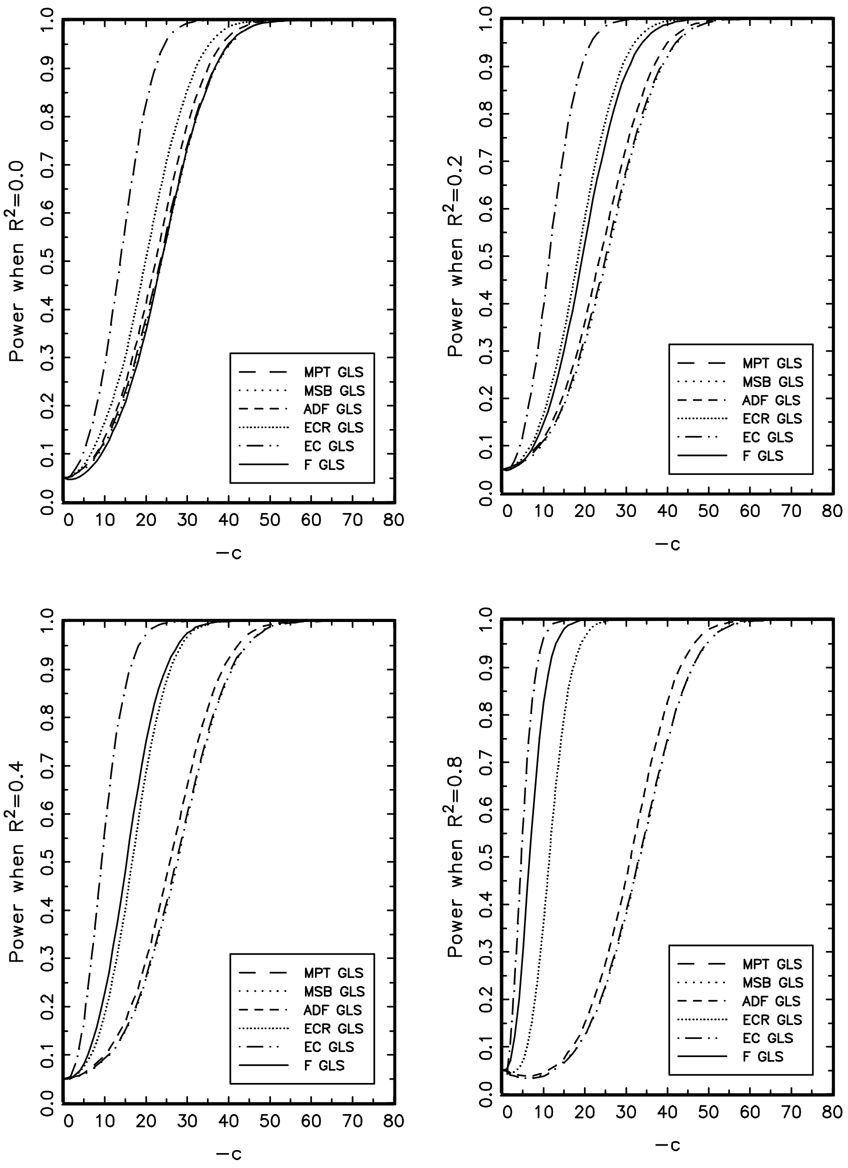

Figure 4e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 3$ and different values of R^2. Detrended Case.

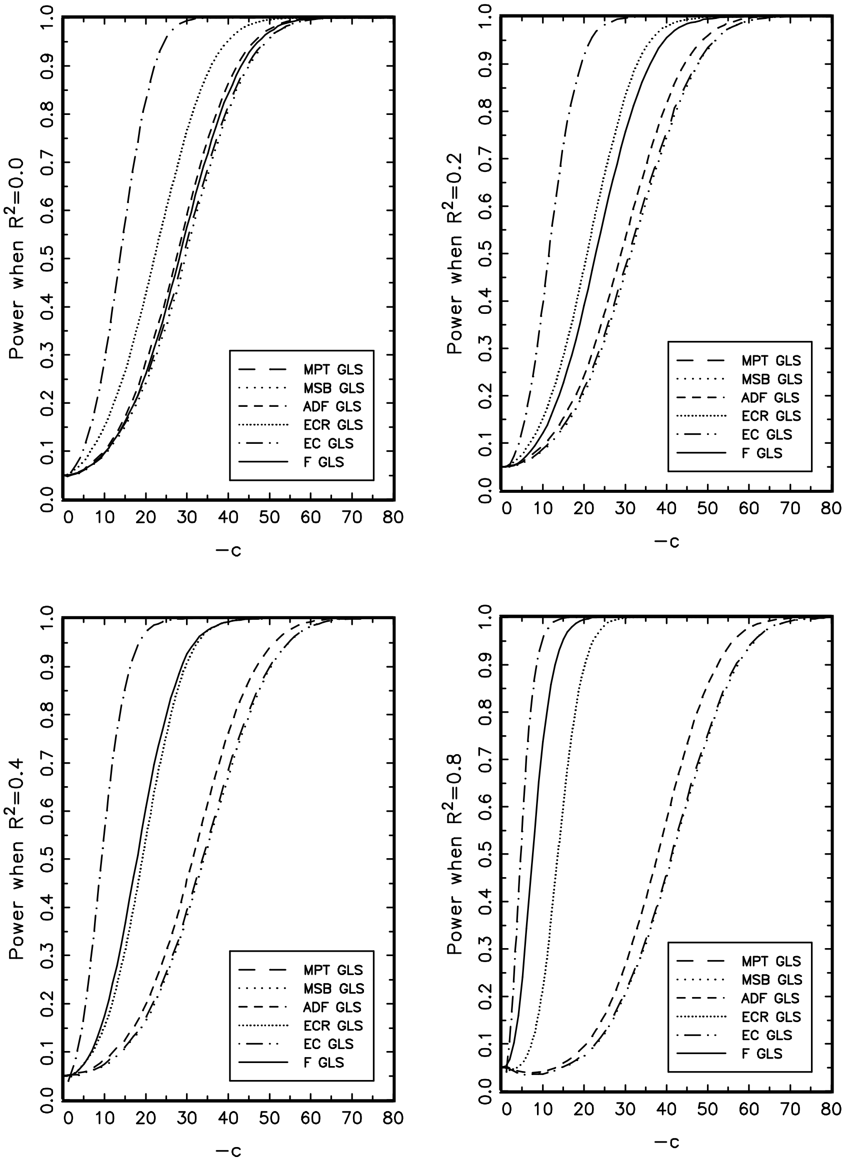

Figure 4f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 5$ and different values of R^2. Detrended Case.

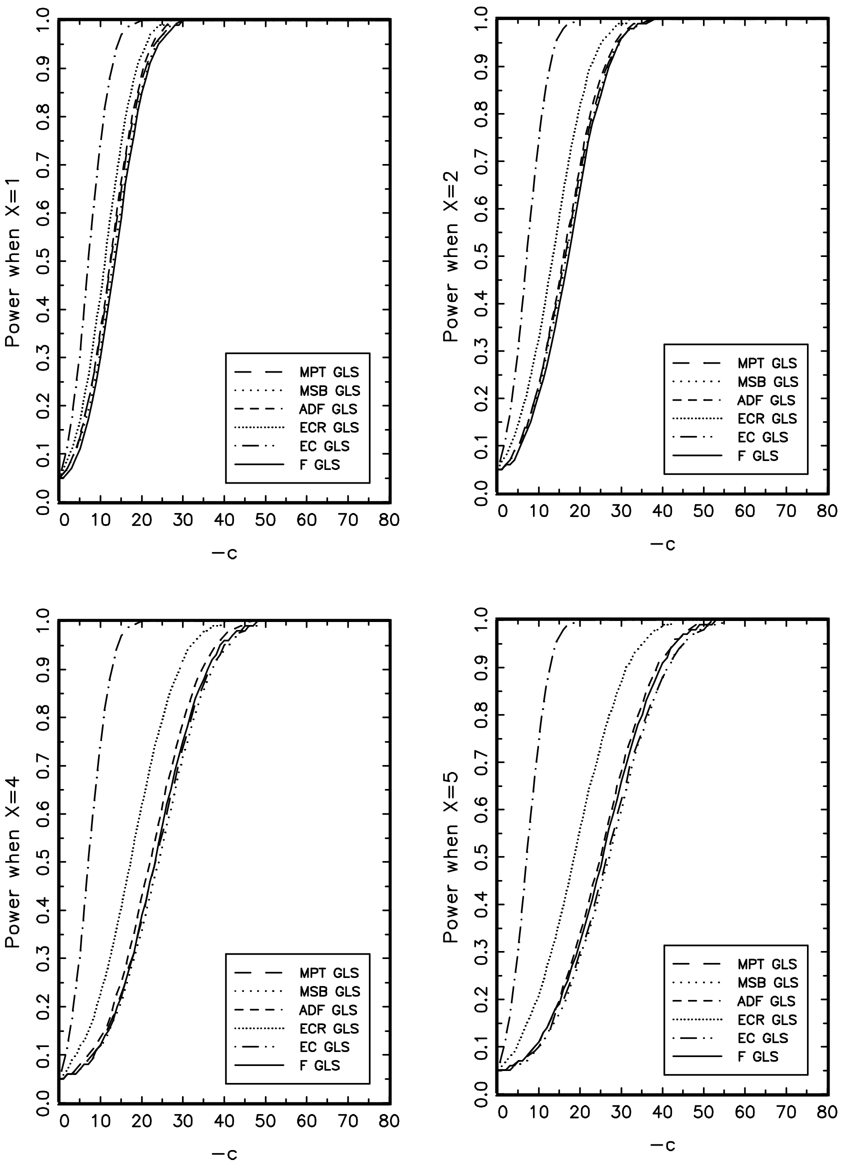

Figure 5a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.0$ and different values of x. Demeaned Case.

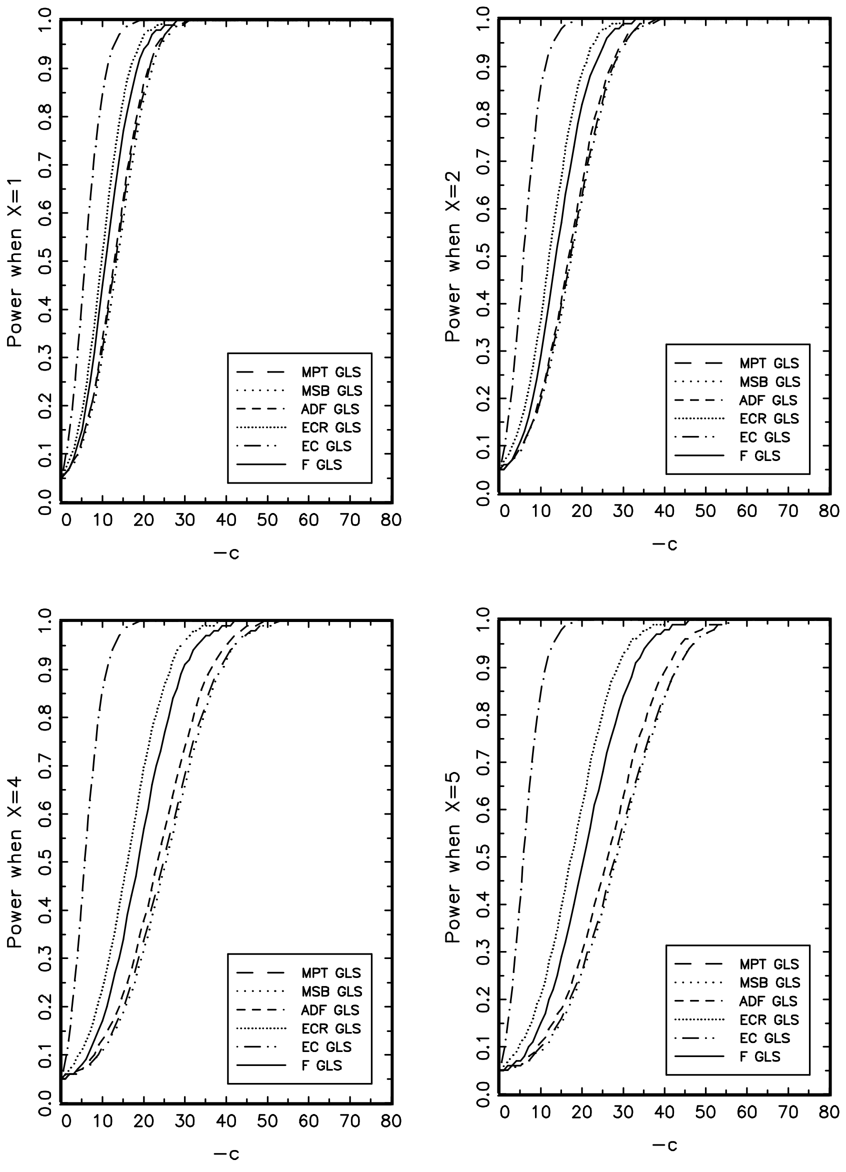

Figure 5b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.2 and different values of x. Demeaned Case.

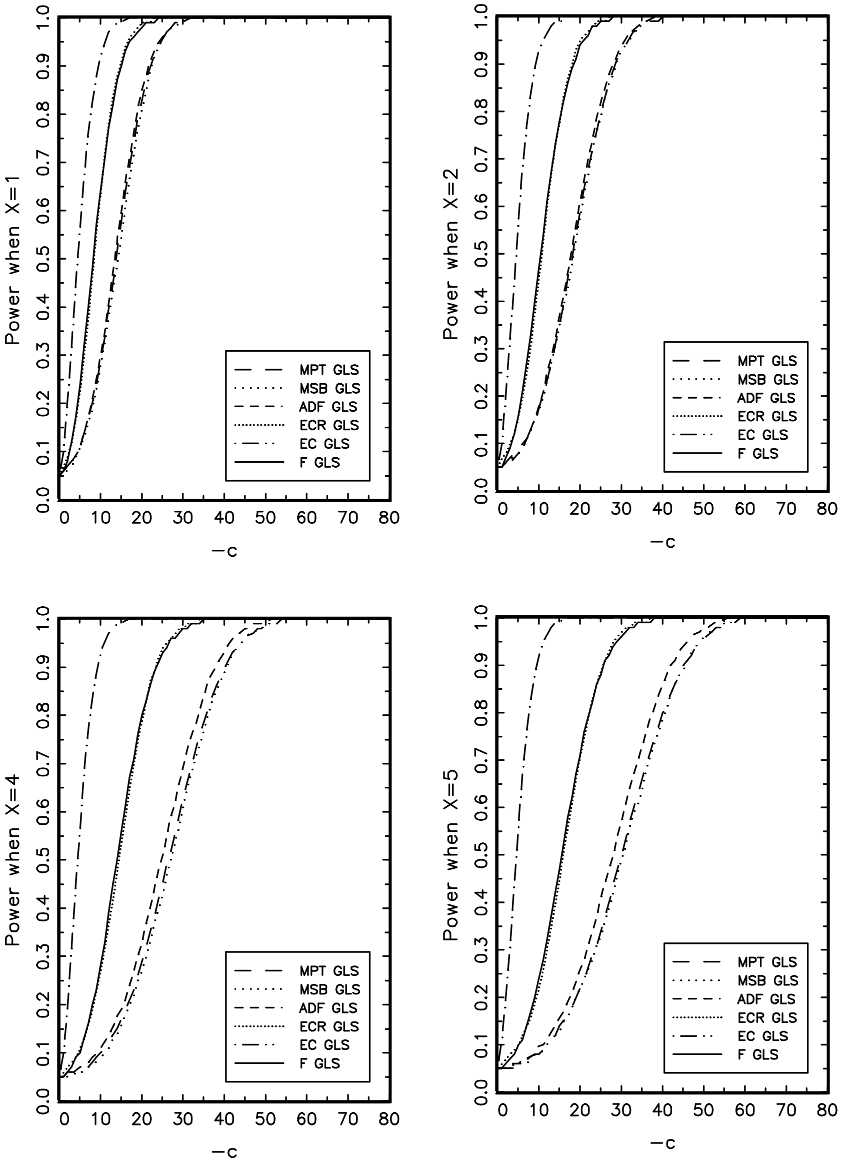

Figure 5c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.4$ and different values of x. Demeaned Case.

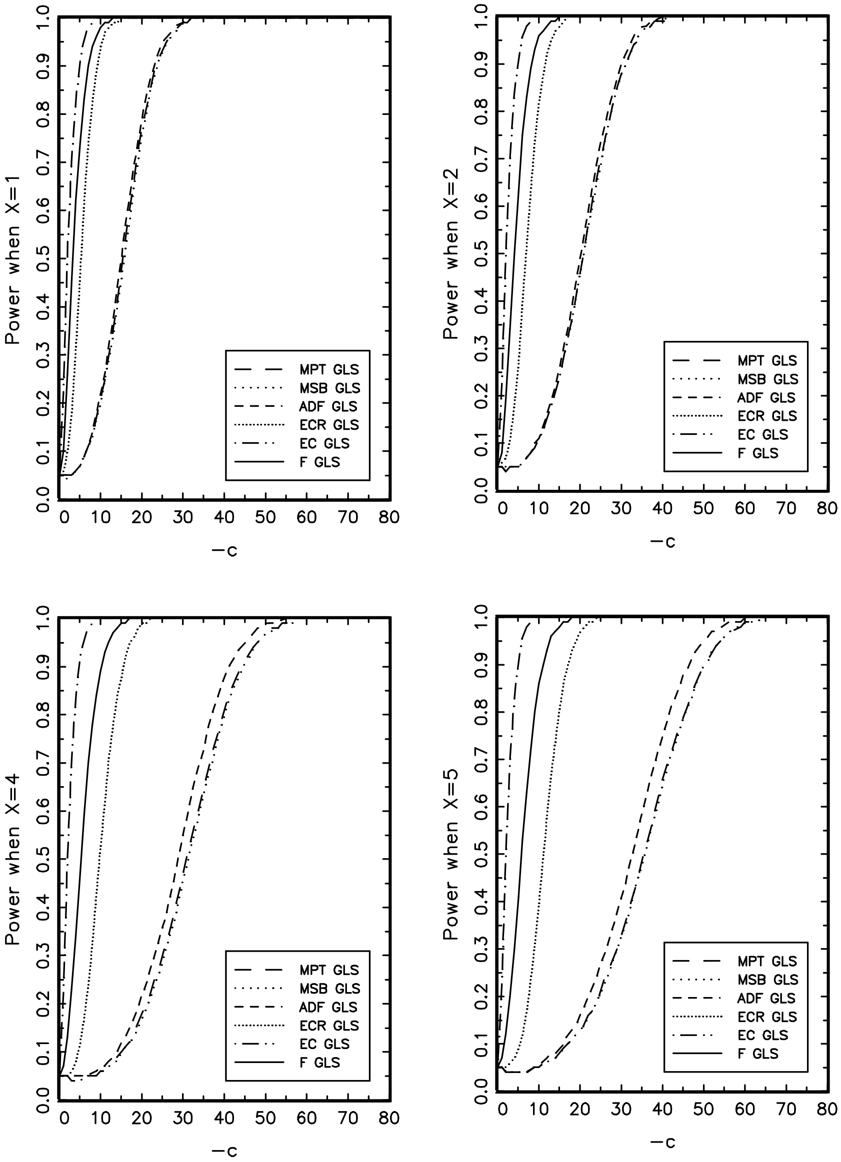

Figure 5d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.8$ and different values of x. Demeaned Case.

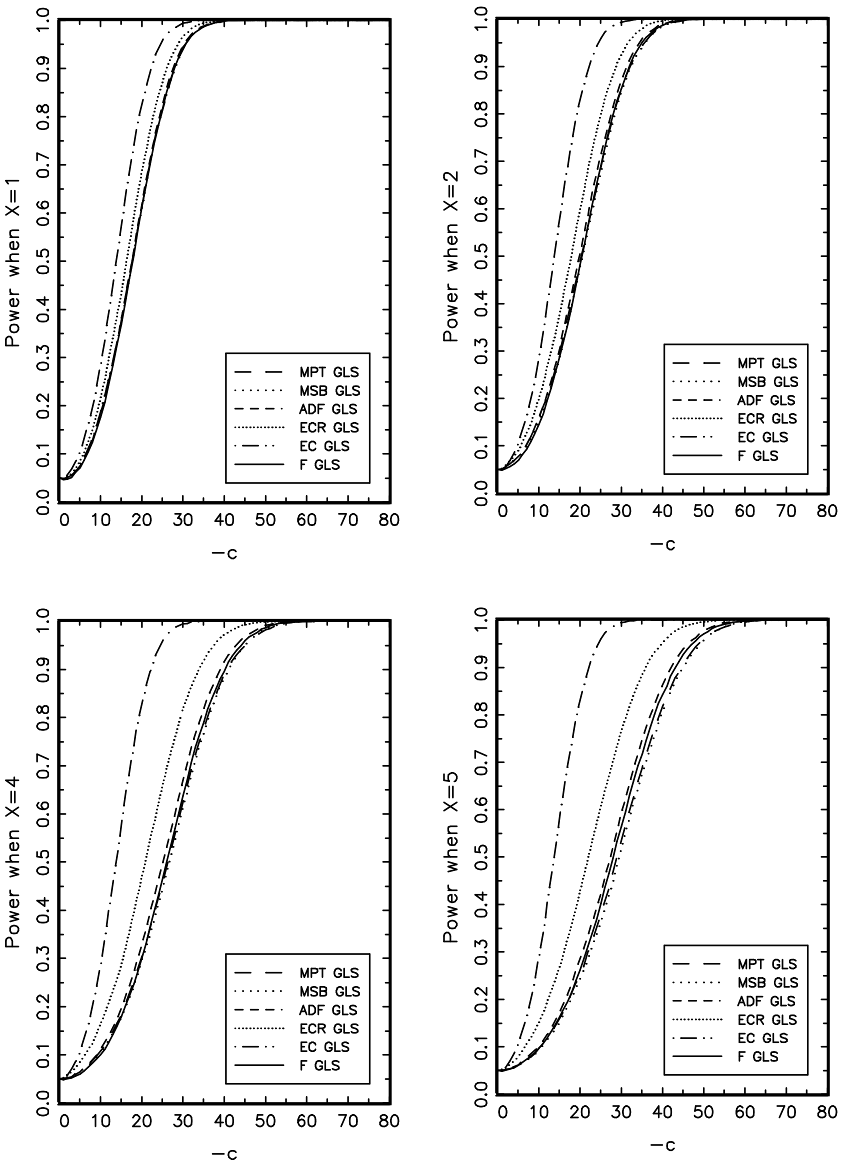

Figure 5e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.0 and different values of x. Detrended Case.

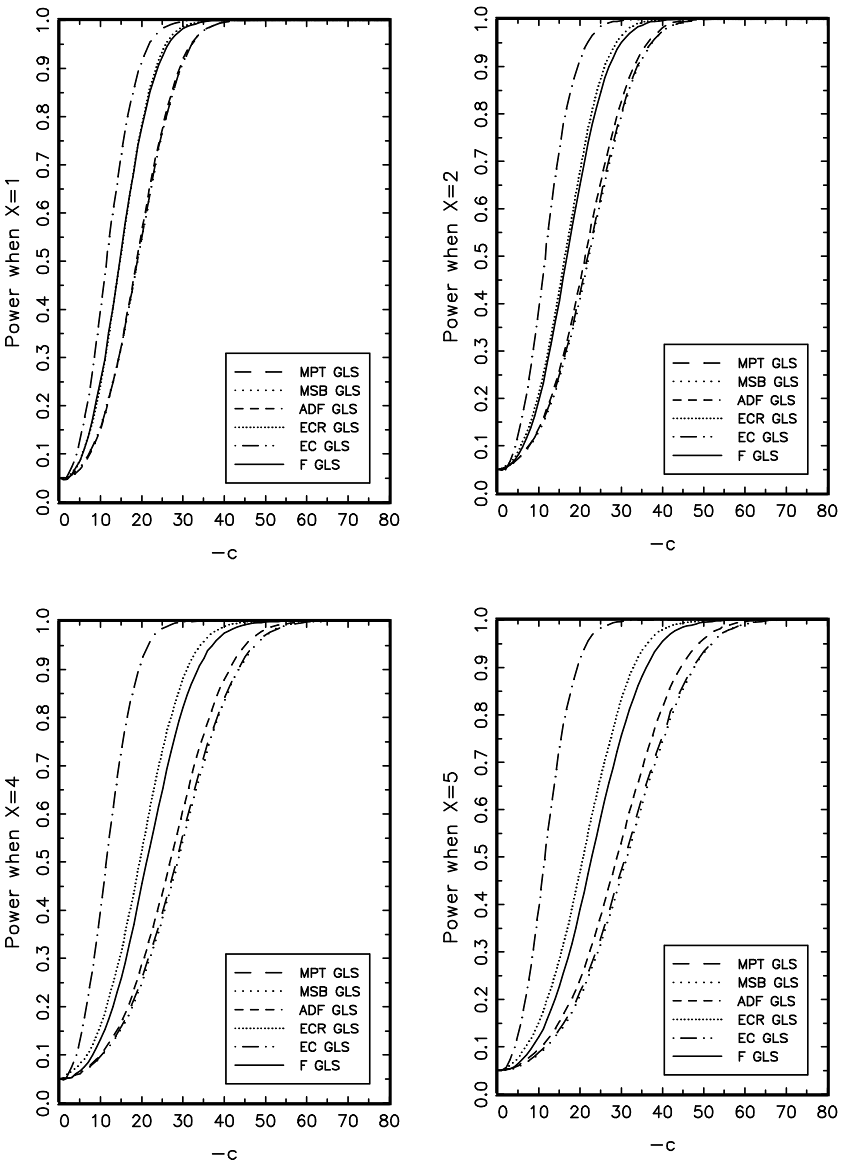

Figure 5f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.2 and different values of x. Detrended Case.

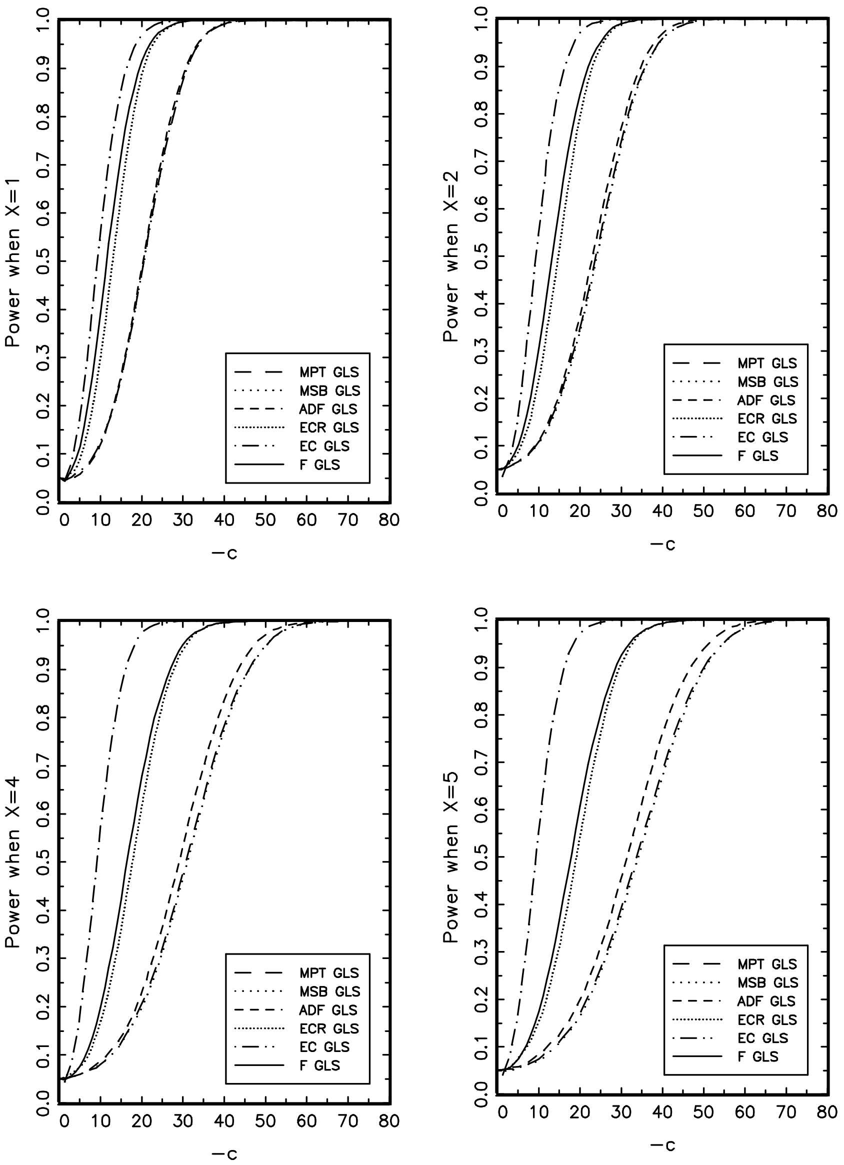

Figure 5g. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.4$ and different values of x. Detrended Case.

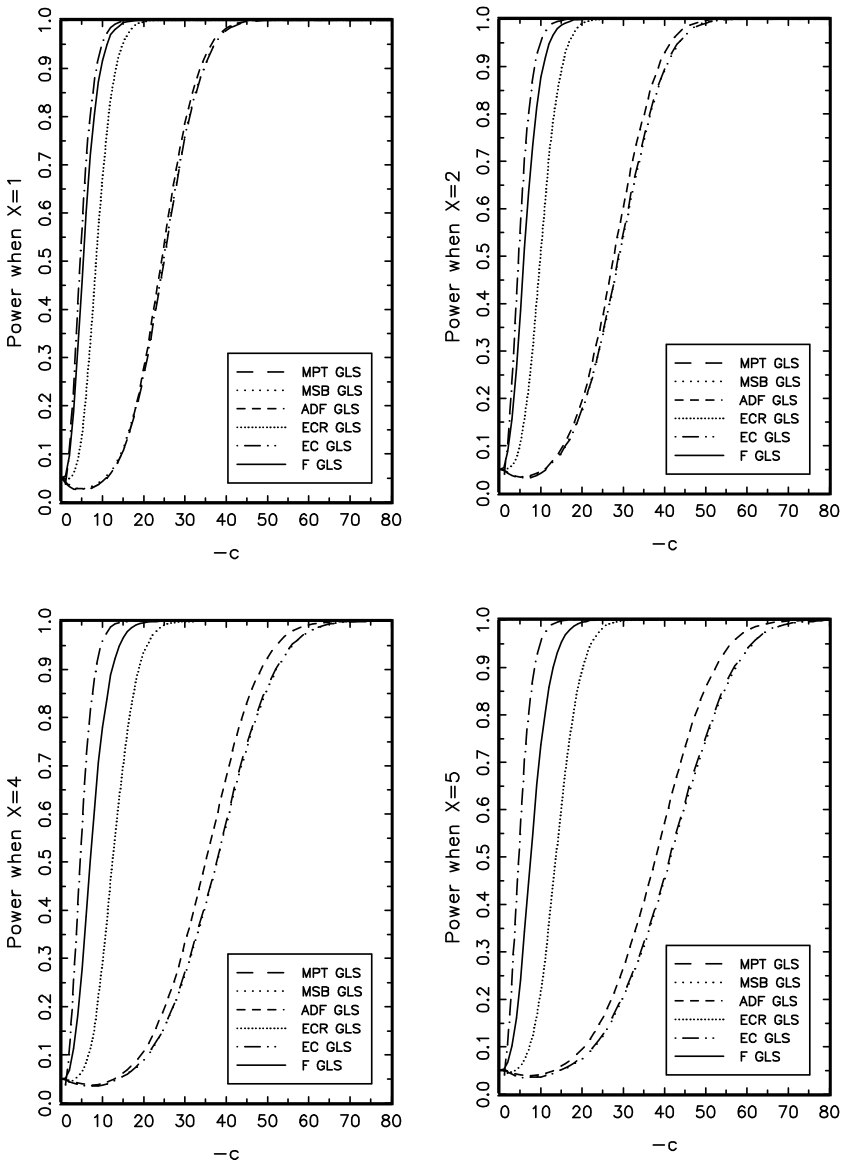

Figure 5h. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.8 and different values of x. Detrended Case.

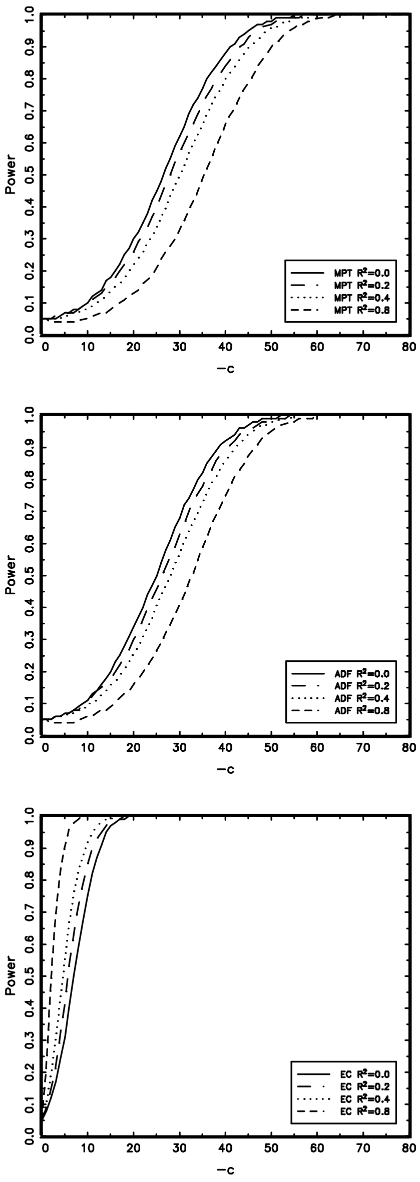

Figure 6a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 1$ and different values of R^2. Demeaned Case.

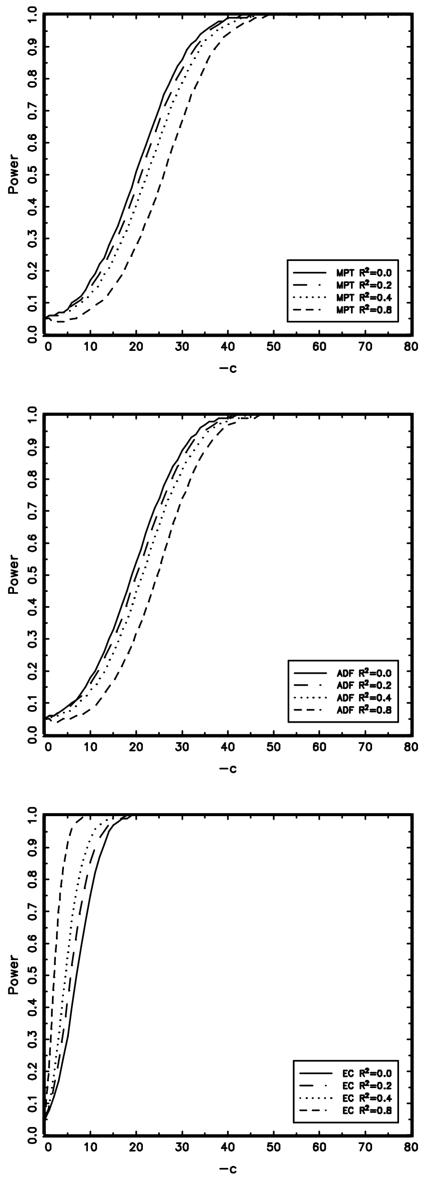

Figure 6b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 3$ and different values of R^2. Demeaned Case.

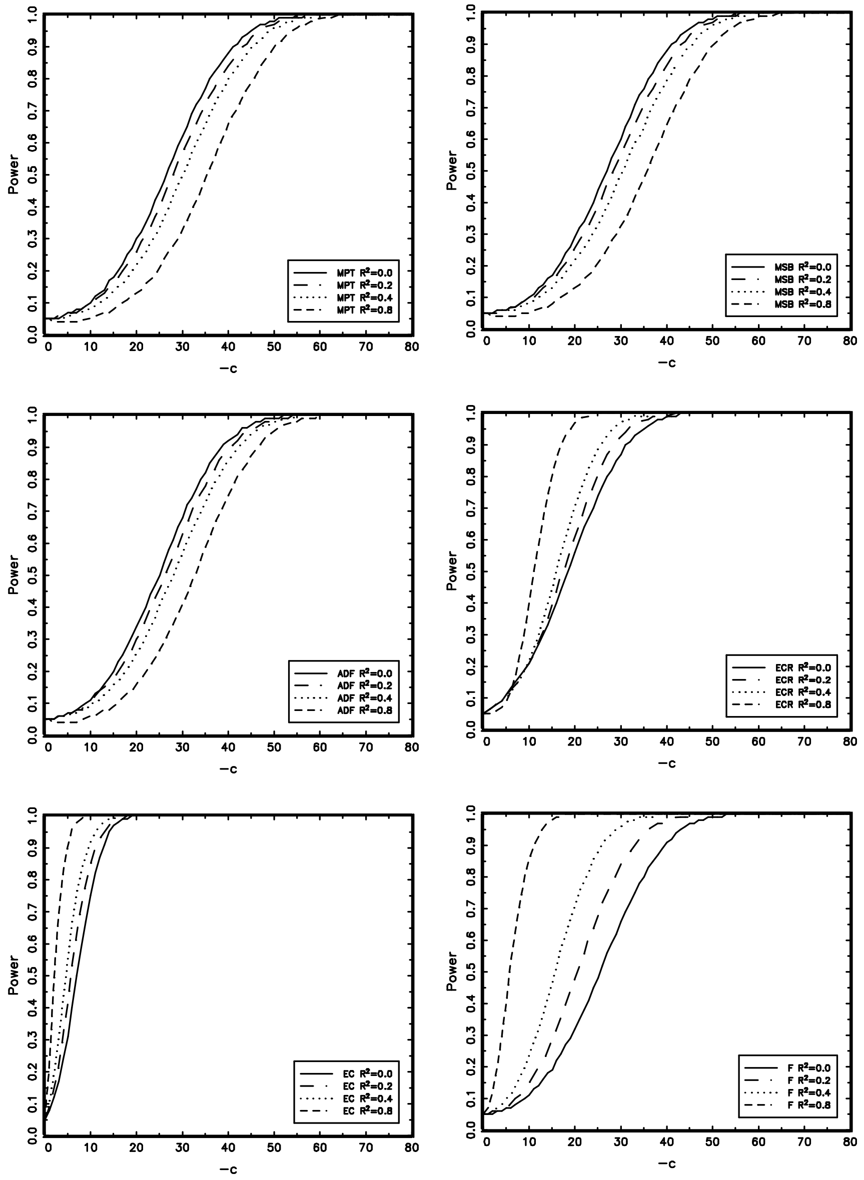

Figure 6c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 5$ and different values of R^2. Demeaned Case.

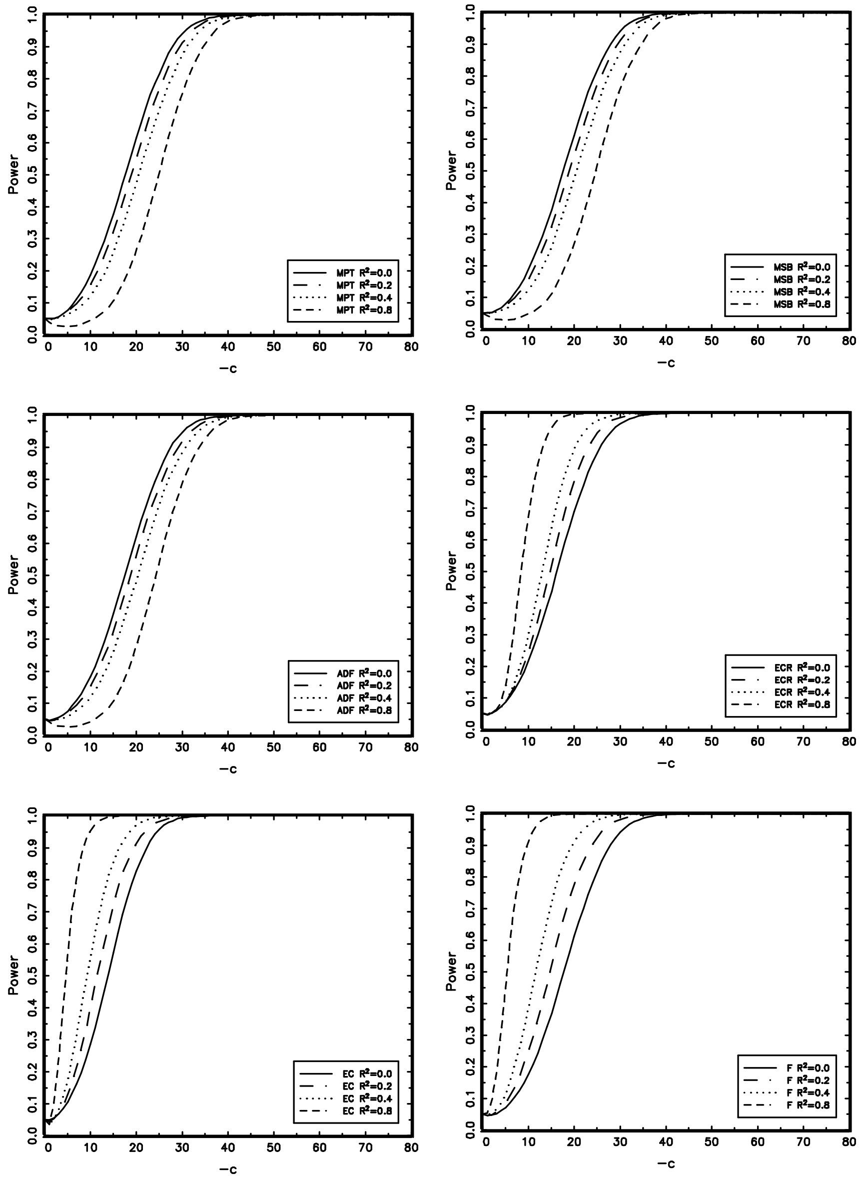

Figure 6d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 1$ and different values of R^2. Detrended Case.

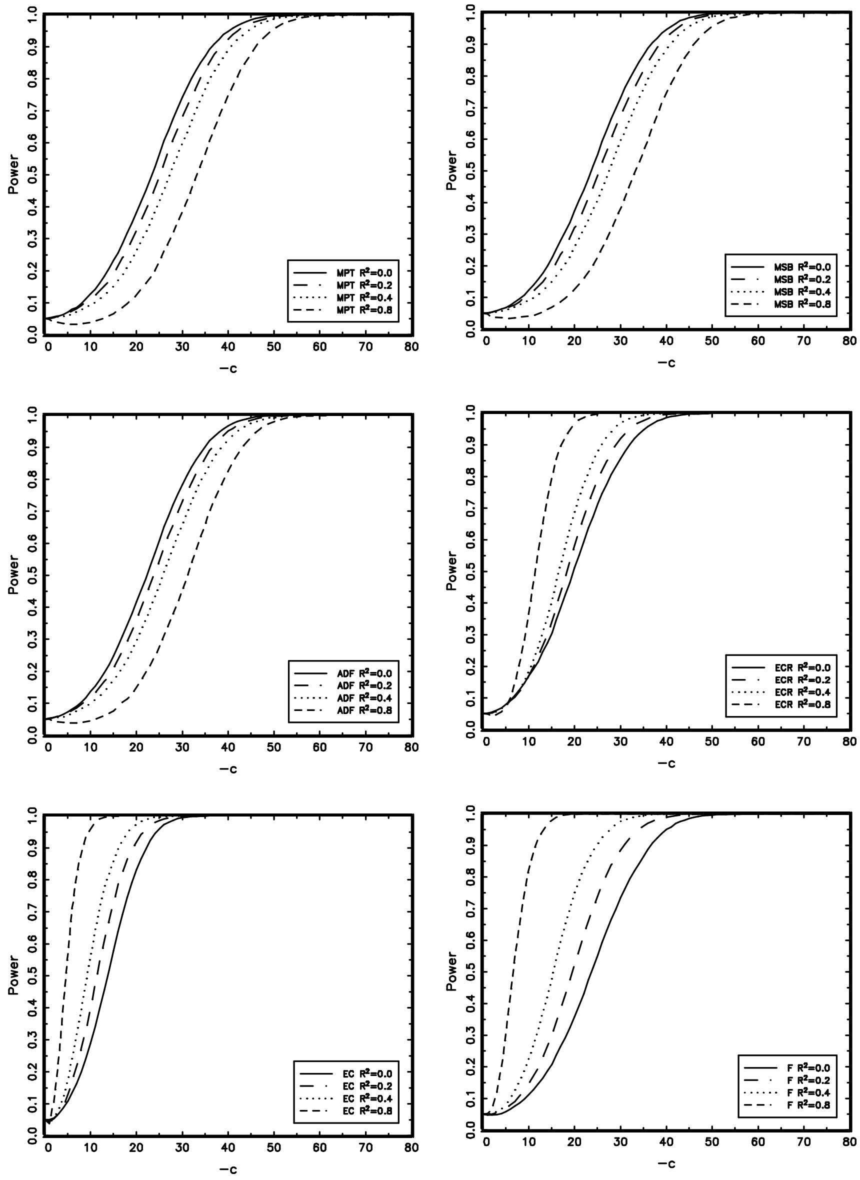

Figure 6e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 3$ and different values of R^2. Detrended Case.

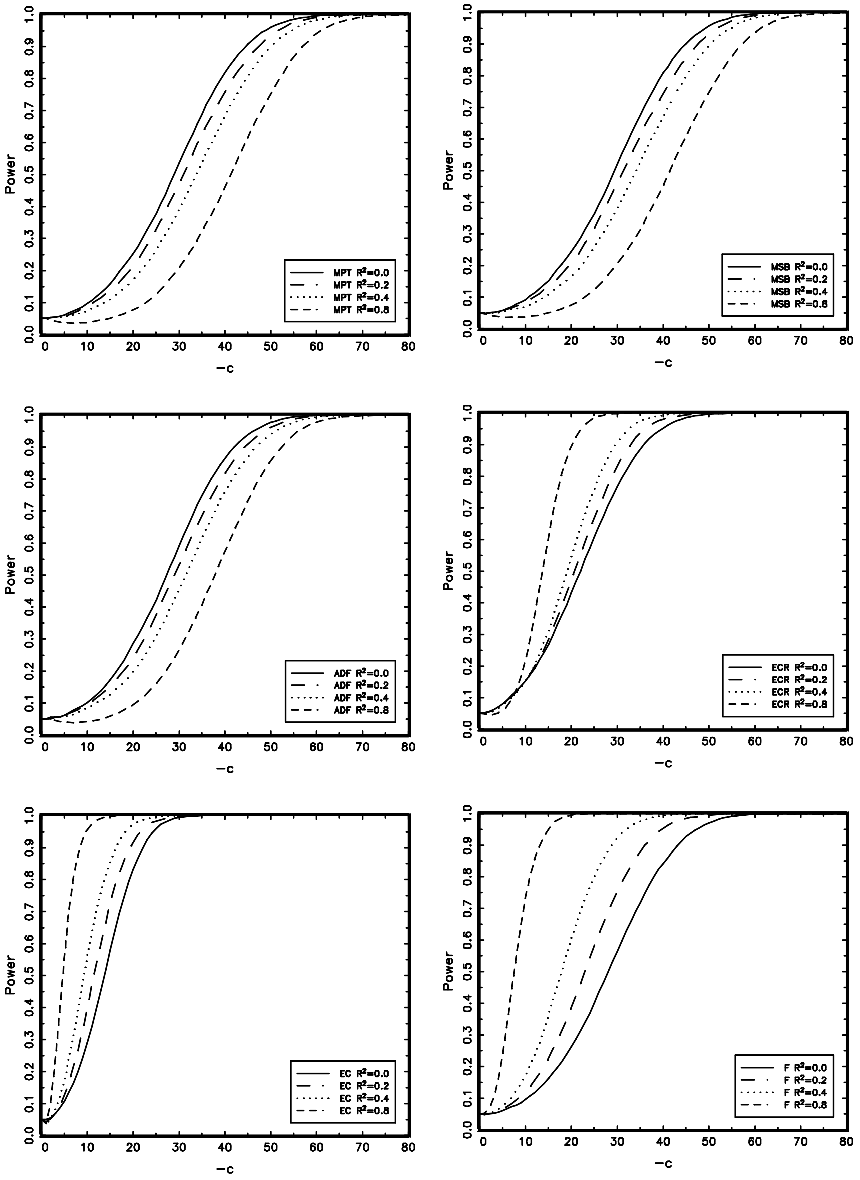

Figure 6f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $x = 5$ and different values of R^2. Detrended Case.

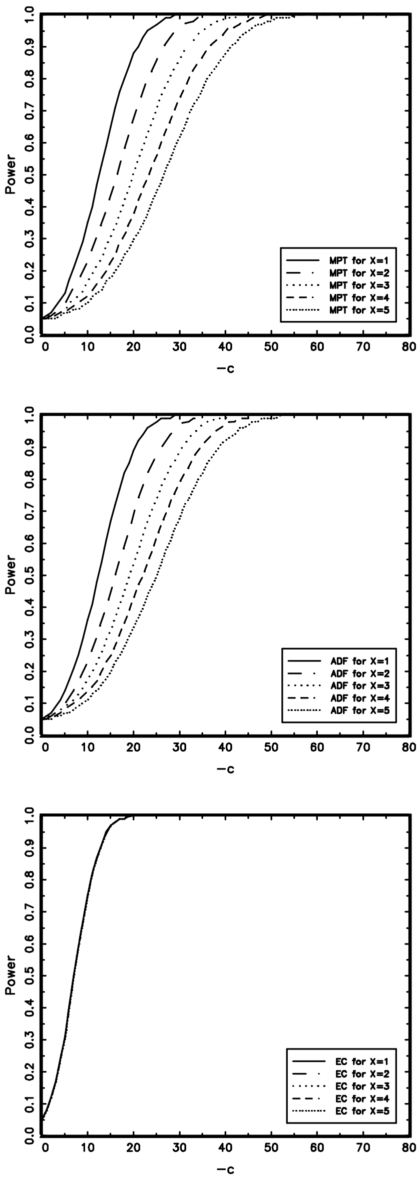

Figure 7a. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.0 and different values of x. Demeaned Case.

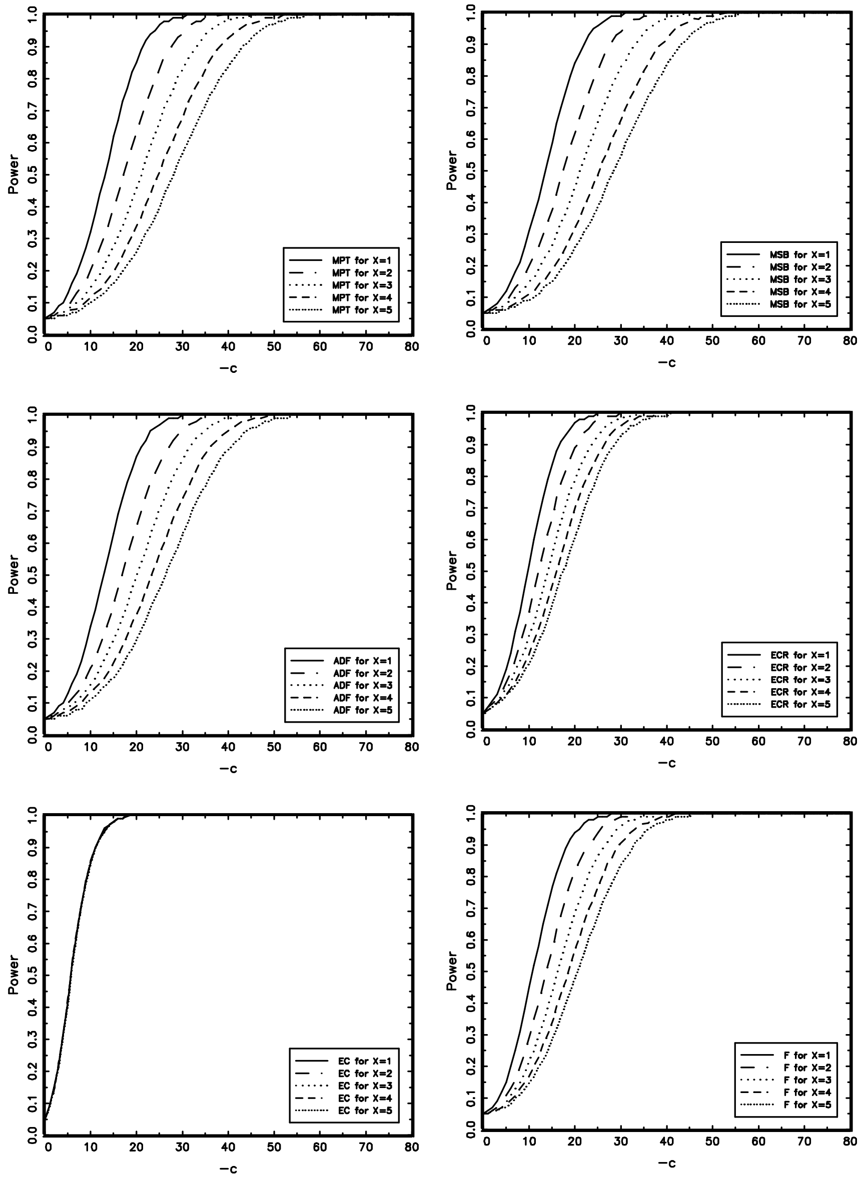

Figure 7b. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.2$ and different values of x. Demeaned Case.

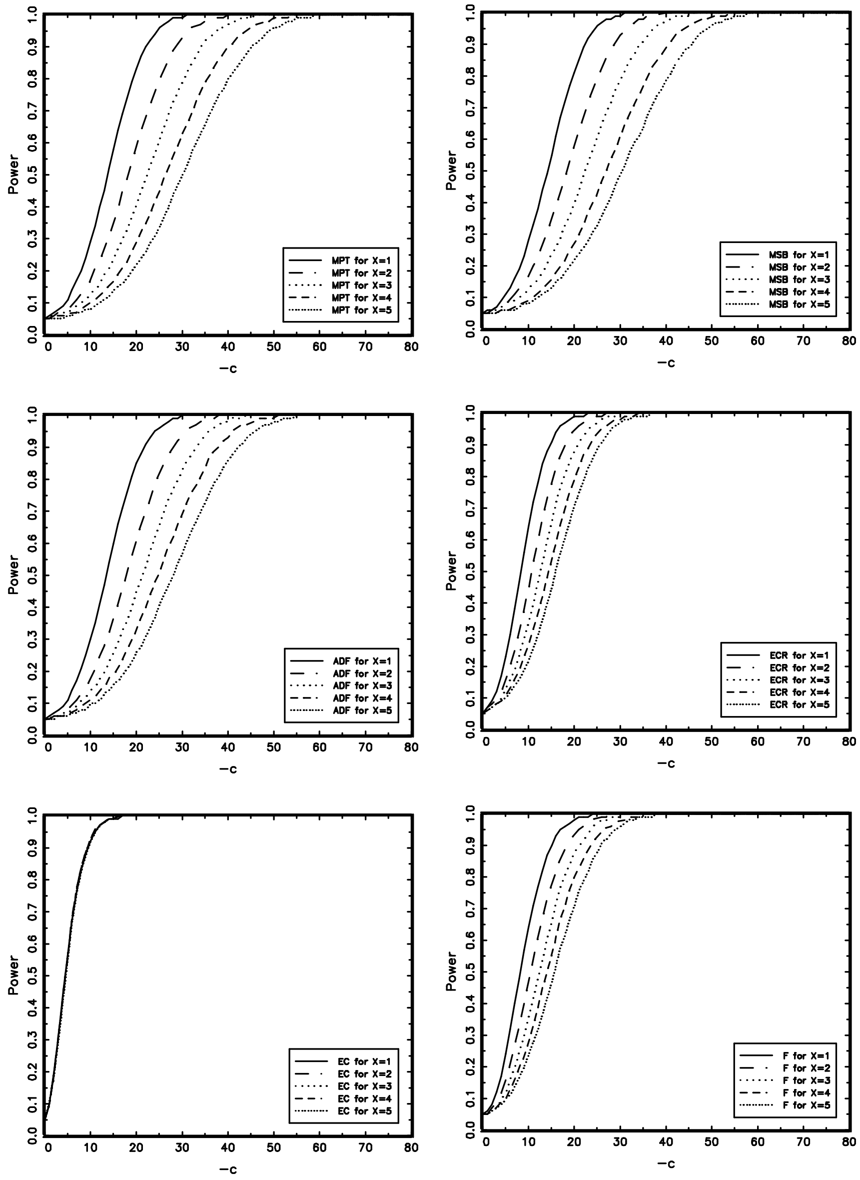

Figure 7c. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.4 and different values of x. Demeaned Case.

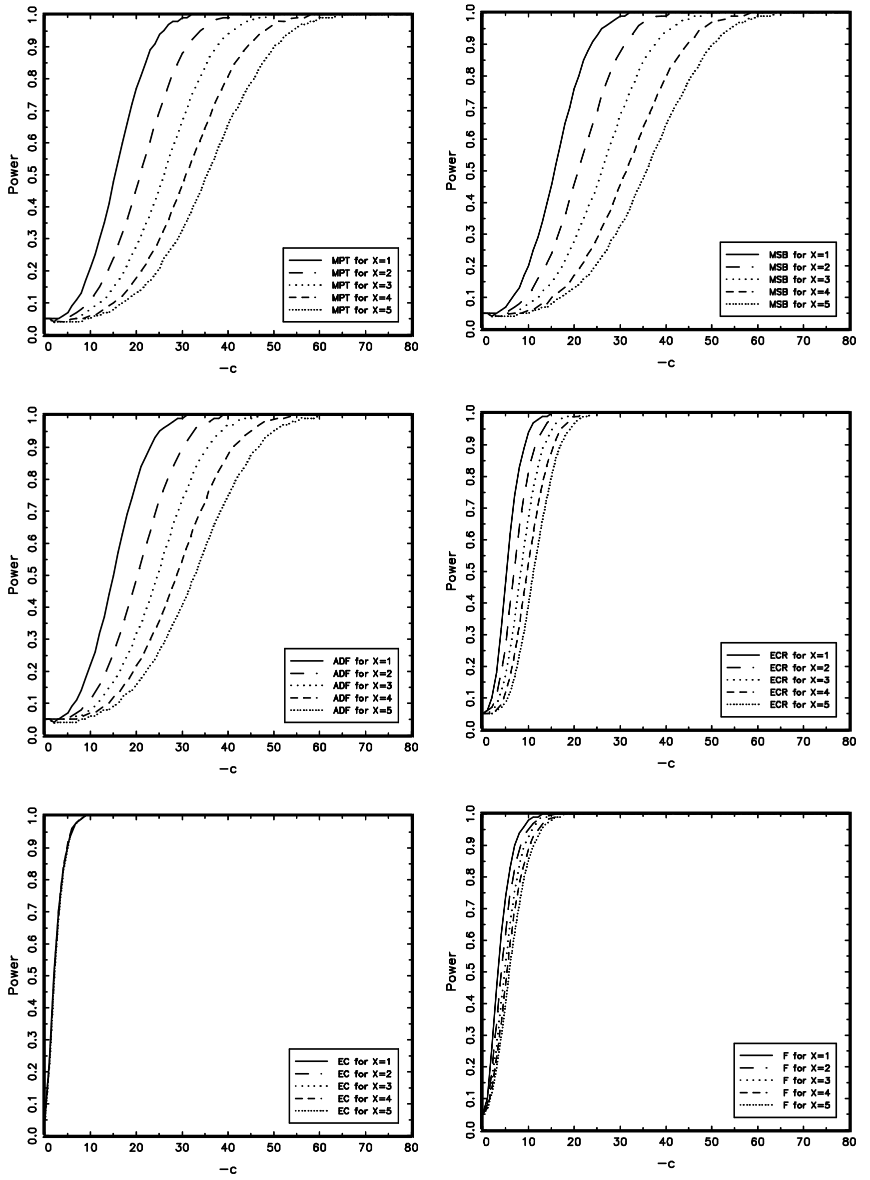

Figure 7d. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.8$ and different values of x. Demeaned Case.

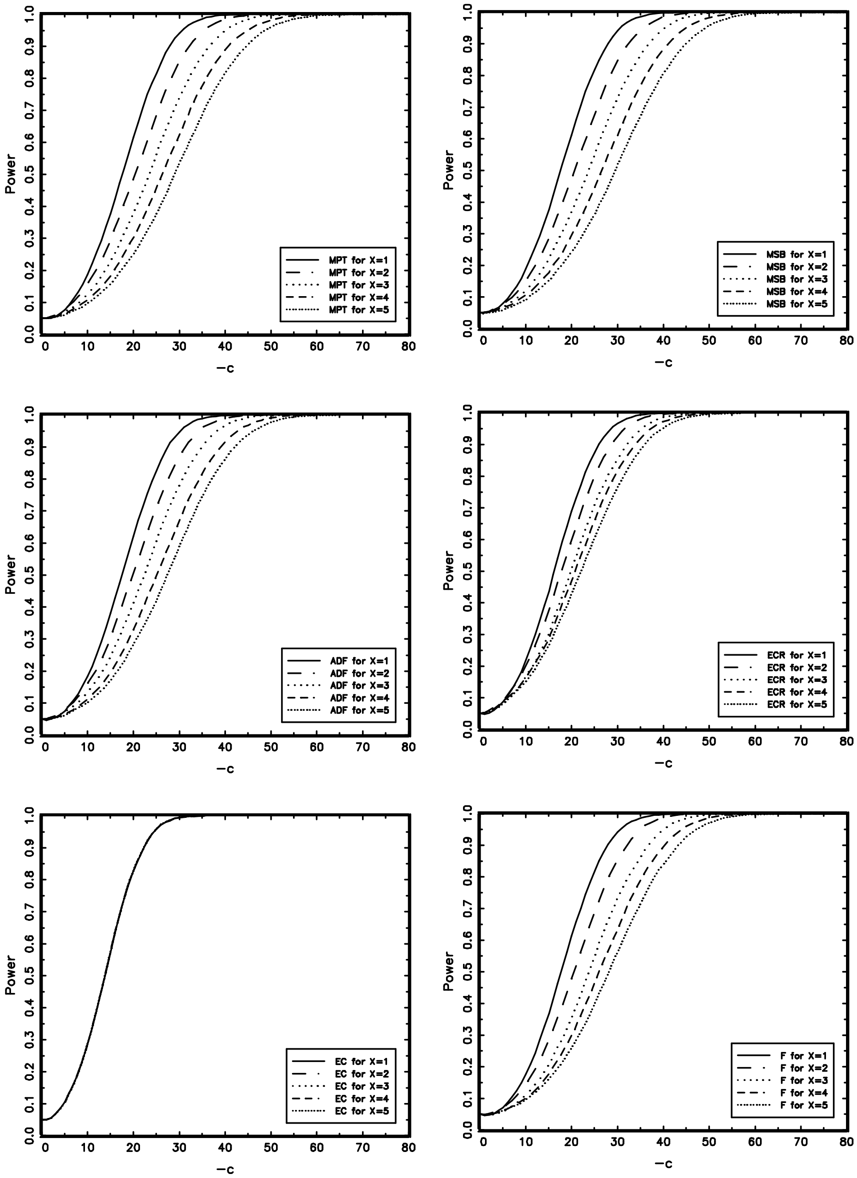

Figure 7e. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.0$ and different values of x. Detrended Case.

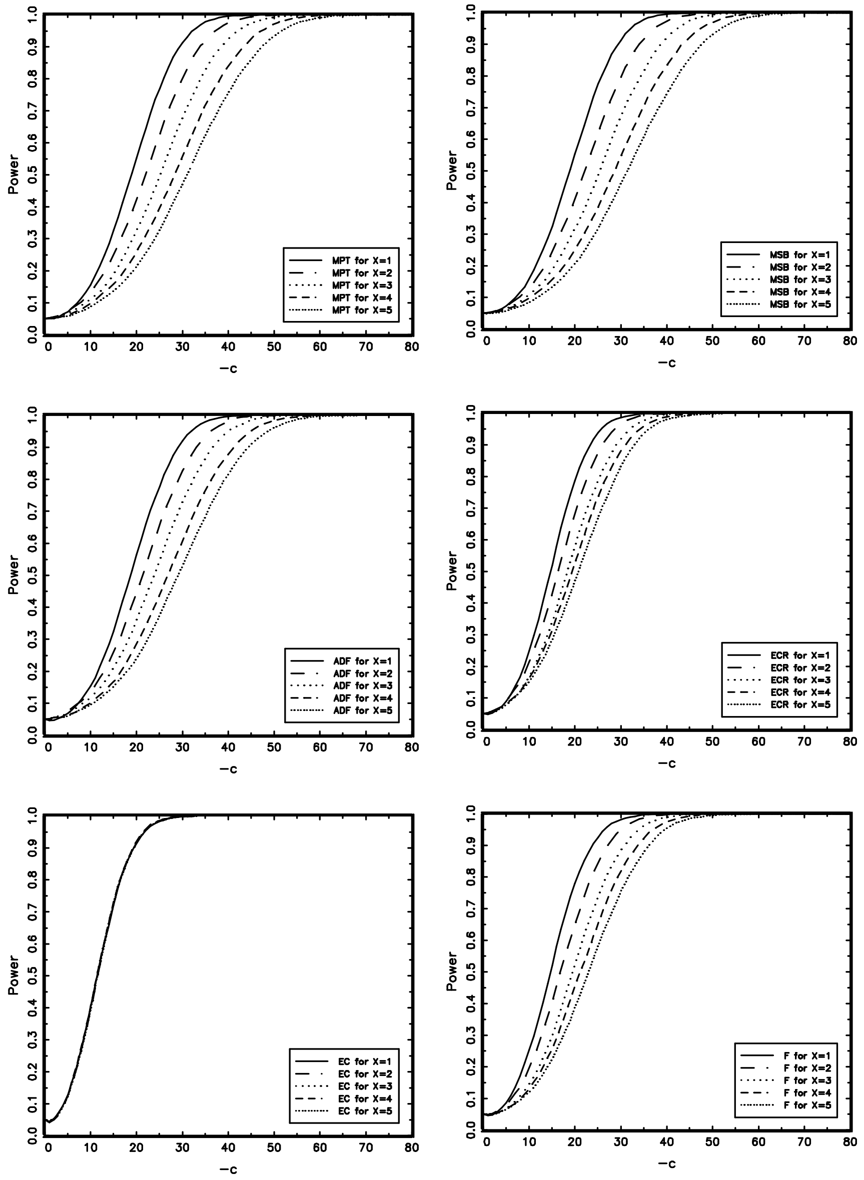

Figure 7f. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.2$ and different values of x. Detrended Case.

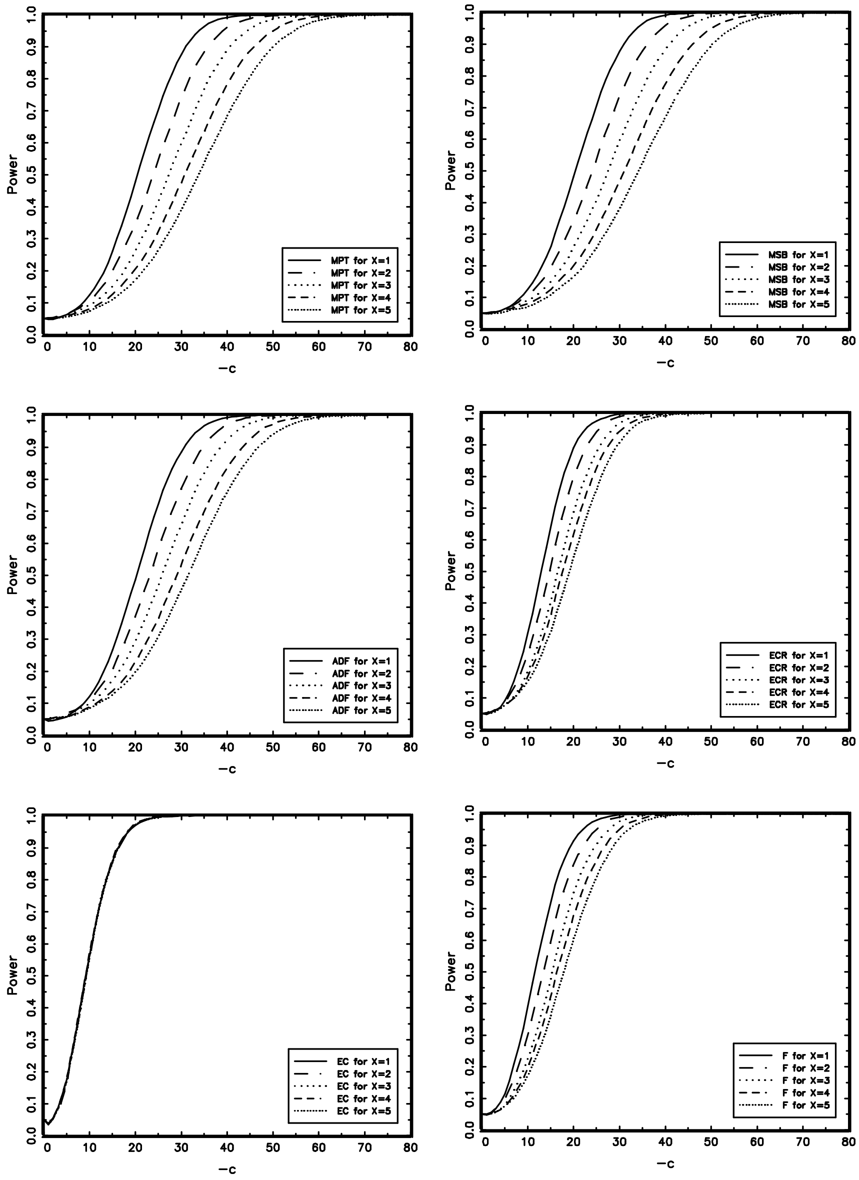

Figure 7g. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for R^2= 0.4 and different values of x. Detrended Case.

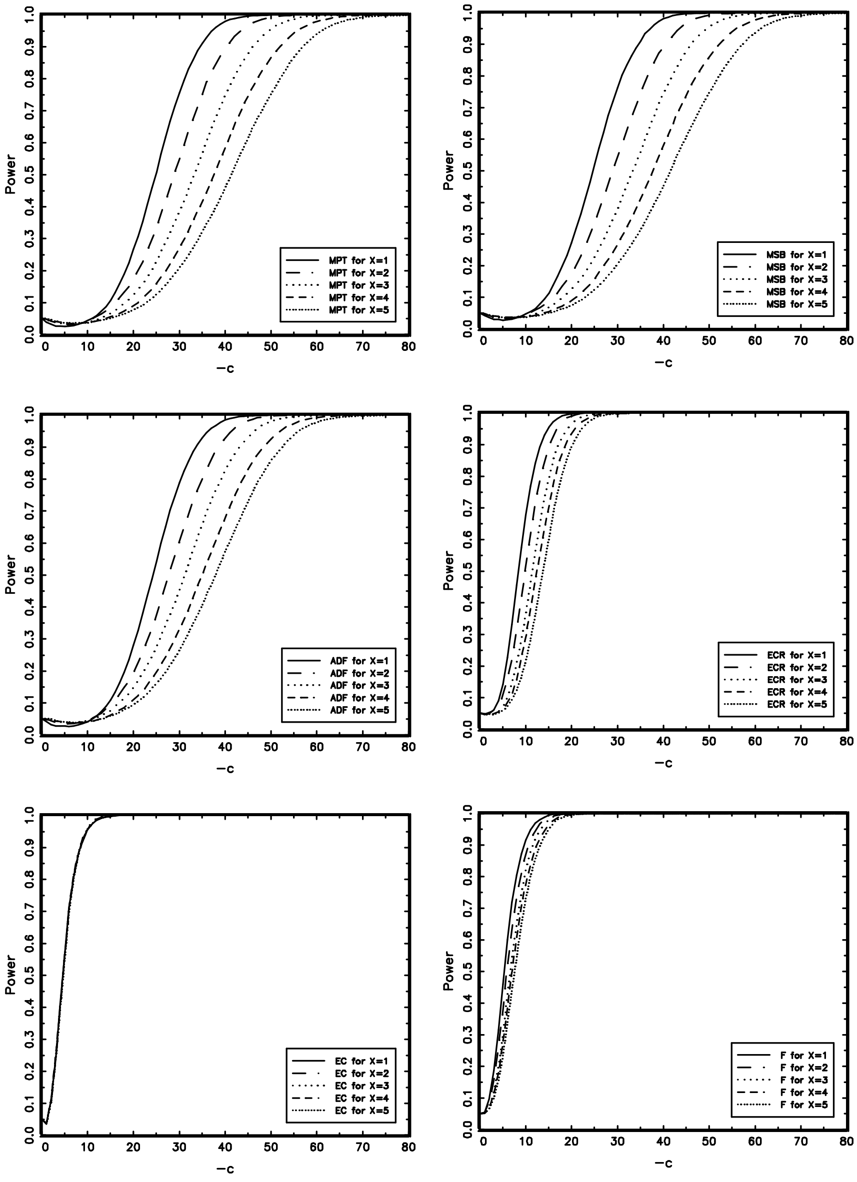

Figure 7h. Asymptotic Power Functions of MP_T^{GLS}, MSB^{GLS}, ADF^{GLS}, ECR^{GLS}, EC^{GLS} and F^{GLS} for $R^2 = 0.8$ and different values of x. Detrended Case.

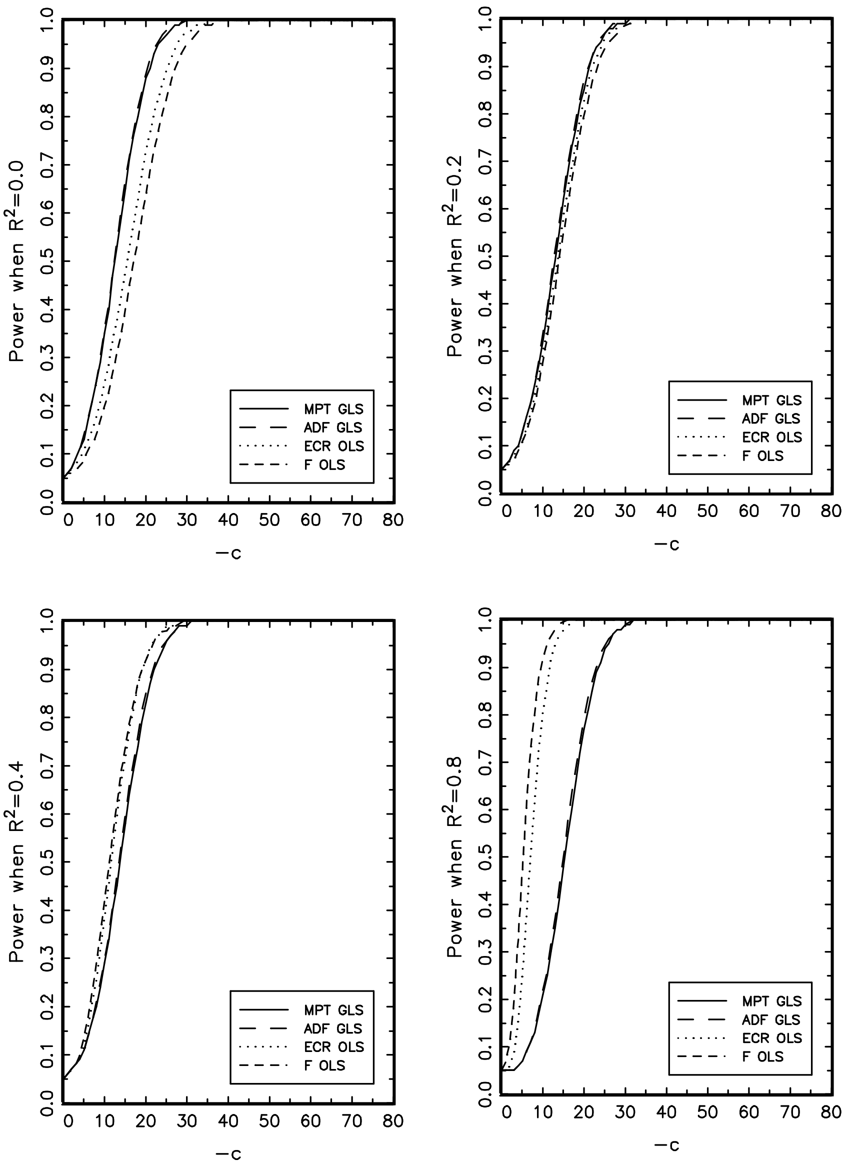

Figure 8a. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 1$ and different values of R^2. Demeaned Case.

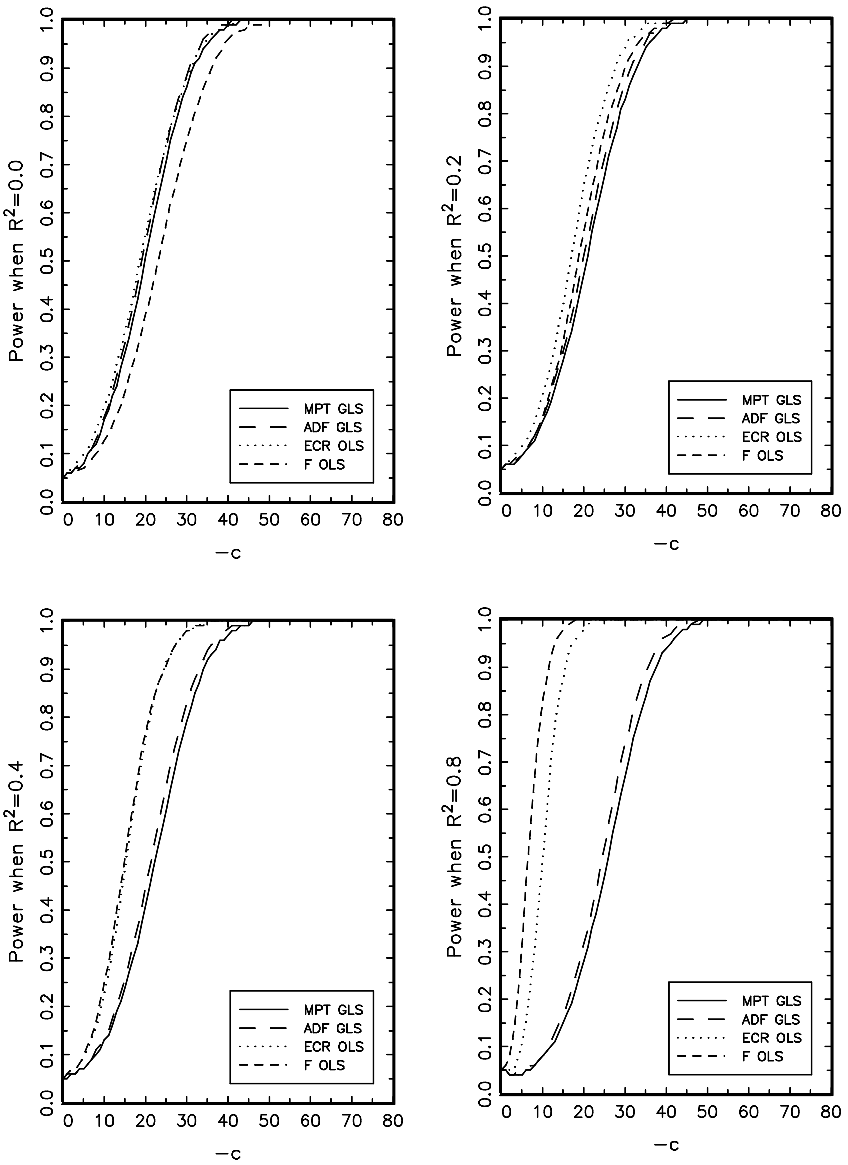

Figure 8b. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 3$ and different values of R^2. Demeaned Case.

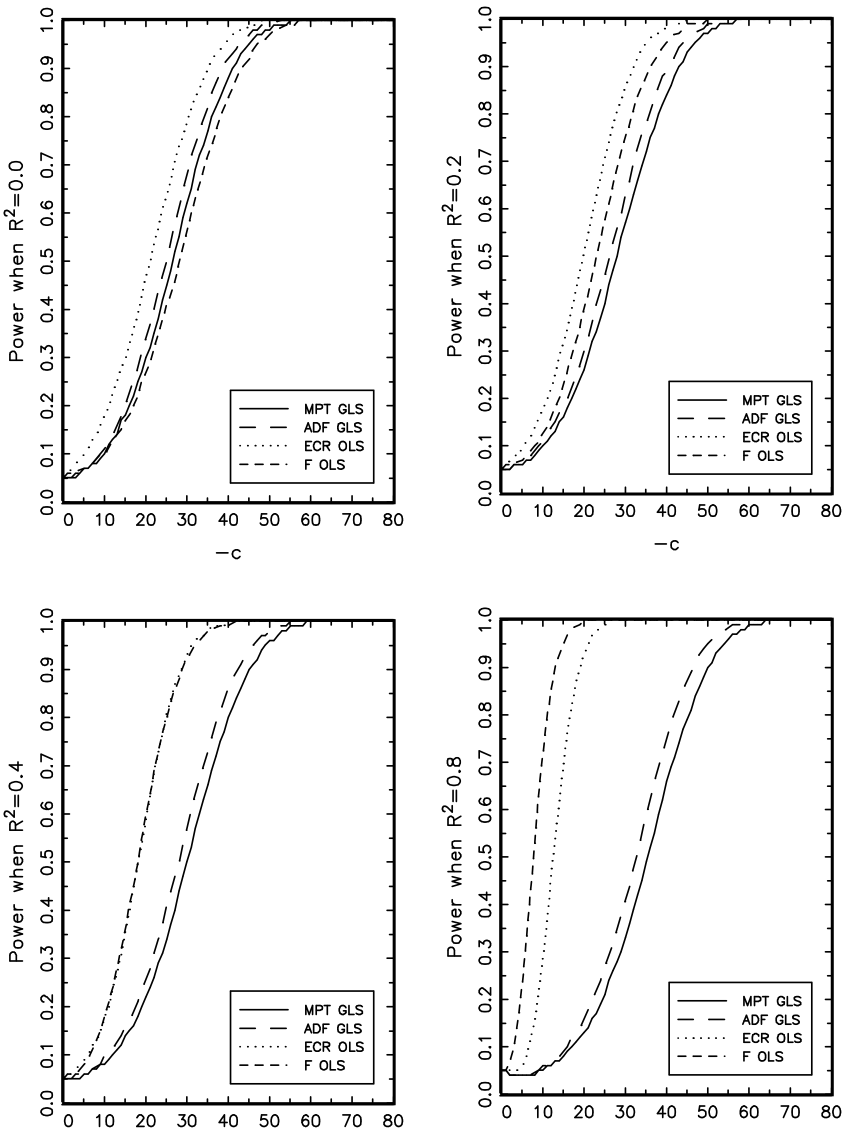

Figure 8c. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 5$ and different values of R^2. Demeaned Case.

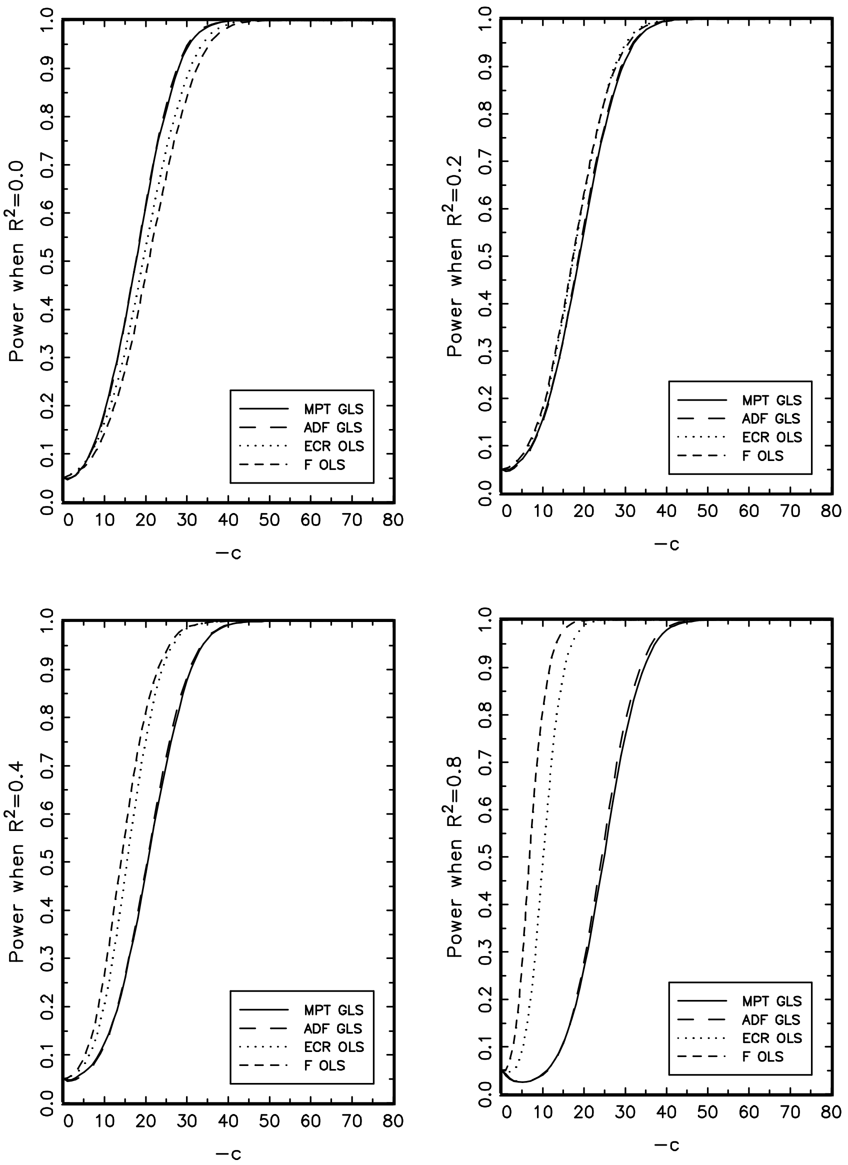

Figure 8d. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 1$ and different values of R^2. Detrended Case.

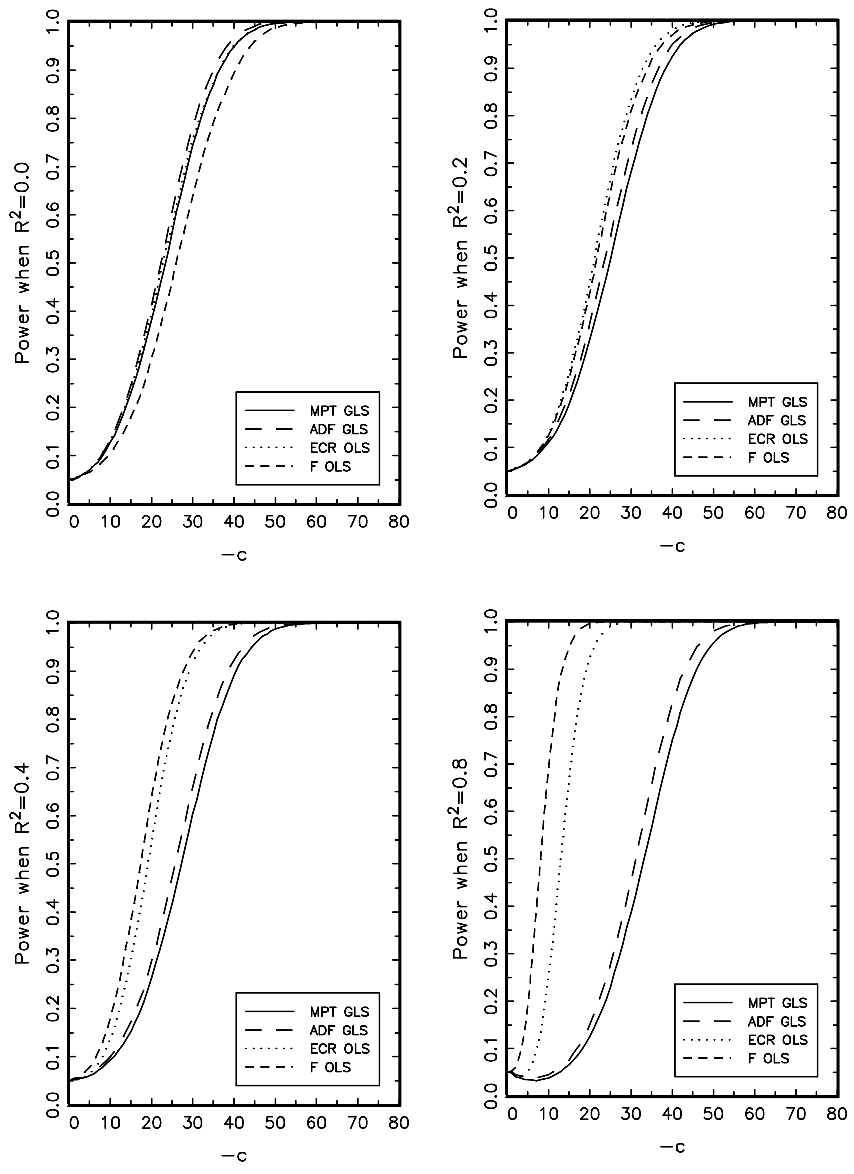

Figure 8e. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 3$ and different values of R^2. Detrended Case.

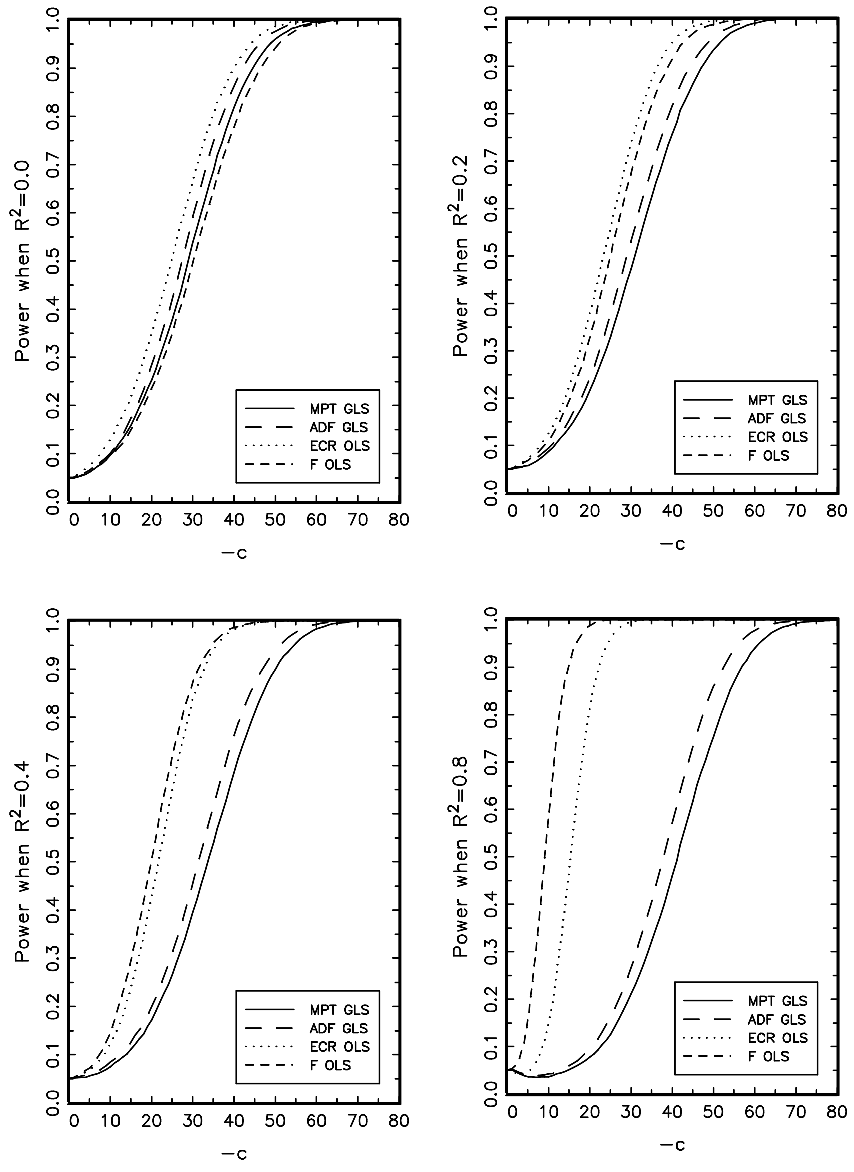

Figure 8f. Asymptotic Power Functions of MP_T^{GLS}, ADF^{GLS}, ECR^{OLS}, and F^{OLS} for $x = 5$ and different values of R^2. Detrended Case.

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Libros

Janina León Castillo y Javier M. Iguiñiz Echeverría (Eds.)
2011 *Desigualdad distributiva en el Perú: Dimensiones.* Lima, Fondo Editorial, Pontificia Universidad Católica del Perú.

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José Rodríguez y Mario Tello (Eds.)
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